Equivalent Circuit Model for a Single Phase Transformer

Jan Kyncl, Adithya Hariram, Zdeněk Novák
Dept. of Electrical Power Engineering
Faculty of Electrical Engineering, Czech Technical University
Prague, Czech Republic
kyncl, hariradi, novakzd5@fel.cvut.cz

***Abstract*—The parameters of power transformers are usually determined from the results of the short-circuit test and open-circuit test, i.e. the user knows active powers, the amplitudes of currents and voltages in both situations. The aim of this paper is to present approximate formulae for "circuit theory" type of equivalent circuits of the chosen diagram. Exact formulae for parallel and serial resistances and self and mutual inductances are not available and probably cannot be obtained in a closed form.**

Keywords — Circuit theory, open circuit test, power transformers, short circuit test.

I. INTRODUCTION

A transformer is mainly a magnetic coupler. Magnetically, a two-winding transformer is a two-port circuit. Only the two currents flowing through the windings can be fixed independently one from another by the outside circuit. Electrostatically, the third current running between the two windings via parasitic capacitances cannot be ignored. This current essentially depends on the inter-winding voltage. The transformer then appears as a three-input multipole as shown in figure 1 [1].

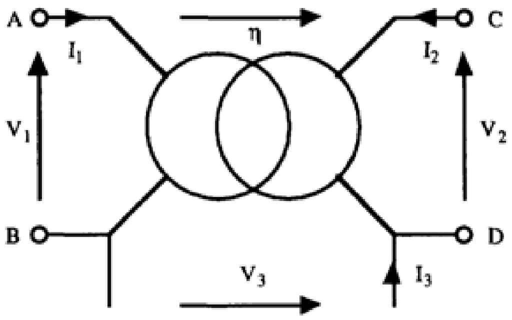

Fig. 1. Two-winding transformer, two-port circuit

The paper basically deals with the power in two winding transformers. For this type of transformers we can presume the losses in the primary winding to be the same as in the secondary one.

Let the ratio of the number of turns in the secondary N_s to the number of turns in the primary N_p be denoted by letter p, as shown in the equation below.

$$p = \frac{N_s}{N_p} \tag{1}$$

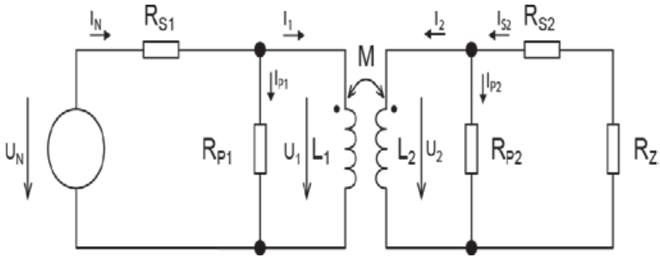

Fig. 2. Representation of loss resistances in two-winding transformer

Let the condition of equal loses be given by

$$R_{s2} = p^2 \cdot R_{s1} \tag{2}$$

We divide these loses equally to the primary and secondary parts, so that we assume

$$R_{p2} = p^2 \cdot R_{p1} \tag{3}$$

The above assumptions decrease the number of the independent parameters of the equivalent circuit. The task is to find the values L_1, L_2, M, R_{s1} and R_{p1}. The short-circuit test and open-circuit test provide us just necessary information for the five unknown values of resistances and inductances in the proposed equivalent circuit.

II. PROPOSED FORMULA

The power factor in transformers is commonly caused by the excessive resistance in the core grounding circuit. Transformers that are normally used in power networks have power factors at a relatively small value, which is usually identified from the short-circuit tests and open-circuit tests. A small value of the power factor tells us that the imaginary part of the impedance is substantially greater than the real part.

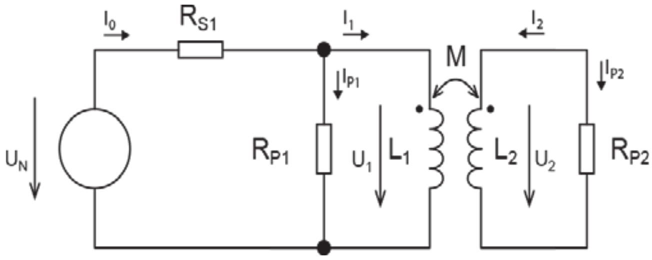

Fig. 3. Open-circuit test in two winding transformer

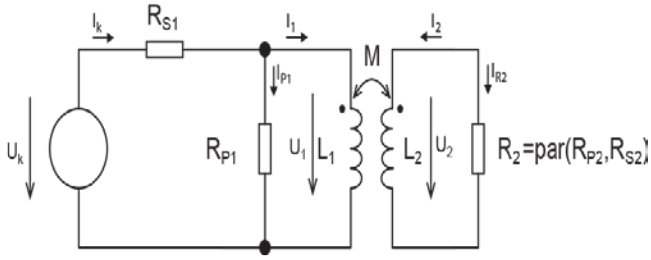

Fig. 4. Short-circuit test in two winding transformer

The current amplitudes for the test were assumed to be functions of L_1, L_2, M, applied voltage and angular frequency ω.

We assume the active losses in the transformer to be uniformly distributed between the primary and secondary windings and

$$M = \sqrt{L_1 \cdot L_2} \tag{4}$$

Using the above relations, the values of R_{s1} and R_{p1} can be approximated separately. Thus by knowing the values of R_{s1} and R_{p1} we can estimate the remaining equivalent circuit parameters.

Resulting formulae:

$$\begin{aligned} L_2 &= \frac{p^2 \cdot u_p}{I_p \cdot \omega} \\ M &= p \cdot \sqrt{\frac{u_p \cdot (I_k \cdot u_p - I_p \cdot u_k)}{I_k \cdot I_p^2 \cdot \omega^2}} \\ L_1 &= \frac{u_p}{I_p \cdot \omega} \\ R_{s1} &= \frac{P_k \cdot u_p^2 - P_p \cdot u_k^2}{2 \cdot I_k^2 \cdot u_p^2 - I_p^2 \cdot u_k^2} \\ R_{p1} &= \frac{2 \cdot (I_p^2 \cdot u_k^2 - 2 \cdot I_k^2 \cdot u_p^2)}{I_p^2 \cdot P_k - 2 \cdot I_k^2 \cdot P_p} \end{aligned} \tag{5}$$

These formulae are not exact, but following experiments proved them to be accurate enough for power engineering purposes.

III. Numerical Experiments

Performing the node analysis on circuits of known parameters under a sinusoidal steady state is a very easy task compared with the inverse task, i.e. for the known values of L_1, L_2, M, R_{s1}, R_{p1}, input voltage and angular frequency we can easily obtain currents, voltages, active and reactive power.

For this we assume the following conditions:

$$\begin{aligned} L_1 &= A_{\mathrm{L}} \cdot N_p^2 \\ L_2 &= A_{\mathrm{L}} \cdot N_s^2 \\ M &= k_{\mathrm{m}} \cdot A_{\mathrm{L}} \cdot n_1 \cdot n_2 \end{aligned} \tag{6}$$

where A_{L} and k_{m} are the inputs for the numerical analysis, and the corresponding next inputs are R_{s1} and R_{p1} given by the relations (2) and (3).

The code was written in the environment of the software Wolfram Mathematica. For a chosen list of parameters A_{L}, k_{m}, R_{s1} and R_{p1} we determine the values of L_1, L_2, M, R_{s1}, R_{p1}.

The conditions of the short-circuit test and open-circuit tests are simulated to obtain inputs of proposed formulae with the errors evaluated as shown in Fig. 5.

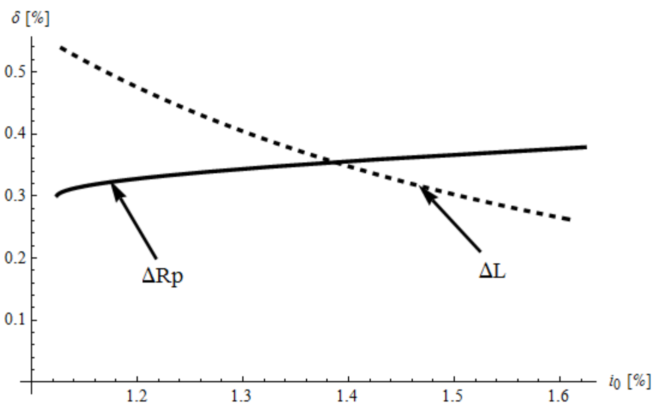

Fig. 5. Error in the estimated parameters for an industrial transformer

References

[1] FRANCOIS BLACHE, JEAN-PIERRE PIERRE KÉRADEC, BRUNO COGITORE, Stray capacitances of two winding transformers: equivalent circuit, measurements, calculation and lowering, Conference Record of the 1994 IEEE Industry Applications Society Annual Meeting, 1994, vol.2, p. 1211 – 1217.