A New Sub-Pixel Detector for X-Corners in Camera Calibration Targets

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ABSTRACT

X-corner patterns are most widely used in camera calibration. In this paper, we propose a new sub-pixel detector for X-corners, which is much simpler than the traditional sub-pixel detection algorithm. In this new algorithm, the pixel position of X-corner is firstly detected by a new operator. Then a second order Taylor polynomial describing the local intensity profile around the corner is educed. The sub-pixel position of X-corner can be determined directly by calculating the saddle point of this intensity profile. Neither preliminary intensity interpolation nor quadratic fitting of the intensity surface is necessary, which greatly reduces the computation load of the detection process. Furthermore, computer simulation results indicate that our new algorithm is slightly more accurate than the traditional algorithm.

Keywords

X-corners, sub-pixel detection, camera calibration

1. INTRODUCTION

Detection of target points is an important task in camera calibration, which will greatly affect the accuracy and stability of 3D machine vision. In recent years, many patterns have been used in camera calibration, such as squares [Zha00a], circles [Hei00a] and crosses [Dev00a] etc. Checkerboards with black-and-white squares are most widely used because the easy sub-pixel detection algorithm for X-corners with high precision [Luc00a]. Traditional algorithm for detecting X-corners first estimates their pixel locations by standard corner detectors (such as Harris [Har01a] and Noble [Nob00a]), then the sub-pixel positions can be determined by fitting quadratic functions to the local intensity profile around the corners and computing their extremal points. The main shortcoming of this algorithm is that the fitting of local intensity surface complicate the detection process. In 2001, Lucchese proposed a new simplified algorithm finding the extremal points by a morphological shrinking operation on the local intensity profile [Luc00a]. This algorithm requires a preliminary interpolation of intensity over the 2×2-pixel neighborhood of the detected corners, which means that shortening the interpolation interval will improve the precision of detection, but the computation load will increase at the same time.

In this paper, we propose a new sub-pixel detector for X-corners. We firstly use a new operator based on Hessian matrix to find the pixel position of X-corner. Then a second order Taylor polynomial describing the local intensity profile around the corner is educed. The sub-pixel position of the corner can be determined directly by calculating the saddle point of this profile. Our new algorithm does not need surface fitting or intensity interpolation, which can simplify the computation greatly.

This paper is organized as follows. Section 2 reviews briefly the traditional sub-pixel algorithm for detecting X-corners. Section 3 describes our new sub-pixel detector in detail. Section 4 provides the computer simulation results. The contrast of accuracy and robustness between the new algorithm and the

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traditional algorithm is used to validate our new algorithm.

2. TRADITIONAL ALGORITHM

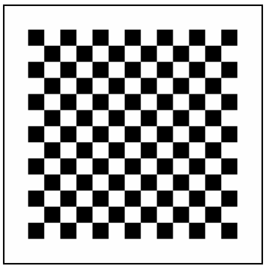

Figure 1: X-corners pattern

Fig. 1 shows a planar target with X-corners widely used in camera calibration. Traditional algorithm for detecting X-corners first finds their pixel positions by Harris detector based on a Hessian matrix looking for the auto-correlation matrix:

$$\mathbf{M}=\begin{bmatrix}\left(\frac{\partial \mathrm{I}}{\partial x}\right)^2\otimes w & \left(\frac{\partial \mathrm{I}}{\partial x}\cdot\frac{\partial \mathrm{I}}{\partial y}\right)\otimes w\\ \left(\frac{\partial \mathrm{I}}{\partial x}\cdot\frac{\partial \mathrm{I}}{\partial y}\right)\otimes w & \left(\frac{\partial \mathrm{I}}{\partial y}\right)^2\otimes w\end{bmatrix},$$

where w is a Gauss smoothing operator. Harris corner detector is expressed as

$$R=\det(\mathbf{M})-\lambda(\mathrm{trace}(\mathbf{M}))^2 .$$

The X-corner is just the local peak point of R. Considering the local intensity around one X-corner in the image which has been smoothed by a Gauss low pass filter, we can see that the 3D shape of the intensity profile is just like a saddle (see Fig. 2c). The saddle point of this surface is just the X-corner to be detected. For each X-corner, a quadratic fitting of the local intensity profile can be obtained. The function can be expressed as

$$\mathrm{F}(x,y)=ax^2+bxy+cy^2+dx+ey+f \qquad (1)$$

This function turns out to be a hyperbolic, so the position of the saddle point can be determined by calculating the intersection of the two lines as follows:

$$\begin{cases}2ax+by+d=0\\ bx+2cy+e=0\end{cases}.$$

In general, this traditional algorithm allows for accuracies of the order of a few hundredths of a pixel [Luc00a], which can satisfy most applications of 3D machine vision. But the preliminary interpolation of intensity and the latter surface fitting aggravate the computation load of this algorithm. In section 3, we can see that our new algorithm is more simple and practical for X-corner sub-pixel detection.

3. NEW SUB-PIXEL DETECTOR

Model for X-Corner Intensity Profile

The model for the intensity of an ideal X-corner (see Fig. 2a) can be expressed as

$$f(x,y)=\begin{cases}0, & xy\le 0\\ 1, & xy>0\end{cases}.$$

Because of the effect of noises, the X-corners in practical target image are not consonant with this ideal model, so preliminary low pass filter is necessary in image processing, which can reduce the effect of noises and smooth the intensity profile around the corners. We select a standard Gauss operator here:

$$\mathrm{g}(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}} .$$

Then the model for the intensity profile of an X-corner smoothed by Gauss operator is given by

$$\mathrm{r}(x,y)=\mathrm{g}(x,y)\otimes\mathrm{f}(x,y)=\Phi(x)\Phi(y)+\Phi(-x)\Phi(-y) \qquad (2)$$

where

$$\Phi(x)=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{x}e^{-\frac{t^2}{2\sigma^2}}dt .$$

Operator for X-Corner Detection

Before calculating the sub-pixel location of X-corner, we need an operator to determine its pixel position. Harris detector is a universal operator for detecting all kinds of corners. But for X-corner, we can use a new shape operator deduced by the eigenvalues of Hessian matrix of the image function. Haralick has discussed that the eigenvalues of Hessian matrix correspond to the minimum and maximum second directional derivatives of a surface, and their associated eigenvectors are the directions in which the second directional derivative is extremized and are orthogonal to each other [Har00a]. Considering the neighborhood of a pixel in an image function as a surface patch, Hessian matrix is expressed as

$$\mathbf{H}=\begin{bmatrix}r_{xx} & r_{xy}\\ r_{xy} & r_{yy}\end{bmatrix},$$

where r_{xx}, r_{xy}, r_{yy} are the second order partial derivatives of the image function $\mathrm{r}(x,y)$. The two eigenvalues of Hessian matrix are

$$\lambda_1=\frac{1}{2}\left(r_{xx}+r_{yy}+D\right),\ \lambda_2=\frac{1}{2}\left(r_{xx}+r_{yy}-D\right).$$

Corresponding normalized eigenvectors are

$$\mathbf{n}_1=\begin{bmatrix}\sqrt{\frac{1}{2}\left(1-\frac{f_{yy}-f_{xx}}{D}\right)} & \sqrt{\frac{1}{2}\left(1+\frac{f_{yy}-f_{xx}}{D}\right)}\end{bmatrix}^T,$$

$$\mathbf{n}_2=\begin{bmatrix}\sqrt{\frac{1}{2}\left(1+\frac{f_{yy}-f_{xx}}{D}\right)} & -\sqrt{\frac{1}{2}\left(1-\frac{f_{yy}-f_{xx}}{D}\right)}\end{bmatrix}^T,$$

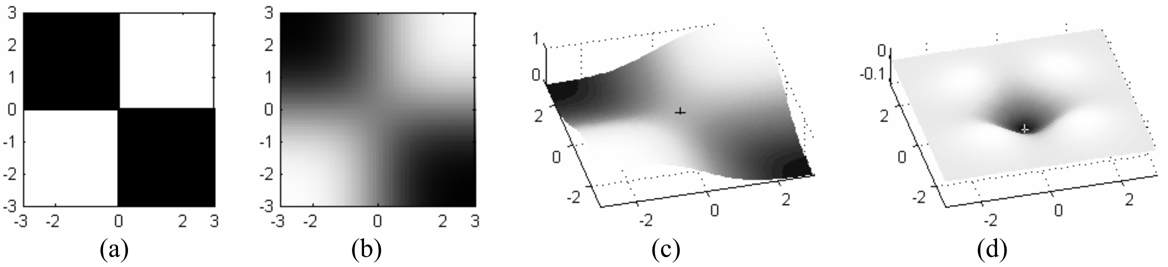

Figure 2: (a) An ideal X-corner consists of four neighboring black-and-white squares. (b) Smoothed X-corner. (c) 3D intensity profile around X-corner. The saddle point corresponding to the X-corner is marked with a '+'. (d) 3D profile of S around X-corner. The negative extremum point corresponding to the X-corner is marked with a '+'.

where

$$D = \sqrt{(r_{xx} - r_{yy})^2 + 4r_{xy}^2}\ .$$

For an X-corner point as Eq. 2, the maximum eigenvalue λ_1 of corresponding Hessian matrix is positive and the minimum eigenvalue λ_2 is negative, so we can get a new simple operator to detect the pixel position of an X-corner:

$$S = \lambda_1 \cdot \lambda_2 = r_{xx} r_{yy} - r_{xy}^2\ .$$

Fig. 2d shows the 3D shape of S around an X-corner. The corner to be detected is the local negative extremum point of S. Based on this constraint, we can determine the pixel position (x_0, y_0) of an X-corner.

Sub-Pixel Detection

It is obvious that the real sub-pixel position of the X-corner must locate in the vicinity of (x_0, y_0). We suppose that its real position is $(x_0 + s, y_0 + t)$, where $(s,t) \in [-0.5, 0.5] \times [-0.5, 0.5]$. For each X-corner, we can use a second Taylor polynomial to describe the local intensity profile around it. Hence,

$$\mathrm{r}(x_0 + s, y_0 + t) = r + (s \quad t)\begin{pmatrix} r_x \\ r_y \end{pmatrix} + \frac{1}{2}(s \quad t)\begin{pmatrix} r_{xx} & r_{xy} \\ r_{xy} & r_{yy} \end{pmatrix}\begin{pmatrix} s \\ t \end{pmatrix} \qquad (3)$$

where r is the value of the image function $\mathrm{r}(x,y)$ at (x_0, y_0), r_x, r_y are the first order partial derivatives of $\mathrm{r}(x,y)$ at (x_0, y_0), r_{xx}, r_{xy}, r_{yy} are the second order partial derivatives of $\mathrm{r}(x,y)$ at (x_0, y_0).

As confirmed above, an X-corner is just the saddle point of the intensity profile. We can set the first order derivatives of Eq. 3 along s and t to zero:

$$\begin{cases} r_{xx}s + r_{xy}t + r_x = 0 \\ r_{xy}s + r_{yy}t + r_y = 0 \end{cases}.$$

Then we can get the sub-pixel position $(x_0 + s, y_0 + t)$ of the X-corner to be detected, where

$$s = \frac{r_y r_{xy} - r_x r_{yy}}{r_{xx} r_{yy} - r_{xy}^2},\ t = \frac{r_x r_{xy} - r_y r_{xx}}{r_{xx} r_{yy} - r_{xy}^2}.$$

Detection of X-corners in Discrete Space

For discrete image, only one modification has to be made, which is the implement of convolution in discrete space. Here we select standard Gauss kernel as the convolution mask. It is given by

$$\mathrm{g}(m,n) = \frac{1}{2\pi\sigma^2} e^{-\frac{m^2+n^2}{2\sigma^2}}\ .$$

The dimension of the mask is $(2N+1) \times (2N+1)$, where N is given by 4[σ]. Then the partial derivatives of discrete image can be obtained by convolving the original image with corresponding differential Gauss kernels.

4. EXPERIMENTAL RESULTS

The proposed algorithm has been tested on computer simulated data. We built a planar target image with 144 X-corners as shown in Fig. 1. The size of each square is 32×32 pixels, and the overall size of this image is 512×512 pixels. In order to simulate the real camera calibration, we get a second image derived from this original image through a projective operation as follows:

$$\mathbf{x}_2 = \mathbf{M}\mathbf{x}_1\ ,$$

where

$$\mathbf{M} = \begin{bmatrix} 0.918 & 0.180 & -25.037 \\ -0.109 & 0.997 & 28.723 \\ 0 & 0 & 0.923 \end{bmatrix}.$$

Moreover, zero-mean white Gaussian noises are added to the resulting image to mimic the practical situation. Fig. 3 shows the resulting images. The standard deviation σ of the Gauss smoothing operator is set as 3, and the size of the kernel is 25×25. For each noise level, we perform 100 independent trials, and the results shown are the average. The contrast of the estimated errors between the traditional algorithm and the new algorithm is summarized in Table 1, which indicates that our new algorithm is slightly more accurate and robust.

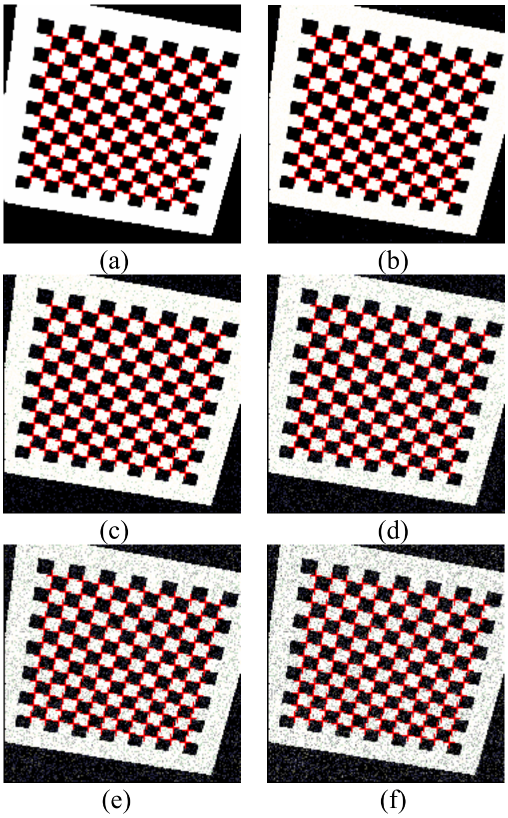

Figure 3: Images of the simulated target, where X-corners are marked as '+'. (a) Noise level $\sigma_n = 0$; (b) $\sigma_n = 0.04$; (c) $\sigma_n = 0.08$; (d) $\sigma_n = 0.12$; (e) $\sigma_n = 0.16$; (f) $\sigma_n = 0.20$.

Noise Level (σ_n)	Estimated Errors (σ_s)	
	Traditional Algorithm	New Algorithm
0	0.0088	0.0086
0.04	0.0331	0.0327
0.08	0.0644	0.0638
0.12	0.0958	0.0949
0.16	0.1279	0.1268
0.20	0.1598	0.1585

Table 1: The estimated errors of the detection for X-corners by two different algorithms: σ_s is defined as

$$\sigma_s = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left\|\hat{x}_k - x_k\right\|^2},$$

where x_k is the true position of X-corner, and $\hat{x}_k$ is the calculated position.

5. CONCLUSION

In this paper, we have proposed a new sub-pixel detector for X-corners in camera calibration targets. This algorithm consists of a new X-corner operator, followed by a second order Taylor polynomial describing the local intensity profile around the X-corner. The sub-pixel position of X-corner can be determined directly by calculating the saddle point of this polynomial. Neither intensity interpolation nor surface fitting is necessary, which simplifies the detection process greatly. Compared with the traditional method, our new algorithm is slightly more accurate and robust.

6. ACKNOWLEDGMENTS

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