Particle-based T-Spline Level Set Evolution for 3D Object Reconstruction with Range and Volume Constraints

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Introduction

3D object reconstruction with range and volume constraints

- Implicitly defined surface
- Evolution
- Point set surface
- Range and volume constraints

Introduction - Properties

Implicitly defined surfaces

- Adapt of topology
- Compute normals
- Range constraints

Evolution

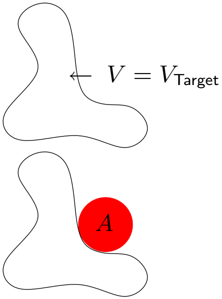
- Linearization of volume constraints

Point set surfaces

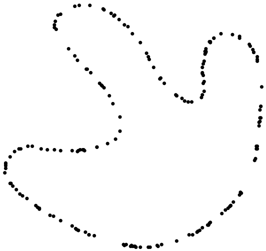
- Sample points for discretization

Introduction - Related work (non-exhausting)

- Volume constraints (Funck et al.'06,'07)
- Range constraints (Flöry et al.'07)
- Point set surfaces (Alexa et al.'07)
- Implicitly defined surfaces (Osher et al.'02)



Given: An unorganized point cloud



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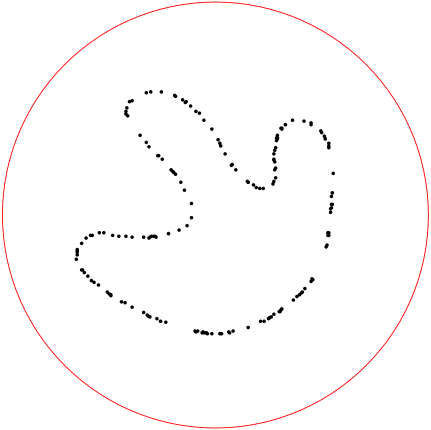

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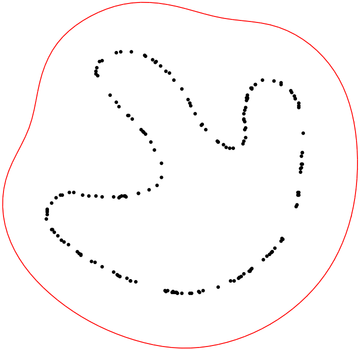

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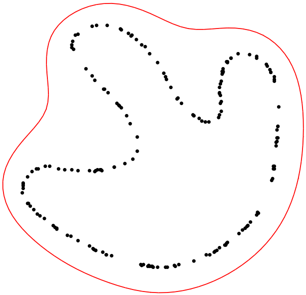

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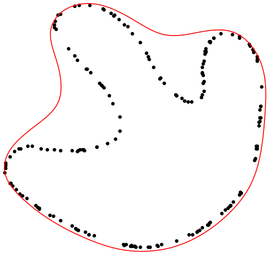

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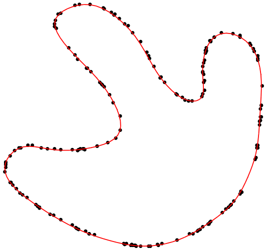
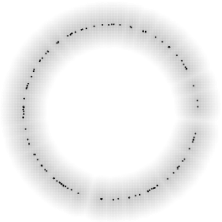


Speed function

$$v(\mathbf{x}) = v(\kappa, \vec{\mathbf{n}}, d)$$

- $\mathbf{x}$ point on the surface.
- κ and $\vec{\mathbf{n}}$ are geometric properties of the surface at $\mathbf{x}$.
- d is the unsigned distance function.



T-spline

T-spline function (introduced by Sederberg et al.'03)

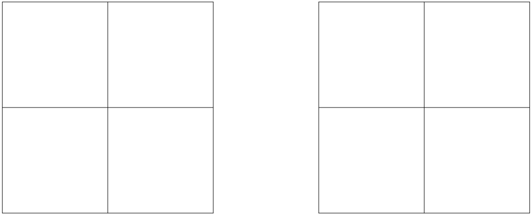
$$f(\mathbf{x},\tau) = \sum_{i=1}^{n} T_i(\mathbf{x})\, c_i(\tau) \quad \mathbf{x} \in \Omega \subset \mathbb{R}^3$$

where τ is a time-variable.

Zero level-set

$$\Gamma(f,\tau) = \{\mathbf{x} \in \Omega | f(\mathbf{x},\tau) = 0\}$$

Grids of a tensor product B-spline and a T-spline:



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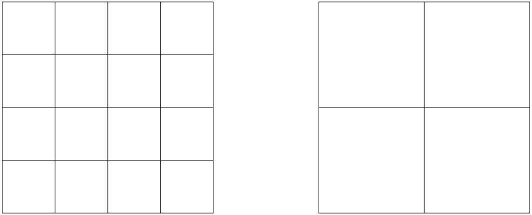

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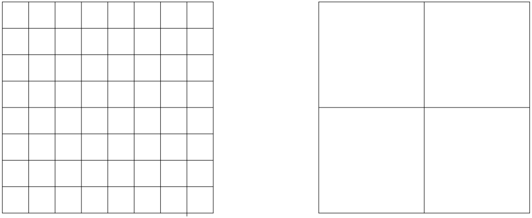

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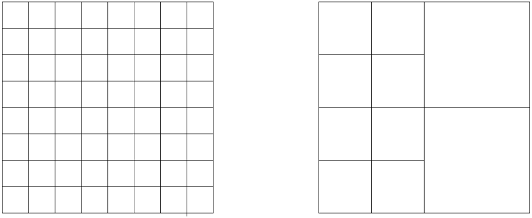

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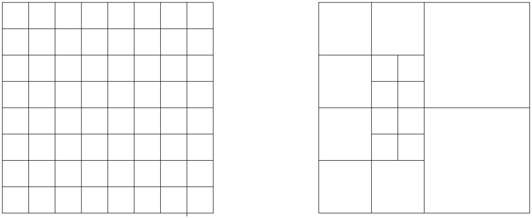

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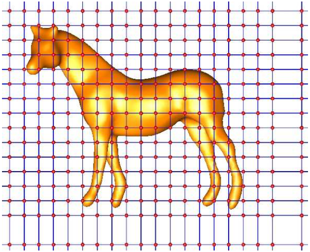
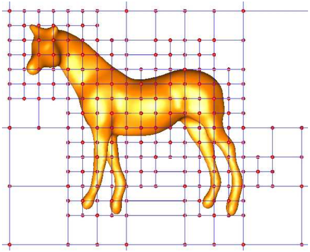


5148 B-spline control points



1484 T-spline control points

Evolution

Least-squares problem

$$\int_{\mathbf{x}\in\Gamma} \Bigg(\underbrace{\begin{array}{l}\text{actual normal}\\ \text{velocity at } \mathbf{x}\end{array}}_{*} - \begin{array}{l}\text{value of speed}\\ \text{function at } \mathbf{x}\end{array} \Bigg)^2 dA \to \min$$

$$* = -\frac{\dot{f}}{|\nabla f|} = \sum_{i=1}^{n} (\ldots)\dot{c}_i$$

Discretization

$$\sum_{j=1}^{N} \Bigg(\begin{array}{l}\text{actual normal}\\ \text{velocity at } \mathbf{x}_j\end{array} - \begin{array}{l}\text{value of speed}\\ \text{function at } \mathbf{x}_j\end{array} \Bigg)^2 \to \min$$

Particle $\mathbf{x}_j \in \Gamma$.

Algorithm

- Initialization (initial T-spline zero-level set and particles, pre-computation of the unsigned distance field function).
- Evolution of the implicitly defined surface (one time step).
- Projection of the particles.
- Local resampling of the particles if necessary.
- Final refinement.

Particles: Requirements and goals

- We need sample points along the zero level-set.
- Sample should be dense enough (close to uniform).
- Sample points should be fast to compute.
- Local resampling.
- Correct topology.

Particle sampling

Marching triangulation (Hartmann '98).

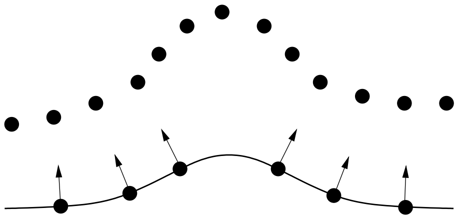
- Needs information on normals (provided by the implicit surface)
- Can be done locally or globally
- Distance between neighbouring sample points in the initial particle set $\approx \rho$

ρ is a user defined constant called "feature-size".

Criteria for resampling

Distance between neighbouring sample points

$$|\mathbf{x}_i - \mathbf{x}_j| > 2\,\rho$$



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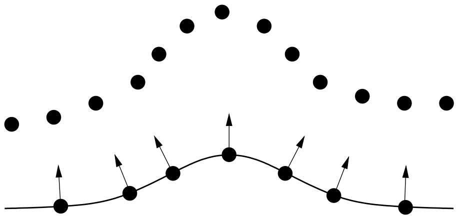
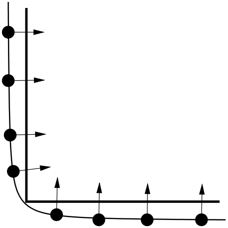


Criteria for resampling

Normals of neighbouring sample points

$$\mathbf{n}_i \cdot \mathbf{n}_j < \epsilon$$

$\mathbf{n}_i$ is the normal of f at $\mathbf{x}_i$



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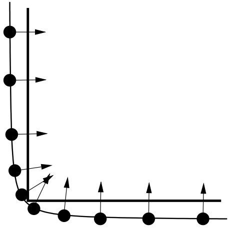
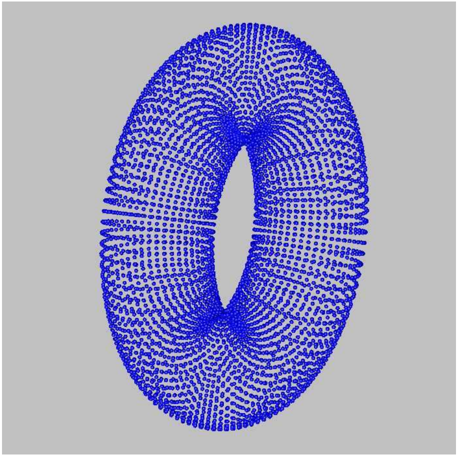
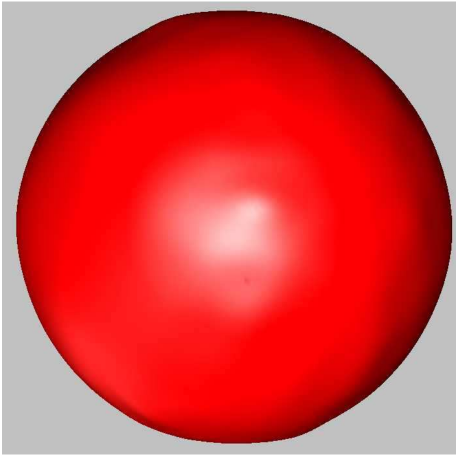


Example 1



4896 target points

Example 1



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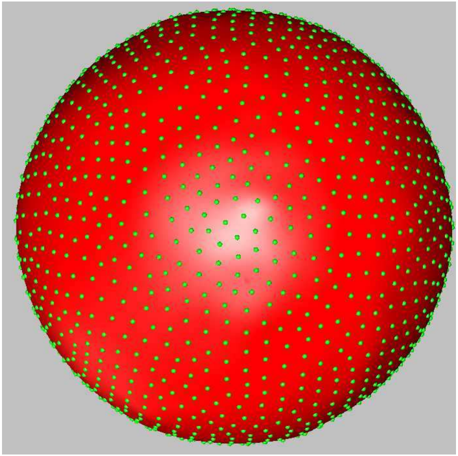
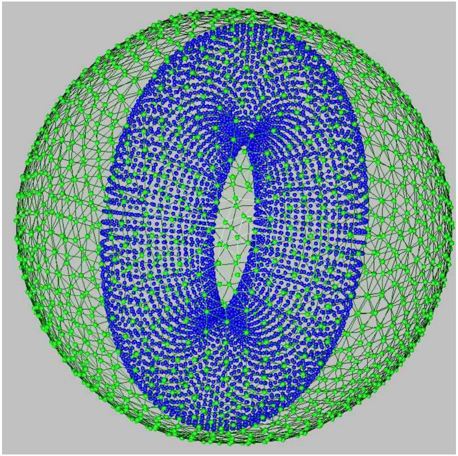


1544 particles

Example 1



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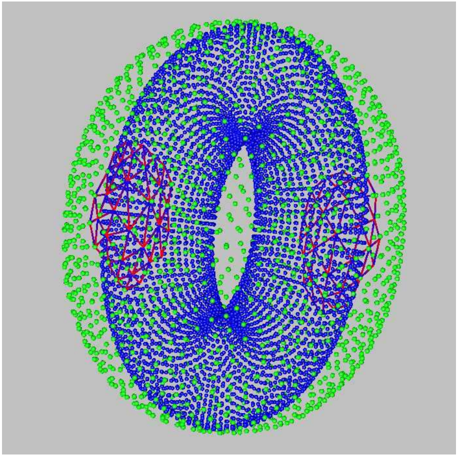

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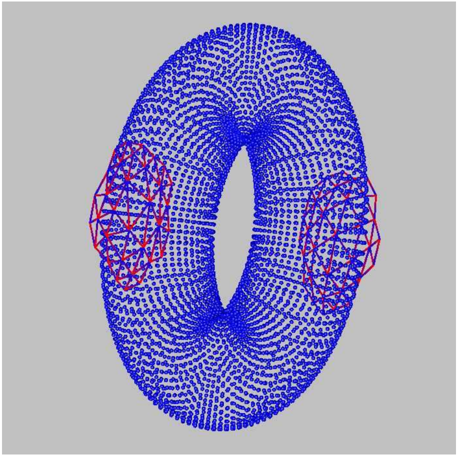

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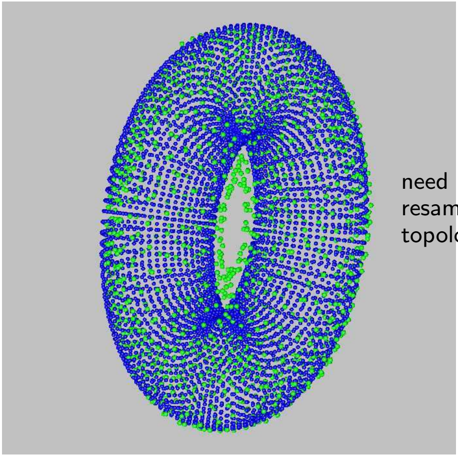
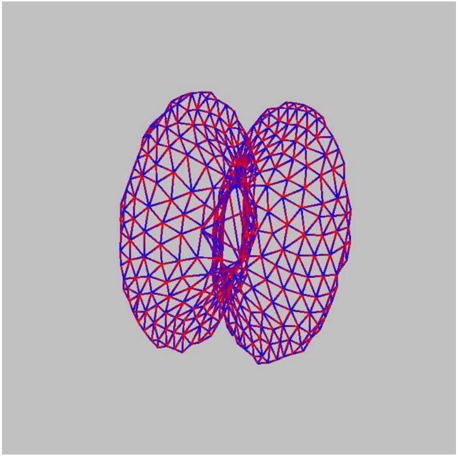


need local resampling for topological change

Example 1



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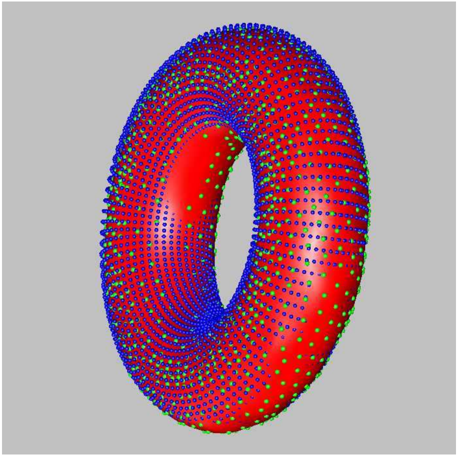
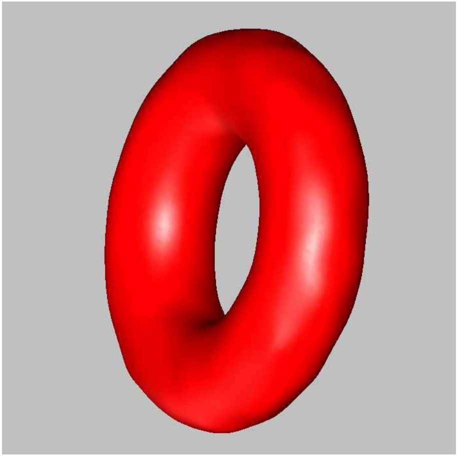
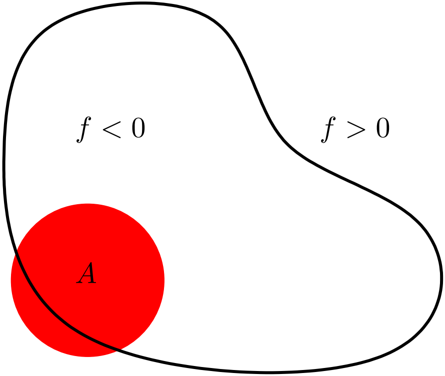


1289 particles

Example 1



- T-spline ensures that $f(\mathbf{x}, \tau) \leq 0$ for $\mathbf{x}$ inside $\Gamma(f, \tau)$.
- Define a constraint that forces a region A to lie inside or outside of the zero level-set.
- $f(\mathbf{x}_i) \leq 0 \quad \mathbf{x}_i \in A$

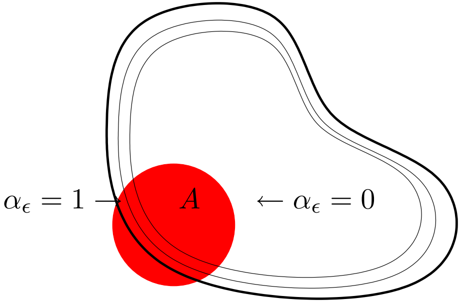

$f < 0$
$f > 0$
A

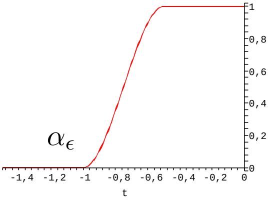
Least-squares problem

$$\sum_{j=1}^{N_0} \left(\begin{array}{l} \text{actual normal} \\ \text{velocity at } \mathbf{x}_j \end{array} - \begin{array}{l} \text{actual value of the} \\ \text{T-spline function at } \mathbf{x}_j \end{array} \right)^2 \alpha_\epsilon(f(\mathbf{x}_j, \tau)) \to \min$$

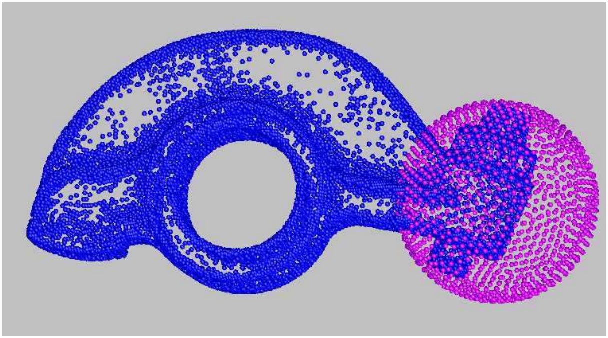
Sample points $\mathbf{x}_j \in A$
$\alpha_\epsilon(f(\mathbf{x}_j, \tau))$ activator function.

Activator function





Example 2



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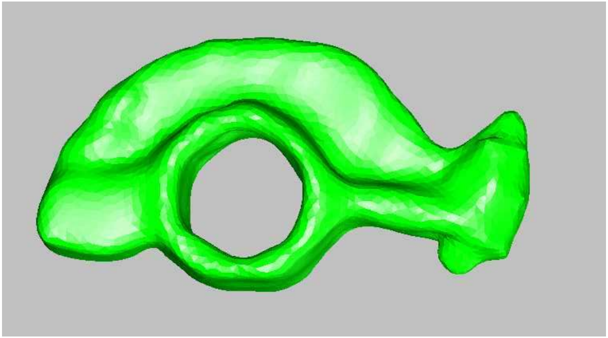

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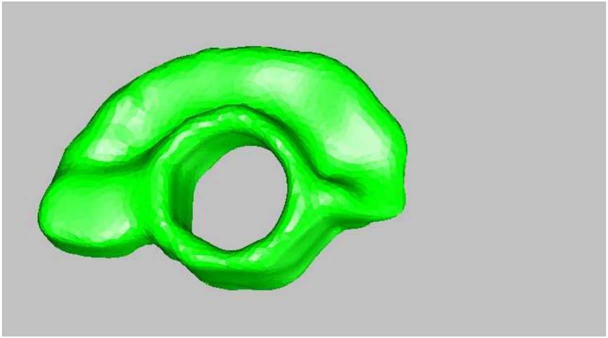


Volume constraint

- Specify a volume function $V(\tau)$
- Control the volume change $\dot{V}(\tau)$ during the evolution
- Volume preservation $\dot{V}(\tau) = 0$

Formulation as linear constraint

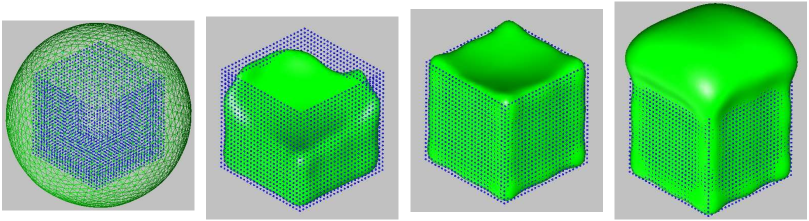
$$\int_{\mathbf{x}\in\Gamma} \underbrace{\begin{pmatrix} \text{actual normal} \\ \text{velocity at } \mathbf{x} \end{pmatrix}}_{*} dA = \dot{V}$$

where

$$* = -\frac{\dot{f}}{|\nabla f|} = \sum_{i=1}^{n} (\ldots)\dot{c}_i$$

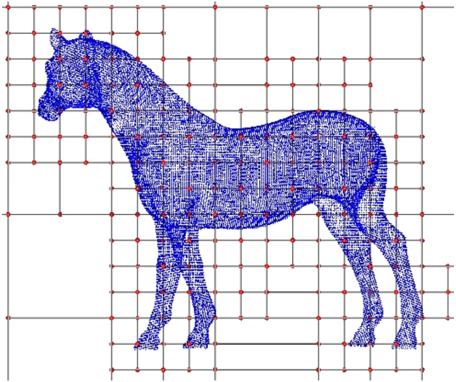
Use Lagrangian multipliers to add the constraint to the evolution.

Example 3



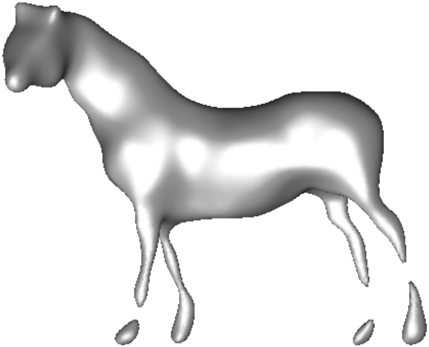
input data | $V = 0.08$ | $V = 0.125$ | $V = 0.2$

Example 4



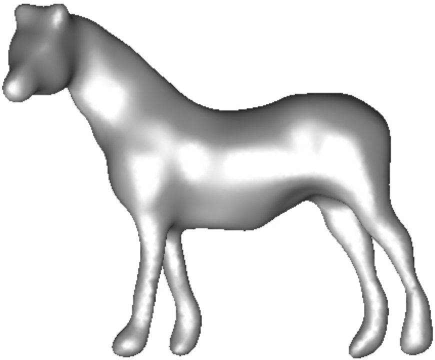
input data (48485 points) and grid

Example 4



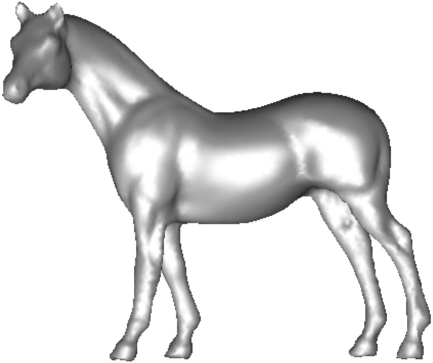
without constraint

Example 4



using the data points for the range constraint
time: ≈ 36 sec

Example 4



after displacement mapping
time: ≈ 3 sec

Summary and conclusion

- Combination of the evolution of implicitly defined surfaces and point set surfaces.
- Strategies for resampling the point set surface.
- Range and volume constraints to represent a-priori knowledge

Thanks for your attention!