Calculation of the Magnetic Force Acting on a Particle in the Magnetic Field

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***Abstract*— This publication presents computation of magnetic force acting on particle moving in the field created by two toroidal wires with currents. Equivalent dipole method is used.**

***Index Terms*—Dielectrics and electrical insulation, dielectrics, dielectrophoresis.**

I. Introduction

Recently, microfluidics combined with the ability of magnetic manipulations has concerned much attention due to its great possibility for biomedical applications, such as mixing, transport and separation of biomolecules. Rapid cell sorting is an example of the recently arising fields of cellular therapy and in biotechnology. Current clinical apparatuses are based on immunoaffinity columns, or on high-gradient magnetic separation columns using either micrometer polymeric beads doped with magnetite, or nanometer iron-dextran colloids, attached to targeting antibodies [3]. There are many methods for supplying drugs to specific locations and magnetic drug targeting (MDT) is a one of the approaches for tumor treatment because of its high targeting effectiveness. MDT contains required medical medium in compound magnetic nanoparticles, introducing these into the blood stream entering a tumor and using a highly inhomogeneous magnetic field in order to locate the magnetic particles within the chosen area [1].

The applications of magnetic particles in analysis system permits one to quantitatively investigate biomolecules in a straightforward way, therefore it is significant to separate and sort magnetic particles from solution efficiently. In recent times, new migration effects induced by diverse external fields have been used for the separation and characterization of micro- and nanoparticles. Particularly, magnetophoresis, which one can define as the migration of magnetic particles under the strongly inhomogeneous magnetic field, is useful for the classification throughout the specification of the magnetic permeability of particles and the separation of microparticles from liquid. A distinctive benefit of the magnetophoresis is that it generates no heat inside bulk liquid, what makes it distinctive from the electrophoresis and dielectrophoresis. This is only true when magnetic field has so low frequency that it does not generate eddy currents [5]. Hence, the magnetic separation is essentially non-invasive method for soft particles such as cells and cell composites, which can be by strong magnetic fields acting on them destroyed. Additional benefit is that the magnetic field can penetrate various substances such as glasses and other synthetic materials much easier than electric field.

Hence, the sources of the magnetophoretic force can be placed entirely outside a microchannel and they are therefore not in contact with vessel and solution. This substantially simplifies construction of the appropriate arrangements used to separation of particles according to required properties. Magnetophoresis is the phenomenon where the gradient of a magnetic field causes movement of some magnetic material objects due to a force on the magnetic moment induced by the same field. The induced magnetic moment also produces its own magnetic fields and they can thus interact [2].

II. Main Equations

It can be shown that equivalent dipole moment of the magnetic sphere is given by relation [4]

$$m = 4\pi r_0^3 K_{\mathrm{CM}}\left(\mu_2, \mu_1\right) H \tag{1}$$

where Clausius-Mossotti factor is given by

$$K_{\mathrm{CM}}\left(\mu_2, \mu_1\right) = \frac{\mu_2 - \mu_1}{\mu_2 + 2\mu_1} \tag{2}$$

When magnetic particle is placed in inhomogeneous field on the equivalent magnetic dipole acts a force, which is dependent from m and from magnetic field H

$$\mathbf{F}_{\mathrm{d}} = \mu_1\left(\mathbf{m}\cdot\nabla\right)\mathbf{H} \tag{3}$$

Taking into account (1) and because $H\nabla H = 1/2\nabla H^2$, the force acting on particle has the value

$$\mathbf{F}_{\mathrm{d}} = 4\pi r_0^3 \mu_1 K_{\mathrm{CM}}\left(\mu_2, \mu_1\right)\nabla H^2 \tag{4}$$

In the above formula coefficient μ_1 is absolute permeability of the medium with number 1. Sense of the force $\mathbf{F}_{\mathrm{d}}$ depends from the sign of the Clausius-Mossotti factor. When $\mu_2 > \mu_1$ then particle is dragged in to the field with greater gradient and when $\mu_2 < \mu_1$ then it is push out of the stronger field.

III. An Example

As an example let us consider calculation of magnetophoretic force acting on magnetic particle placed between two toroidal wires which are 0.4 μm away each from other. Cross section of each wire is circular with the radius 0.1 μm and toroid diameter is equal 0.8 μm. Constant currents I_0 = 0.25 mA are flowing in opposite directions in order to achieve high magnetic field gradient (Fig. 1). Between these two wires magnetic particle with the radius r_0 = 1/30 μm and relative permeability μ_2 = 10 can move freely along straight lines. All is placed in air with permeability μ_1 = 1. The whole computational area is surrounded by a sphere with radius 2 μm. Clausius-Mossotti factor has for this example value K_{CM} = 9/12. First, we have to solve magnetic field equation [6]

$$\nabla \times \left(\frac{1}{\mu_{\mathrm{w}} \mu_0} \mathbf{B} \right) = \mathbf{J} \qquad \mathbf{B} = \nabla \times \mathbf{A} \tag{5}$$

by finite element method. First the potential **A** next magnetic field strength was calculated. Magnetic field has axial symmetry respectively axes *y*

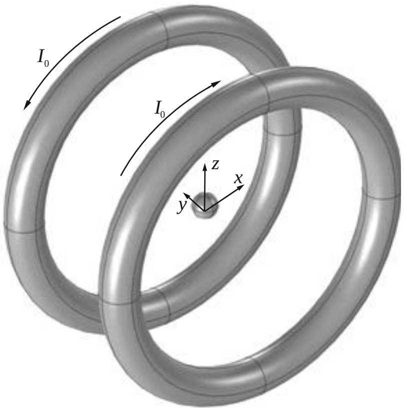

Fig. 1. Two wires with currents and magnetic particle inside.

In the Fig.2 the component $\partial H_x/\partial x$ is plotted and in Fig.3 the component $\partial H_x/\partial x$ as the particle moves along *x* and *y* axes, respectively.

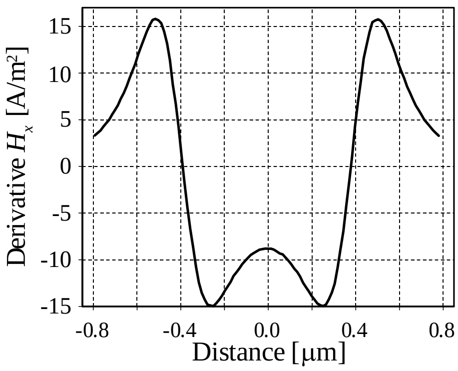

Fig. 2. Plot of the partial derivative $\partial H_x/\partial x$ along the line $x \in (-0.82, +0.82)$ μm.

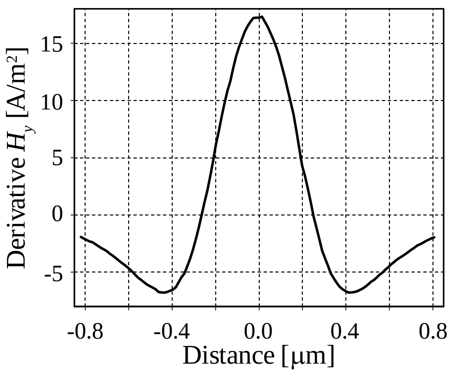

Fig. 3. Plot of the partial derivative $\partial H_y/\partial x$ along the line $y \in (-0.82, +0.82)$ μm.

In Fig. 4 and Fig.5 there are shown F_x and F_y components of the force acting on particle. The most abrupt changes have place when particle moves under the wires.

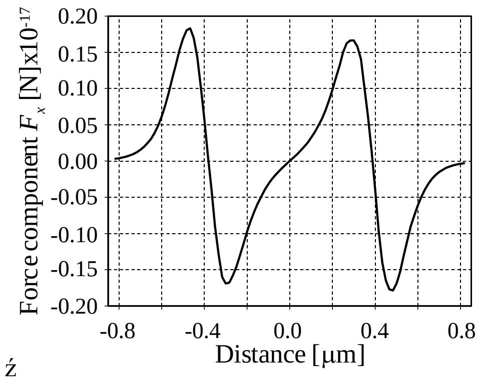

Fig. 4. Plot of the F_x along the line $x \in (-0.82, +0.82)$ μm.

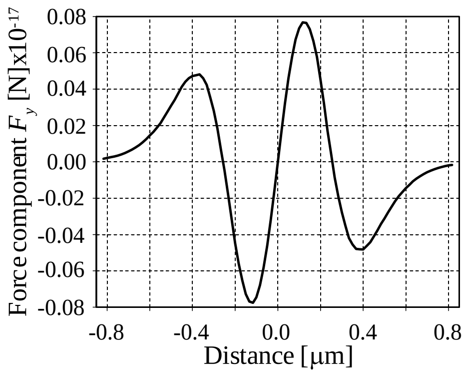

Fig. 5. Plot of the F_y along the line $y \in (-0.82, +0.82)$ μm.

In this publication calculation of the forces acting on magnetic particle immersed in fluid with the permeability μ_0. Equivalent dipole method is used. The main advantage of the equivalent dipole method is that distribution of the magnetic field is calculated without presence of particle in computational space. This significantly simplifies the computations. The other method, which can be used in this context, the Maxwell stress method needs division of the space on the finite elements together with the particles. This can substantially increase the finite element number and lead to weakly conditioned system matrix.

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