Analytical-numerical method with an application of the nonlinear boundary condition

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Abstract **– Certain electromagnetic field problems regarding structures with nonlinear conductivity are considered. The problems are solved by an analytical-numerical method that applies the nonlinear boundary condition. The power balance is verified to check the accuracy of the solution.**

Keywords – analytical-numerical method; nonlinear boundary condition; nonlinear conductivity

I. FORMULATION OF THE NONLINEAR PROBLEMS

In order to present the analytical-numerical method that applies the nonlinear boundary condition, conductor configurations of linear and cylindrical symmetry are considered (Fig. 1). The structures contain layers of nonlinear conductivity that imposes a *J–E* relationship of the form:

$$J(E) = \gamma(E)E = \sum_{k=1,3,5,...}^{m} \gamma_k E^k . \quad (1)$$

The coefficients γ_k have been selected so that a current density component saturation effect is present (Fig. 1c), which can be noticed in some high-temperature superconductors [1]. It is assumed that $l >> h >> X_2$, in the linear symmetry problem. In both configurations it is assumed that the field depends only on one spatial coordinate (denoted as u – meaning x or r respectively for Cartesian and cylindrical coordinates) and time. A known total current time function is imposed with the Neumann boundary condition:

$$\frac{\partial A(t,u)}{\partial u} = -\mu \frac{I \sin(\omega_1 t)}{o}, \quad (2)$$

where $\omega_1 = 2\pi f_1$ and $f_1 = 60$Hz hence displacement currents are omitted. The parameter o depends on the shape of the boundary ($2h$ for the linear symmetry problem and $2\pi R_2$ for the problem of cylindrical symmetry). The differential equation describing the field distribution is [4]:

$$\nabla^2 A = \mu \sum_{k=1,3,...}^{m} \gamma_k \left(\frac{\partial A}{\partial t}\right)^k . \quad (3)$$

Equation (3) can be obtained from Maxwell's equations by applying the Lorentz gauge. For certain analytical formulae to be derived, the following magnetic vector potential component expansion is applied:

$$A = \sum_{i=1}^{n} \kappa^{i-1} A_i , \quad n \to \infty . \quad (4)$$

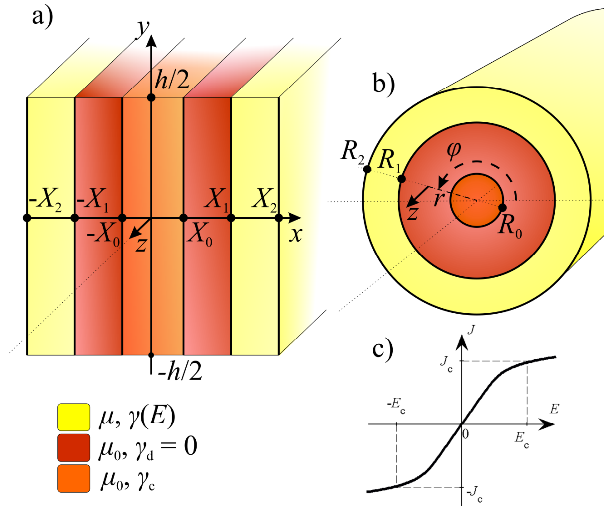

Fig. 1. Nonlinear problems in the electromagnetic field: a) tape configuration of linear symmetry, b) tubular conductor of cylindrical symmetry, c) nonlinear *J–E* curve

κ is a finite non-zero real-valued parameter. The solution is more accurate when more components of (4) are included. This has been shown in the authors' previous papers e.g. [2, 4] and is also verified in chapter IV.

II. NONLINEAR BOUNDARY CONDITION

The idea of applying the nonlinear boundary condition is that the effect of the nonlinear region is to be estimated and the area of calculations is reduced to the linear region alone. When this is done – the linear region electromagnetic field distribution can be calculated first. Afterwards, the nonlinear field is considered with the use of the boundary values of the linear region. The Neumann boundary is replaced by a nonlinear boundary condition at the linear region boundary $u = u_1$ ($x = X_1$ for the linear symmetry and $r = R_1$ for the tubular conductor), where for every time harmonic the dependence is assumed [5]:

$$\underline{\lambda}_{\mathrm{p}h} \underline{A}_h + \underline{\lambda}_{\mathrm{d}h} \frac{\mathrm{d}\underline{A}_h}{\mathrm{d}u} + \sum_{k=1}^{M_h} \underline{\lambda}_{\mathrm{p}h,k} \left(\prod_{\eta=1}^{h_{\max}} \left|\underline{A}_\eta\right|^{G_{h,k,\eta}} \right) \mathrm{e}^{\mathrm{j}\left(\sum_{\eta=1}^{h_{\max}} g_{h,k,\eta} \arg(\underline{A}_\eta) \right)} = \underline{\Xi}_h , \quad (5)$$

where $G_{h,k,\eta} \in \mathbb{N}_0$, $g_{h,k,\eta} \in \mathbb{Z}$ and M_h is the number of nonlinear dependencies on $A(t, u_1)$.

The $\underline{\lambda}_k$ coefficients are calculated with the help of:

– analytical dependencies derived by certain components of the Poincaré method [4],
– numerical-symbolic computation implemented in Visual C++ [3].

III. Electromagnetic Field Distribution

The electromagnetic field distribution for the linear region is obtained by means of the nonlinear boundary condition (5). The boundary value harmonic components $\underline{A}_h(u_1)$ and $\frac{\mathrm{d}\underline{A}_h(u_1)}{\mathrm{d}u}$ are applied in an analytical-numerical method [2, 4, 5] to obtain the nonlinear region electromagnetic field distribution. The time functions of the magnetic field strength and current density components, in the nonlinear regions, are presented in Fig. 2. The nonlinear *J–E* curve causes the arising of higher time harmonics in the components of the electromagnetic field even though the total current was assumed in a time periodic form consisting of only one time harmonic (imposed by (2)).

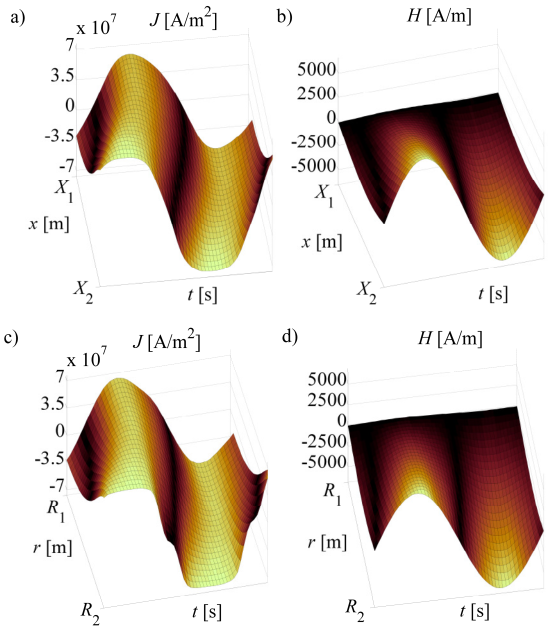

Fig. 2. Electromagnetic field distribution in the nonlinear conductor – a) current density (tape configuration), b) magnetic field strength (tape configuration), c) current density (tubular conductor), d) magnetic field strength (tubular conductor)

IV. Power Balance

In order to check the solutions, the electromagnetic field power balance has been verified. The instantaneous power has been calculated by means of the Poynting vector:

$$p_{\mathrm{P}} = p_{\mathrm{P}0} + \sum_{h=2,4,6,\ldots}^{h_{p\max}} p_{\mathrm{P}h}\cos(h\omega_1 t + \theta_{\mathrm{P}h}) = -\oint\!\!\oint_{\Omega} (\vec{E}\times\vec{H})\,\mathrm{d}\vec{\Omega}, \quad (6)$$

and with the use of volume integrals:

$$p_V = p_{V0} + \sum_{h=2,4,6,\ldots}^{h_{p\max}} p_{Vh}\cos(h\omega_1 t + \theta_{Vh}) = \\ = \frac{1}{2}\frac{\partial}{\partial t}\iiint_V \mu\left|\vec{H}\right|^2 \mathrm{d}V + \iiint_V \vec{J}\,\vec{E}\,\mathrm{d}V. \quad (7)$$

The balance fulfillment error is calculated for each instantaneous power component by the formula:

$$e_{ph}(t) = \left|1 - \frac{p_{Vh}}{p_{\mathrm{P}h}}\right| \cdot 100\%. \quad (8)$$

The calculation results are presented on Figure 3 for various numbers of components in the (4) series (denoted by n).

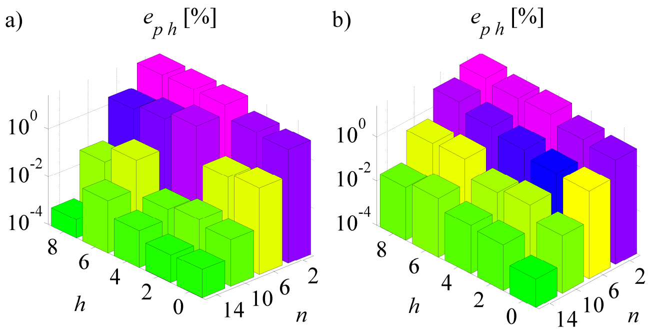

Fig. 3. Obtained error values of power balance fulfillment for various n: a) tape configuration, b) tubular conductor

It can be noticed that the result is improved with the use of more components of the (4) series. For $n = 10$ the obtained error values were below 1%.

V. Conclusions

Certain electromagnetic field nonlinear boundary value problems have been solved by applying an analytical-numerical method with the use of the nonlinear boundary condition. The method uses analytical formulae that are derived with the application of (4). The nonlinear boundary condition coefficients are computed by a numerical-symbolic algorithm implemented in Visual C++. After the linear region electromagnetic field distribution had been obtained, the nonlinear region was computed by an analytical-numerical method [2, 4, 5]. The obtained time functions of the field components contained higher time harmonics arising as an effect of nonlinearity.

The power balance has been verified for the solution. The calculated error values were less than 1% for $n = 10$ and have improved with additional components added to the (4) series. A solution with a very low error value can be used in the future e.g. to check the efficiency of numerical methods.

It has been shown that the nonlinear boundary condition may be used to estimate the effect of the nonlinear region.

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