

# Setting the Parameters of the LFT Shape Matching Algorithm

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## ABSTRACT

The LFT algorithm (Large Fat Tetrahedron) is used to detect congruent subsets amongst unordered point sets and forms the kernel of a partial shape matching method. Although the method yields several advantages and is relatively efficient, its performance depends highly on the choice of various geometrical threshold parameters, as *e.g.*, for the length difference of two edges. We present an overview of the key parameters of the algorithm and their influence on the computation, a guide line to provide an initial value for the parameters and we propose an approach to their automatic adjustment.

## Keywords

Scan view registration, partial shape matching, fat tetrahedron, threshold parameters

## 1. INTRODUCTION

The LFT algorithm (Large Fat Tetrahedron) was designed to detect approximately congruent tetrahedrons in two point sets [Vergeest 2010]. If such tetrahedrons are found, they might indicate the overlapping region of partially matching shapes. The assumption was that if a large fat tetrahedron with particular dimensions occurs in point set  $A$ , the probability that a congruent tetrahedron is found in point set  $B$  is small, unless both tetrahedrons reside in the overlap region of  $A$  and  $B$ . Thus, LFTs can serve as indicators of partial shape matching. The algorithm will be briefly described in Section 2.

One important application of partial shape matching is 3D scanning of physical objects. To construct a geometric model from a physical object, multiple scan views are taken, each consisting of range data, *i.e.* 3D points representing the object's surface. Since the orientation of the object relative to the scanning device is different for different scan views, the collection of points from all scan views do not as such represent the object's surface. First the points

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need to be aligned to each other, that is be transformed to a common coordinate system. The process of aligning the scan views is called scan view registration. From the aligned point sets the surface of the object can be reconstructed, either fully or partially, depending on the coverage of the scan views. If the surface can be fully reconstructed, it can be assumed to represent the boundary of a volume, or solid model. Then a solid model can be derived, which can serve as input to a CAD (Computer-Aided Design) system for further modeling and processing. Basing a design on an existing object or on reuse of precedent models is an important paradigm in some industries, such as industrial design engineering. Such a method can be successful only when even occasional users of scanning devices can easily operate the system. However, the registration task is, even nowadays, still an impeding factor. In practice the user could be a stylist who has manually created a clay model of a future household device. Whereas taking the scan views of the clay model is a commonsense task to him/her, the registration of view pairs is not. The scanning system's manufacturer normally offers an interactive software package, allowing the user to designate correspondences he/she observes amongst the scan views, as to provide a starting position for a shape matching algorithm, typically based on the ICP (Iterative Closest Point) method. The user has to align each scan view with the set of scan views aligned previously. Generally this way of operating

the scanner is perceived as slow and tedious, both by incidental users and by trained operators.

Several approaches have been reported to the problem of partial shape matching. From here on we assume that the input data consists of unordered point sets only. That is, we will not rely on preprocesses that generate surface meshes, nor on additional information such as color, texture or material properties of the scanned object. We focus on the kernel problem of matching two point sets. Most methods make use of geometric descriptors and/or feature points. The geometric descriptor can take many forms, including moments, FFT coefficients, spin images etc [Johnson 1999]. Defining a feature using integrated quantities rather than using derivatives reduces the influence of noise [Gelfand 2005]. Another approach to diminish sensitivity to noise and data outliers is taken by [Aiger 2008]. He collects sets of 4 planar points in each of the two point clouds. If a particular set from one point cloud is approximately congruent with one from the other point cloud, a candidate corresponding pair of 4-points sets is found. If the 4-points set is relatively wide, then the method is less sensitive to noise. We refer to [Gelfand 2005] and references therein for a more extended description of registration methods.

Our approach is inspired by the 4-points congruent sets as in [Aiger 2008]. We look for 4-points sets which define a large fat tetrahedron (LFT). The assumption is that the geometry of a large tetrahedron is relatively rare and therefore can serve to detect correspondences in the two point clouds. However, since true correspondence exists in the overlap region only, an upper bound must be set to the size of the tetrahedrons. Secondly, since the number of fat tetrahedrons in point sets can be very large, a straightforward comparison of two sets of tetrahedrons (each derived from one point cloud) would not be efficient. Our algorithm derives a limited number of fat tetrahedrons from one point cloud. Then each tetrahedron is tested for being approximately congruent to any point neighborhood in the other point set. In [Vergeest 2010] we have speculated about the advantages of the LFT algorithm compared to other strategies. However, lacking a benchmark platform we cannot demonstrate this. In the next section the LFT algorithm will be briefly described. In section 3 we present the influence of parameters on the computational performance of the method. Conclusions and recommendations are given in section 4.

## 2. THE LFT ALGORITHM

Let two point sets  $A$  and  $B$  be given, originating from sampling of a portion of the surface of a three-dimensional object. There may exist subsets  $A' \subseteq A$

and  $B' \subseteq B$  such that  $A'$  and  $B'$  are samples of the same subsurface of the object.  $A'$  and  $B'$  are then said to represent an overlap region of the samples.

Let a set of sets  $B_i$  be a partitioning of  $B$  defined as follows. A three-dimensional grid is constructed, aligned with a bounding box of  $B$ . The grid has the size of the bounding box of  $B$ . The block-shaped grid elements, or cells, all have the same size and have index  $i$ ,  $i = 1, \dots, G$ , where  $G$  is the number of cells of  $B$ . Each cell encloses zero or more points of  $B$ . Each point of  $B$  is enclosed by exactly one cell.  $B_i$  is the set of points enclosed by cell indexed  $i$ .

Let  $A'$  and  $B'$  be the largest overlap of  $A$  and  $B$ , informally defined as follows. Assuming that  $A$  and  $B$  are range images of a physical object, let  $S_A$  and  $S_B$  informally be defined as the portions of the surface of the physical object represented by  $A$  and  $B$ , respectively. Then, both  $A'$  and  $B'$  represent all or some part of the physical overlap surface  $S_A \cap S_B$ . Depending on the extent of  $S_A \cap S_B$ ,  $A'$  and  $B'$  each may contain zero up to as many points as the cardinality of  $A$  and  $B$ , respectively.

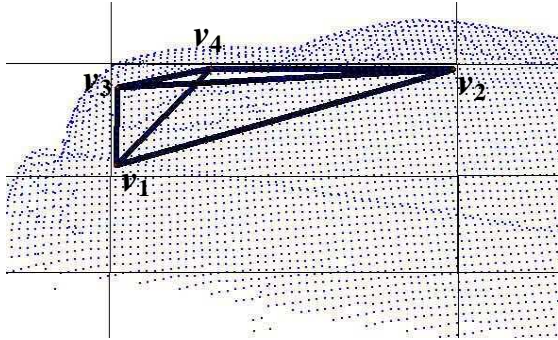
Let  $B'_i = B_i \cap B'$ , that is the portion of cell  $B_i$  coinciding with the overlap region. Our search strategy is based on the assumption that the overlap region is connected and has the extent of at least the size of a cell. In such cases there might exist sets  $B_i$  containing multiple points of  $B'_i$ . A property of any point of  $B'$  is that its Euclidian distance to  $A$  is relatively small, provided that  $A$  and  $B$  are defined in the same coordinate system. However, since  $A$  and  $B$  originate from independent sampling processes, they will in general be defined in different coordinate systems. The difference between the two coordinate systems can be described by a rigid body transformation  $M$ , such that  $MB$  and  $A$  are defined in the same coordinate system, where  $MB$  is the set of points of  $B$  to which transformation  $M$  has been applied. We name this transformation the *matching transformation*.

Since neither the overlap region, nor the matching transformation  $M$  are known, we determine which of the sets  $B_i$  is fully or partly contained in  $B'$ . We do so by constructing the largest and fattest tetrahedron in each cell and test each such LFT against congruency with 4-points sets of  $A$ .

### 2.1 Finding the LFT

When a small set of points of  $B_i$  is close to  $A$  and if these points are sufficiently non-planar, then the transformation to match this set with  $A$  is a relatively good candidate of the  $M$  we are looking for. Relying on this principle we base the algorithm on matching 4 points to  $A$ , where the 4 points are contained in the same cell. The 4 points, denoted  $v_1, v_2, v_3, v_4$ , are

selected from  $B_i$  such that they form an LFT as follows:  $v_1$  and  $v_2$  are the points in  $B_i$  which are furthest apart.  $v_3$  is the point in  $B_i$  furthest from the line through  $v_1$  and  $v_2$ , that is it maximizes  $|(v_3 - v_1) \times (v_2 - v_1)|$ .  $v_4$  is the point in  $B_i$  furthest from the plane defined by  $v_1, v_2$  and  $v_3$ , that is it maximizes  $|((v_3 - v_1) \times (v_2 - v_1)) \cdot (v_4 - v_1)|$ . An example of an LFT is shown in Figure 1.



**Figure 1. Point cloud  $B$ , its cell structure and the LFT contained in a cell. Data are from the Stanford Bunny [Stanford 2010].**

## 2.2 Calculating the transformation $M$

Once an LFT has been determined in a particular cell of point cloud  $B$ , we look for potential corresponding 4-points sets in  $A$ . As mentioned, when the LFT resides in the overlap region of shapes  $A$  and  $B$  then there should exist 4 points in  $A$  representing a tetrahedron with dimensions equal to those of the LFT, that is up to some precision since the point sets  $A$  and  $B$  are obtained as independent measurements. The deviation between “corresponding” points can be expected to be as large as half of the scan spacing practiced. We have applied two methods to detect (approximately) congruent 4-points sets in  $A$ . We look for points  $a_1, a_2, a_3$  and  $a_4$  in  $A$  such that there exists a transformation  $M$  with  $Mv_i \approx a_i$  for  $i=1, \dots, 4$ . The edge lengths of the LFT are denoted  $l_{ij} = |v_i - v_j|$ .

### Method 1

For each point in  $A$ , name it  $a_1$

Translate the LFT such that  $v_1 = a_1$

Search for points in  $A$  at distance  $l_{12}$  from  $a_1$  and name them  $a_2$

For each such  $a_2$

Search for points in  $A$  at distance  $l_{13}$  from  $v_1$  and at distance  $l_{23}$  from  $v_2$  and name them  $a_3$

For each such  $a_3$

Rotate the LFT about  $v_1$  such that  $v_2$  gets closest to  $a_2$

Rotate the LFT about axis  $(v_1, v_2)$  such that  $v_3$  gets closest to  $a_3$

If then  $v_4$  is close to any point in  $A$  (called  $a_4$ ) the accumulated transformations so far applied to the LFT represent a candidate  $M$

End of method 1

Alternatively we can explicitly test congruency by comparing the six edge lengths of the LFT to the corresponding distances between candidate points  $a_1, a_2, a_3, a_4$ . We define  $\delta_1$  as the threshold value for length comparisons.

### Method 2

For each point in  $A$ , name it  $a_1$

For each point in  $A$ , name it  $a_2$

If  $|\text{dist}(a_1, a_2) - l_{12}| < \delta_1$

For each point in  $A$ , name it  $a_3$

If  $|\text{dist}(a_1, a_3) - l_{13}| < \delta_1$  and

$|\text{dist}(a_2, a_3) - l_{23}| < \delta_1$

For each point in  $A$ , name it  $a_4$

If  $|\text{dist}(a_1, a_4) - l_{14}| < \delta_1$  and

$|\text{dist}(a_2, a_4) - l_{24}| < \delta_1$  and

$|\text{dist}(a_3, a_4) - l_{34}| < \delta_1$

Compute  $M$  from the  $v_i$  and  $a_i$ .

End of method 2

In method 2 the calculation of the transform  $M$  is postponed until the congruency is fully checked. One way to obtain  $M$  (as we implemented it) is to concatenate the translations and rotations in exactly the same way as done in method 1; see [Vergeest 2010] for the explicit equation. The two sets  $(v_1, v_2, v_3)$  and  $(a_1, a_2, a_3)$  are sufficient to determine  $M$ , but they do not lead to a set of linear equations with a unique solution since the congruency is approximate only. However, a solution based on minimizing  $\sum |Mv_i - a_i|^2$  would be feasible and accurate (not implemented).

## 2.3 Computing the degree of overlap

Typically thousands of candidate  $M$  transforms are found, depending on threshold  $\delta_1$ . If  $\delta_1$  is increased the number of candidates will rise steeply, as discussed later. We need to test whether or not a particular  $M$  is the matching transform. If an LFT  $L$  is contained in  $B'$  then the directed Hausdorff distance of  $ML$  to  $A$  will be (by definition) small if  $M$  is the matching transformation. It can be expected that then a significant portion of the points  $MB_i$  (from the current cell) will be close to  $A$  as well. Conversely, when many points  $MB_i$  appear close to  $A$  the probability that  $M$  is the matching transform is large.

In our algorithm, all points in cell  $B_i$  are subjected to  $M$  and their distance to  $A$  is determined. If a sufficient fraction  $f$  of the points are closer than  $\delta_2$  to  $A$  then  $M$  is saved as a candidate transformation. As a final step, each of the candidate transformations is used to compute the set  $MB$ , involving all points of  $B$ . The degree of matching of  $B$  to  $A$  is defined as  $N$ , the number of points in  $MB$  closer than  $\delta_3$  to  $A$ . The transformation producing the largest  $N$  is the outcome of the method.

### 3. PERFORMANCE AND PARAMETERS

As reported in [Vergeest 2010] the algorithm has been successfully applied to practical scan view registration. A typical CPU time of partial shape matching was 500s, which could be reduced to about 10s in a CUDA-GPU implementation [Kooijman 2009].

We have now studied the influence of the parameters  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$  and  $f$  on the computational performance of the algorithm for method 2. The granularity of the subdivision into  $G$  cells is also of influence to the algorithm. Not all cells produce an acceptable LFT. We have set a lower limit to the number of points from  $B$  contained in a cell; if the cell contains too few points we do not consider it. If a particular LFT is too small or too thin, it is discarded. Therefore, typically 10% of the cells produce an LFT for further processing. We focused on method 2 since its implementation is relatively simple and it will be compared to its CUDA implementation in the near future. An upper bound of the complexity of the algorithm is  $C \propto GP^6$ , where  $P$  is the number of points occurring in a scan view (we assumed that  $A$  and  $B$  are of comparable size). In the search process, for each cell, each point in  $A$  is visited at least twice in order to form the line segment  $a_1a_2$ . When the test against  $l_{12}$  is passed, another loop over all elements of  $A$  is made to find candidates  $a_3$  and finally one more loop to find  $a_4$  (the maximal cost is proportional to  $GP^4$  so far). If  $a_4$  is found, then all points in the cell are compared to  $A$  (cost proportional to  $P^2$ ) and possibly another check of all  $B$  against  $A$  is performed, as described in section 2.3. The main terms of the cost  $C$  are:

$$C \propto P^2 + \beta_1 P^3 + \beta_2 P^4 + (\beta_3 + \beta_4) P^6. \quad (1)$$

$\beta_1$  is the probability that the  $l_{12}$  test is passed,  $\beta_2$  is proportional to the probability that the  $l_{13}$  and  $l_{23}$  test are passed and  $\beta_3$  is proportional to the probability that the  $l_{14}$ ,  $l_{24}$  and  $l_{34}$  tests are passed.  $\beta_4$  depends on parameter  $f$  and the degree of overlap found of points in the current cell with  $A$ .

If we would set  $\delta_1=0$  then no LFT would practically pass the  $l_{12}$  test and terms 2 and 3 would vanish, or

$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ . If aforementioned fraction  $f$  and all  $\delta_i$  are large then  $C$  behaves like an  $6^{\text{th}}$  power function of  $P$  for large  $P$ ; the number of candidate  $M$  transforms would be very large and many of them have to be checked by Hausdorff twice, namely once involving  $MB_i$  and possibly once more involving  $MB$ .

To achieve efficient partial shape matching the algorithm should detect the matching  $M$  (therefore, the parameters should not be too small), without superfluous candidates (therefore, the parameters should not be too large).

To gain insight in the effects of the parameters we have performed numerical tests on one particular set of  $A$  and  $B$ , with cardinality 3188 and 2407, respectively. These sets are down-sampled versions of the Bunny data from the Stanford Scan Data Repository [Stanford 2010]. The data points are relatively evenly spaced, about 2mm apart, on a surface with a diameter of approximately 150mm.

Table 1 gives an impression of the computational expense of the algorithm for Method 2. The 10 runs differ only by the parameter  $\delta_1$ . The cell division was  $G = 5 \times 5 \times 5 = 125$  equally sized blocks of  $24 \times 31 \times 27$ mm. Out of these, 77 were empty and 7 cells contained an LFT that was sufficiently large and fat. Out of these 7, only one LFT appeared to be located in the overlap region and could produce the correct matching transform. This particular LFT had  $l_{12} = 40.6$ mm and a base triangle of height 20.2mm (that is the distance of point  $v_3$  to edge  $v_1v_2$ ) and thickness 3.1mm (distance of  $v_4$  to the base). The algorithm includes threshold parameters for thickness to accept LFTs and also for the minimum number of points contained in cells that are considered as carrier of an LFT. For the runs of Table 1, the limitation parameters were chosen 3.0mm for height and 96 for the number of points in a cell. The latter number was calculated as  $n_B / G^{2/3} = 2407/25 \cong 96$ , which reflects the fact that the data points represent a dimensionality 2 boundary rather than a volumetric content. The cell located in the overlap region contained 100 points. We have set fraction parameter  $f = 0.9$ . If 90% or more of the cell points after transformation got closer than  $\delta_2$  to any point of  $A$  then the LFT yielded "cell OK" in the table.  $\delta_2$  was set to 1.5mm. The threshold for computing the overlap explicitly was set to  $\delta_3 = 1.5$ mm. When the correct overlap (and thus the correct matching transform) was found, the associated LFT was classified as "all OK".

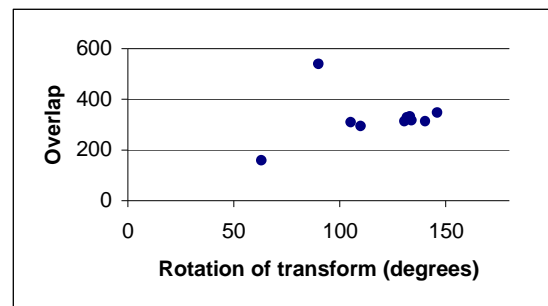
From the table we see that  $\delta_1 = 0.67$  is approximately the lowest threshold for which at least one of the 7 LFTs produced the correct matching transform  $M$ . The number of points from  $MB$  closer than  $\delta_3 = 1.5$ mm to  $A$  was 540 (22.4%), reflecting the size of the overlap region (we were able to judge the

$n_A = 3188, n_B = 2407, G = 125, \text{LFTs} = 7, \delta_2 = 1.5\text{mm}, \delta_3 = 1.5\text{mm}, f = 0.9$									
$\delta_1$ (mm)	$a_{12}$	$a_{12}$ OK	$a_{123}$	$a_{123}$ OK	$a_{1234}$	$a_{1234}$ OK	cell OK	all OK	CPU(s)
0.00	$7.1 \times 10^7$	0	0	0	0	0	0	0	2
0.15	$7.1 \times 10^7$	$2.0 \times 10^5$	$6.3 \times 10^8$	$6.5 \times 10^3$	$2.0 \times 10^7$	10	0	0	18
0.30	$7.1 \times 10^7$	$4.0 \times 10^5$	$1.3 \times 10^9$	$5.1 \times 10^4$	$1.7 \times 10^8$	438	0	0	40
0.45	$7.1 \times 10^7$	$5.9 \times 10^5$	$1.9 \times 10^9$	$1.8 \times 10^5$	$5.6 \times 10^8$	4345	2	0	95
0.60	$7.1 \times 10^7$	$7.9 \times 10^5$	$2.5 \times 10^9$	$4.2 \times 10^5$	$1.3 \times 10^9$	22,900	4	0	330
0.66	$7.1 \times 10^7$	$8.7 \times 10^5$	$2.8 \times 10^9$	$5.6 \times 10^5$	$1.8 \times 10^9$	36,638	6	0	410
0.67	$7.1 \times 10^7$	$8.9 \times 10^5$	$2.8 \times 10^9$	$6.9 \times 10^5$	$1.9 \times 10^9$	45,274	10	1	460
0.75	$7.1 \times 10^7$	$9.9 \times 10^5$	$3.2 \times 10^9$	$8.2 \times 10^5$	$2.6 \times 10^9$	83,868	15	2	780
1.05	$7.1 \times 10^7$	$1.4 \times 10^6$	$4.4 \times 10^9$	$2.3 \times 10^6$	$7.2 \times 10^9$	$5.8 \times 10^5$	64	4	4800
1.50	$7.1 \times 10^7$	$2.0 \times 10^6$	$6.3 \times 10^9$	$6.6 \times 10^6$	$2.1 \times 10^{10}$	$4.5 \times 10^6$	371	26	36,700

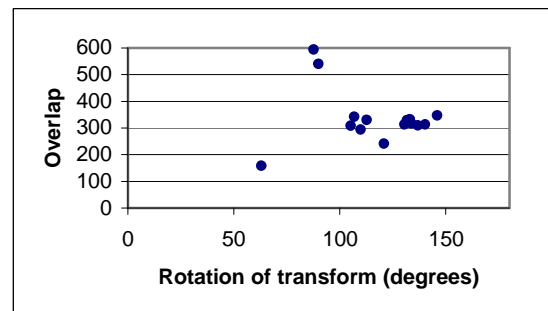
**Table 1. Complexity terms of length tests by Method 2 as function of threshold  $\delta_1$ . Data are from the Stanford Bunny.**

correctness of the transformation of these particular scan views based on ground knowledge from alternative shape matching methods). Out of the 100 points in the particular cell containing the “correct” LFT, 97 were at close distance to  $A$  (within threshold  $\delta_2 = 1.5\text{mm}$ ).

Obviously, the 4 vertices  $v_1 \dots v_4$  of the LFT itself belong to that set of 97 points, since they are close to the points  $a_1 \dots a_4$ . For the same LFT one different 4-points set in  $A$  was found leading to a transformation passing “Cell OK”. With that transformation 91 points of the cell were close to  $A$  but so were only 159 points of  $B$ , making it very unlikely that the transformation was the matching transform. Another LFT generated 8 transforms passing “Cell OK” but turned out not the matching transform. The total of 10 cases of “Cell OK” is depicted in Figure 2. The number of overlap points in  $B$  is plotted against the rotation exerted by the transform. The latter quantity was chosen as it characterizes the transform, although there is, of course, not a one-to-one relationship between the angle and the transformation matrix. The orientation of scan view  $B$  relative to  $A$  is 89.9 degrees, according to the solution found at  $\delta_1 = 0.67$ . The number of  $l_{12}$  tests is, independently of  $\delta_1$ , equal to  $n_A^2$  times the number of LFTs considered. For the runs of Table 1 this number, labeled  $a_{12}$ , was  $3188^2 \times 7 = 7.1 \times 10^7$ . For  $\delta_1 = 0.67$ , 1.3% of the  $l_{12}$  comparisons passed the test, labeled “ $a_{12}$  OK”. This percentage is collective over all accepted LFTs in the run. The number of ( $l_{13}, l_{23}$ ) tests is  $n_A$  times the “ $a_{12}$  OK” cases, or  $2.8 \times 10^9$ . This number depends on the third power of  $n_A$ . Further,  $\beta_1$  in equation (1) is dependent on  $\delta_1$ .



**Figure 2. Number of points in  $MB$  close to  $A$  versus rotation of  $M$  for  $\delta_1 = 0.67$  and  $f=0.9$ . The plot contains 10 data points.**

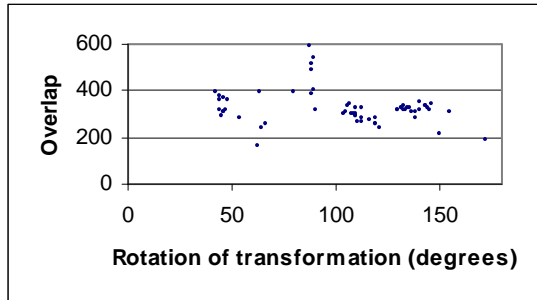


**Figure 3. Number of points in  $MB$  close to  $A$  versus rotation of  $M$  for  $\delta_1 = 0.75$  and  $f=0.9$ . The plot contains 15 data points.**

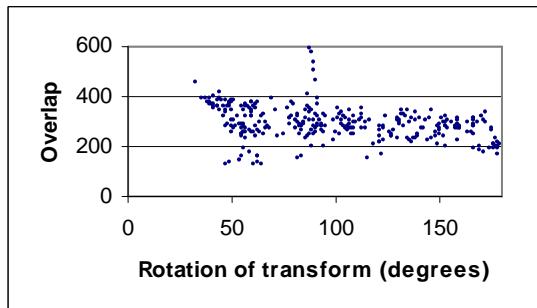
For the particular runs in Table 1 it turned out that  $\beta_1 = 0$  for  $\delta_1 \leq 0.66$  and  $\beta_1$  is rising for increasing  $\delta_1$ . For  $\delta_1 = 0.67$ , 0.02% of the  $a_{123}$  tests was positive. This percentage is proportional to  $\beta_2$  and depends on  $\delta_1$ . From the  $1.9 \times 10^9$   $a_{1234}$  tests, 0.002% or 45,274 resulted positively. This fraction affects  $\beta_3$ . Finally, the number of exhaustive tests of distance  $MB$  to  $A$  depends on the fraction of “cell OK” (0.02% in the run for  $\delta_1 = 0.67$ ). This fraction is proportional to  $\beta_4$ .

The increase of computation time with increasing  $\delta_1$  is obvious from the rightmost column of Table 1. The choice of  $\delta_1$  seems most critical, whereas  $\delta_2$  and  $f$  affect the number of distance evaluations, which will be increasingly critical for large  $n_A$  and  $n_B$ .

The effect of changing  $f$  from 0.9 to 0.8 is shown in Figure 5, which should be compared to Figure 3. The number of candidates passing the  $f$  threshold rises from 15 to 308. Instead of having only two correct transformations for  $f=0.9$ , there are 4 of them for  $f=0.8$ . They show up as a narrow peak in Figure 5 near  $90^\circ$ .



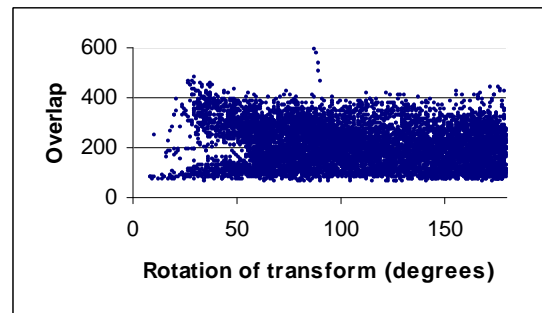
**Figure 4.** Number of points in  $MB$  close to  $A$  versus rotation of  $M$  for  $\delta_1 = 1.05$  and  $f = 0.9$ . The plot contains 64 data points.



**Figure 5.** Number of points in  $MB$  close to  $A$  versus rotation of  $M$  for  $\delta_1 = 0.75$  and  $f = 0.8$ . The plot contains 308 data points.

Lowering  $f$  to 0.5 does not lead to more correct transforms, but increases the background of incorrect candidates (Figure 6). In Table 2 the number of candidates and the performance of the algorithm, for the different values of  $f$ , are presented. It should be noted that coefficient  $\beta_4$  increases with decreasing  $f$ , leading to  $6^{\text{th}}$  power behavior of the complexity, see equation (1). The increase of CPU time in Table 2 can be practically completely attributed to the number of full distance computations. When the number of tested cells increases from 308 to 8695 (factor 28.2), the CPU time for  $B$ -to- $A$  distance computation increases by factor  $(2114-780) / (829-780) = 27.2$ . In all cases the number of cell-to- $A$  distance computations is

83,868, which would increase with the  $6^{\text{th}}$  power of number points as well. However, the  $\beta_3$  coefficient is small when  $\delta_1$  is moderate and the number of points in a cell is small.



**Figure 6.** Number of points in  $MB$  close to  $A$  versus rotation of  $M$  for  $\delta_1 = 0.75$  and  $f = 0.5$ . The plot contains 8695 data points.

$n_A = 3188, n_B = 2407, G = 125, \text{LFTs} = 7,$ $\delta_1 = 0.75\text{mm}, \delta_2 = 1.5\text{mm}, \delta_3 = 1.5\text{mm}$				
$f$	$a_{1234}$ OK	cell OK	all OK	CPU(s)
0.9	83,868	15	2	780
0.8	83,868	308	4	829
0.5	83,868	8695	4	2114

**Table 2.** Dependence on  $f$  of the performance of Method 2.

We observed that increasing  $\delta_1$  and/or decreasing  $f$  puts a dramatic load on the computation. Further, having approximately 1.6 times more unordered points in the sets, rises the CPU times by a factor 20.8, see Table 3 for details. For a pure  $6^{\text{th}}$  order behavior one would expect a factor 16.8, but there are other factors such as the number of accepted LFTs, which changed from 7 to 9.

#### 4. CONCLUSIONS

The two most critical factors influencing the performance of the algorithm are the point set sizes and  $\delta_1$ . In the test runs we have used down-sampled versions of point sets containing originally 40,200 and 30,400 points. The down-sampling algorithm removed points conservatively from the set such that no two points were less than  $\varepsilon$  length units apart, where  $\varepsilon$  was set to 2mm for the runs in Tables 1 and 2, and  $\varepsilon = 1.5\text{mm}$  for the runs in Table 3. For given  $\varepsilon$  an upper limit of  $\delta_1$  would be  $0.5\varepsilon$  in a worst-case one-dimensional setting. Using a small value for  $\delta_1$  could lead to zero LFT matches.



$n_A = 3188, n_B = 2407, G = 125, \delta_1 = 0.75\text{mm}, \delta_2 = 1.5\text{mm}, \delta_3 = 1.5\text{mm}, f = 0.9$										
$n_A, n_B, \text{LFTs}$	$a_{12}$	$a_{12} \text{ OK}$	$a_{123}$	$a_{123} \text{ OK}$	$a_{1234}$	$a_{1234} \text{ OK}$	cell OK	all OK	CPU(s)	
3188, 2407, 7	$7.1 \times 10^7$	$9.9 \times 10^5$	$3.2 \times 10^9$	$8.2 \times 10^5$	$2.6 \times 10^9$	83,868	15	2	780	
5140, 4127, 9	$2.4 \times 10^8$	$3.1 \times 10^6$	$1.6 \times 10^{10}$	$4.0 \times 10^6$	$2.1 \times 10^{10}$	737,044	703	40	16,240	

**Table 3. Effect of increasing the density of the point sets by approximately a factor 1.6 for Method 2.**

Empirically we have found  $\delta_1=0.66\text{mm}=0.33\epsilon$  as an upper limit in the particular setting of the runs we performed. This could be considered as a rule of thumb for  $\delta_1$ , although it presumes evenly spaced points. The choice of  $G$  has turned out to be critical as well. When we selected  $G=6 \times 6 \times 6=216$ , none of the LFTs yielded a correct transform, unless we increased  $\delta_1$  from 0.67 to 1.05mm. Indeed, refining the cell subdivision could exceptionally imply that fewer cells are fully included in the overlap region. When we lowered  $G$  to  $4 \times 4 \times 4=64$ , none of the LFTs yielded a correct transform, not even at  $\delta_1 = 1.05$ . This could have been expected since the LFTs all exceed the size of the overlap region and are therefore unlikely to fit “correctly” to  $A$  at any place.

The subdivision has been implemented on an arbitrarily orientated evenly spaced grid, namely a grid aligned with the global coordinate system. This subdivision method could be improved significantly to reduce the number cells that should be considered further. If we assume that the overlap region contains at least 20% of the points of  $B$  then the  $5 \times 5 \times 5$  subdivision seems appropriate.

The values of  $\delta_3$  and  $\delta_4$  are less critical, *provided* that  $\delta_1$  is not unnecessarily large. The values we supplied (1.5mm or  $0.75\epsilon$ ) seem reasonable.  $f=0.9$  turned also a good starting value; the correct transforms yielded about 95% overlap of the cell with  $A$ , and we observed that even with increased  $\delta_1$ , it was not useful to set  $f$  lower than 0.9.

As mentioned, these recommendations for initial parameter values are still case-specific. When the scanning process would result in very unevenly distributed points, the parameters should be derived from the highest occurring  $\epsilon$ .

A possible strategy for automatically adjusting the parameters is to set  $\epsilon$  relatively large (e.g. 0.01 times the diagonal of the bounding box of  $A$ ) and perform a run with  $\delta_1 = 0.3\epsilon$ . Then the largest overlap detected can be tested for compactness and connectedness. If the distribution of the overlapping points is not consistent with a connected portion of  $A$  and/or  $B$ , runs with increased  $\delta_1$  can be carried out.

Both the length tests and the distance computations can be implemented with a good degree of parallelization. Unlike purely two-dimensional processes such as image restoration, 3D scan view data cannot be subdivided in portions which can be processed completely independently. Still an acceleration of the computation by a factor of 10 to 100 appears feasible in Cuda implementations that are presently under investigation.

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