Application of Hybrid Boundary Element Method on Modelling of Hemispherical Ground Inhomogeneity

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***Abstract*—The procedure for modelling ground inhomogeneity influence on grounding system based on so-called hybrid boundary element method (HBEM) is given in this paper. The HBEM is a recently proposed numerical method for stationary or quasi-stationary EM field analysis. The obtained results are compared with those based on using the Green's function for the point source inside a semi-spherical inhomogeneity.**

Keywords—Green's function; ground inhomogeneity; grounding systems; hybrid boundary element method; resistance; Method of Moments; Quasi-stationary EM field.

I. INTRODUCTION

There is a certain interest for developing procedures for analyzing influence of ground inhomogeneities modelled as hemispherically-shaped domains on grounding systems in their vicinity. Various ground inhomogeneities (pillar foundations, ponds, small lakes) can be approximated by semi-conducting hemispherical domain. This is the reason that many authors (including some of the co-authors of this paper) recently published papers dealing with this topic [1-7]. The approach is based on using the boundary element method [1], direct solution of the Poisson's electric scalar potential differential equation for two hemispherical and concentrically placed domains placed in the homogeneous ground [2], or application of an approximate solution for the Green's function of the point source inside/outside the hemispherical domain [3] as it has been done in [4-8].

The example solved in this paper is a single wire electrode inside the hemispherical semi-conducting inhomogeneity, placed in homogeneous ground [8]. The single electrode can be assumed as an equivalent of a wire armature cage system. This approximation is carried out using the complex function theory as it has been explained in [9].

The results obtained using the approach mentioned last are going to be verified in this paper with the ones obtained using the recently proposed hybrid boundary element method (HBEM) [10-12]. It is based on the equivalent electrode method [6, 13], and the point-matching method for the potential of the grounding electrode and for normal component of the electric field on the boundary surface between conductive media [14]. The method is also applicable to quasi-stationary grounding systems with complex conductivity of multilayered media. It enables avoiding of numerical integration, which can cause problems when boundary integral equations contain singular and nearly singular integrals as often occurs in practice.

II. THEORETICAL BACKGROUND

The application of the hybrid boundary element method (HBEM) to analysis of the single wire electrode inside the hemispherical semi-conducting inhomogeneity of radii r_S and specific conductivity σ_2, placed in the homogeneous ground having specific conductivity σ_1, is illustrated in Fig. 1. The corresponding cylindrical coordinates r and z have been introduced. The wire electrode's length and cross-section radii are labelled by h and a_h, respectively. The boundary surface between the hemisphere and the surrounding ground, i.e. unknown total charges distribution on it, is modelled by rings of charges Q_n, $n = 1,...N$, (Fig. 1). The rings of radii a_n, $n = 1,...N$ are placed parallel to the ground surface at the depths h_n, $n = 1,...N$. The cross-section dimensions of the ring wires a_{rn}, $n = 1,...N$ are determined applying the procedure of the HBEM.

The longitudinal current along the wire electrode is assumed in polynomial form as in [15]:

$$I(z') = \sum_{m=0}^{M} I_m (z'/h)^m , \tag{1}$$

while the leakage current is

$$I_{\text{leak}}(z') = -\frac{\partial I(z')}{\partial z'} = -\sum_{m=1}^{M} \frac{mI_m}{l_k} (z'/h)^{m-1}. \tag{2}$$

Applying the quasi-stationary image theory, the potential of the system in the cylindrical coordinate system is expressed as in [12], i.e.

$$\begin{aligned}\varphi(r,z) = {} & \frac{1}{4\pi\sigma_2} \int_0^h I_{\text{leak}}(z') \left(\frac{1}{\sqrt{r^2 + (z-z')^2}} + \frac{1}{\sqrt{r^2 + (z+z')^2}} \right) \mathrm{d}z' + \\ & + \sum_{n=1}^{N} \frac{Q_n}{2\pi^2 \varepsilon_0} \left(\frac{K(\pi/2, k_{1n})}{\sqrt{(r+a_n)^2 + (z-h_n)^2}} + \frac{K(\pi/2, k_{2n})}{\sqrt{(r+a_n)^2 + (z+h_n)^2}} \right),\end{aligned} \tag{3}$$

where

$$k_{1n}^2 = \frac{4r\, r_n}{(r+a_n)^2 + (z-h_n)^2}, k_{2n}^2 = \frac{4r\, r_n}{(r+a_n)^2 + (z+h_n)^2},$$

and

$$K(\pi/2,k)=\int_0^{\pi/2}\left(1-k^2\sin^2\alpha\right)^{-1/2}\mathrm{d}\alpha \tag{4}$$

is a complete elliptic integral of the first kind.

Now, there is a total of $M+1$ unknown current coefficients in (1) (I_m , $m=0,1,..,M$), and N charge coefficients in (3) (Q_n $n=1,2,..,N$), i.e. total of $M+N+1$ unknowns.

Based on the procedure from [12], $M+1$ equations can be formed satisfying the boundary condition for the potential given by (2) on the wire electrode surface $\varphi=U$, at the points defined by

$$r_k=a_{\mathrm{e}}\,,\; z_k=\frac{2k-1}{2(M+1)}h\,.\; k=1,2,...,M+1\,. \tag{5}$$

The rest of equations (N) is formed satisfying the boundary condition for the normal component of the electric field at the points placed on the hemispherical boundary surface between two domains

$$E_{Rn}(R=r_{\mathrm{s}}^{+})=\frac{-\sigma_2}{\varepsilon_0(\sigma_1-\sigma_2)}\eta_n,\eta_n=\frac{Q_n}{2\pi a_n\Delta l_n},n=1,2,..,N\,. \tag{6}$$

where $a_{\mathrm{r}n}=\frac{4}{\pi}r_s\sin[\pi/(8N)]$, and $\Delta l_n=r_s\,\pi/(2N)$, $n=1,2,..,N$.

After determining unknowns, it is possible to obtain the total feeding current as

$$I_g=\int_0^h I_{\mathrm{leak}}(z')\,\mathrm{d}z'\,. \tag{7}$$

Finally, the resistance of the grounding system can be determined as

$$R_g=U/I_g\,. \tag{8}$$

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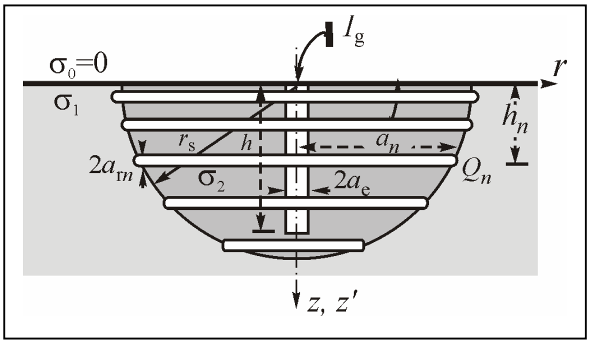

Figure 1. Single electrode in the hemispherical domain and the equivalent system of charges.

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