Computation of Self and Mutual Inductances in Nonlinear Magnetic Systems

Anton Hamler, Marko Jesenik, Miloš Beković and Mladen Trlep
Faculty of Electrical Engineering and Computer Science
University of Maribor
Maribor, Slovenia

***Abstract*—This paper describes possible procedures for computation of static self-inductances and static mutual inductances in nonlinear magnetic systems. The procedures are based on numerical calculation of magnetic field with finite element method. Previously mentioned calculations are carried out for various current excitations. Inductances are calculated with magnetic flux of coils and with magnetic field energy. Single-phase transformer is used as a test model for verification of inductance calculation procedures. Calculated results of inductance values, magnetic fluxes and magnetic field energy are estimated in regards to the measurements of the geometrically and physically identical realistic transformer.**

Keywords— computation inductance, mutual inductance, self-inductance, inductance measurement, transformer

I. Introduction

Coil inductances of electrical devices are one of the fundamental parameters for description of devices with concentrated elements. To increase inductance and magnetic flux linkage in these devices, magnetic circuits made of ferromagnetic or ferrimagnetic materials are used. Magnetic circuits contain nonlinear magnetic properties, which contribute to the nonlinearity of the magnetic system. It means that inductances of the system depend on the intensity of magnetizing the magnetic circuit. Inductance effects are only notable in time-dependent magnetic field, which means that the intensity of magnetizing is always variable.

Many authors [1]-[4] have been already analyzing and calculating inductances. So far, developed methods are still not enough general and accurate. Discussed magnetic systems often contain single coil or exclude permanent magnets. This paper describes two methods for calculation of static self-inductances and static mutual inductances in nonlinear magnetic systems.

II. Calculation of Inductances

A. *Calculation of inductances from magnetic flux*

Flux linkage in nonlinear magnetic system consisting of n coils could be described with (1).

$$\psi_1(i) = L_1(i)\cdot i_1 + M_{12}(i)\cdot i_2 + M_{13}(i)\cdot i_3 + \cdots + M_{1n}(i)\cdot i_n \quad (1)$$

L is self-inductance of a coil, M mutual inductance and i electric current. Index subscribed to self-inductance and first index subscribed to mutual inductance represents the number of a coil in which the flux linkage is observed. The second index subscribed to mutual inductance symbol represents a coil, which is a source of flux linkage caused in another coil given by the first subscribed index. The inductances of nonlinear magnetic system indirectly, through permeability, depend on electrical current i all n coils. For a some coil, flux linkage depends on 2*n parameters that is given in expression (1) for the flux linkage of the first coil. Previously mentioned parameters are self-inductance, n-1 mutual inductances and n currents. The number of influential parameters when calculating inductance can be reduced under assumption that parameters are constants for minor current changes. Therefore, method of current variation is applicable. The method of current variation consists of following steps:

1)Nonlinear calculation of magnetic conditions

Calculation of magnetic conditions with FEM is carried out, including magnetic nonlinearity of materials in the considered system and including terms under which inductance would be determined. It means that only excitations, which assure resultant field, are active. Inductances are calculated for the resultant field.

2)Linearization of magnetic conditions

Linearization of magnetic conditions is carried out in point of operation, which is obtained from results of nonlinear calculation of field in the previous step. Permeability is determined for each element of discretization and it is considered constant in further calculations. With this procedure, magnetic system becomes linear and inductance is independent from electric current.

3)Linear calculation of magnetic conditions

Permeability of element of discretization, determined in the previous step, is used for linear calculation of magnetic conditions with FEM. Electric current of coil, of which inductance is calculated, decreases or increases for Δi, in regards to the value in step 1).

4)Calculation of inductance

Self-inductance can be calculated according to equation (2) and mutual inductance can be calculated according to equation (3).

$$L_1 = \frac{\Delta\psi_1}{\Delta i_1} = \frac{N_1\cdot\Delta\Phi_1}{\Delta i_1} \quad (2)$$

$$M_{12} = \frac{\Delta\psi_1}{\Delta i_2} = \frac{N_1\cdot\Delta\Phi_1}{\Delta i_2} \quad (3)$$

For calculation of mutual inductances of the remaining coils in the system, it should be separately made a change of the current Δi in each and every coil and calculation process should be repeated every time.

B. Calculation of inductances from magnetic field energy

Self-inductances and mutual inductances can be calculated according to (4) if magnetic energy of the magnetic system is calculated.

$$W_{\mathrm{m}} = \int_0^{i_1} L_1(\mu)\cdot i_1 \cdot \mathrm{d}i_1 + \int_0^{i_1} M_{21}(\mu)\cdot i_2 \cdot \mathrm{d}i_1 + \int_0^{i_2} M_{12}(\mu)\cdot i_1 \cdot \mathrm{d}i_2 + \int_0^{i_2} L_2(\mu)\cdot i_2 \cdot \mathrm{d}i_2 \qquad (4)$$

The mentioned calculation is described through the similar steps as described in the capture II, subchapter A. Detailed explanation will be presented in the final paper.

III. Example of Inductance Calculation

Described procedures in chapter II are verified with measurements on a single-phase transformer. Transformer is shown in Fig. 1. Basic transformer data is given in Table I. Magnetic conditions are calculated with programme package OPERA, on the basis of 3D FEM. Self-inductance of the primary coil and mutual inductance has been determined from measured values for voltage U_1 = 220 V, current I_1 = 0.207 A and power P_1 = 16.42 W under no-load according to (5) and (6). Calculation of magnetic flux according to measurements results is carried out with (7). For this example, it is assumed that voltage and current are sinusoidal waves. Values of parameters, evaluated from measurement results and calculations are given in Table II. The measurement values of the inductance as the function of the voltage are given in Fig. 2.

$$L_1 = \frac{\sqrt{(U_1 I_1)^2 - P_1^2}}{2\pi f I_1^2} \qquad (5)$$

$$M_{21} = \frac{U_2}{2\pi f I_1} \qquad (6)$$

$$\hat{\Phi} = \frac{\sqrt{2}\cdot(U_1 - I_1\cdot R_1)}{2\pi f N_1} \cong \frac{\sqrt{2}\cdot U_2}{2\pi f N_2} \qquad (7)$$

IV. Conclusions

The paper shows procedures of inductance calculation, which are based on current variation and on previously calculated flux linkage or magnetic field energy. Both methods give results of similar values. Calculated and measured results show good agreement, which is confirmed with the experimental transformer model. The inductance calculation is based on the novel approaches for calculation of magnetic fluxes of coils and magnetic field energy in nonlinear magnetic systems, which will be explained in the final paper.

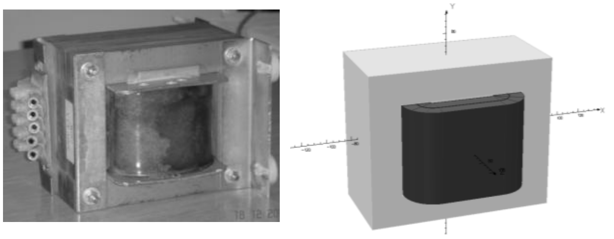

Figure 1: Real transformer and calculating model.

TABLE I. Basic Transformer Data

Parameter	*Primary*	*Secondary*
U(V)	220	120
I(A)	4.1	7.5
R(Ω)	1.1	0.4
N(turns)	248	136
P(W)	248	136
Outer dimensions (mm)	X × Y × Z = 150 × 125 × 74	

TABLE II. Measured and Calculated Values of Parameters under no-load Transformer Operation

Para-meter	**Meas-urement**	**Values from results of field calculation**	
		Calculation from Ψ	***Calculation from Wm***
L_1(H)	3.375864	3.317220	3.320894
M_{21}(H)	1.848776	1.786318	-
W_m(Ws)	0.142847	0.142689	0.142847
Φ_1(Vs)	0.003989	0.003923	0.003928
Φ_2(Vs)	0.003987	0.003853	-

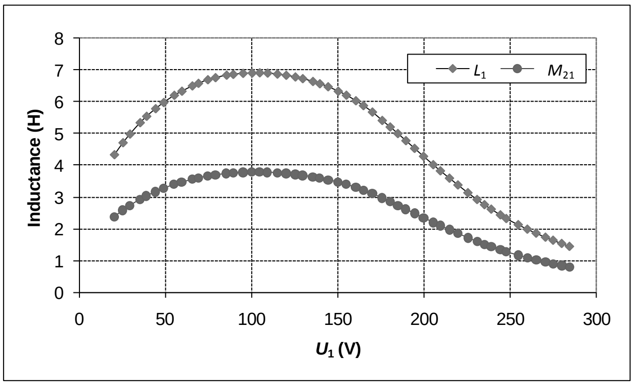

Figure 1: Static inductance dependent from the voltage RMS value.

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