Model Order Reduction Applied to Optimization of Electromagentic Devices

Yuki Sato and Hajime Igarashi
Graduate School of Information Science and Technology
Hokkaido University
Kita 14, Nishi 9, Kita-ku, Sapporo, 060-0814, Japan
yukisato@em-si.eng.hokudai.ac.jp

***Abstract*— This paper introduces the model order reduction (MOR) based on the proper orthogonal decomposition (POD) for optimization of electromagnetic devices. The computational time for optimization can be reduced by using this method. In this work, the shape of an inductor is optimized by using the adaptive genetic algorithm and MOR based on POD. It is shown that the present method provides the same optimization result as the one obtained by the conventional method with shorter computational time.**

Keywords—Model order reduction, proper orthogonal decomposition, optimization.

I. Introduction

In recent years, model order reduction (MOR) based on the proper orthogonal decomposition (POD) has widely been used for the time-domain analysis of fluid and electromagnetic fields [1-4]. In this method, we apply principal component analysis to variance-covariance matrix which consists of solution vectors snapshotted during transient state. We then obtain basis vectors for reduced system from eigenvectors of variance-covariance matrix. By using MOR, we can shorten the computational time in time-domain electromagnetic analysis [2], [3]. However, it has been pointed out that numerical errors can exist in the magnetically saturated region [3]. In order to overcome this problem, the subdomain reduction technique, in which the MOR is applied only to the domain excluding the nonlinear material domain, has been presented [4].

Because MOR can accelerate electromagnetic field analysis, it would be very effective for optimization problems which need multiple field analyses [5-7]. Three-dimension shape optimization has not widely been performed since three-dimension electromagnetic analysis is very time consuming so far. It is expected that MOR allows us to perform three-dimension optimization in practical design processes.

In this paper, we introduce the fast optimization method for three-dimension electromagnetic devices using MOR based on the POD. In this method, we snapshot the solution vectors of various shapes analyzed during the preprocessing of optimization, from which we can obtain the basis vectors to reduce the order of the original system. Moreover, the subdomain reduction is applied to the air region surrounding the design region. The computational time for optimization is expected to be shortened by the present method. In this work, we employ adaptive genetic algorithm (GA) [8], which gives faster convergence than conventional GA, for the optimization method. It is shown that the present method can effectively reduce the computational time for optimization.

II. Model Order Reduction for Optimization

Let us consider a linear equation

$$\mathrm{A}(\boldsymbol{u}_i)\boldsymbol{x}_i = \boldsymbol{b}(\boldsymbol{u}_i) \tag{1}$$

where $\mathrm{A}\in R^{m\times m}$, $\boldsymbol{x}_i$, $\boldsymbol{b}\in R^m$ and $\boldsymbol{u}_\mathrm{i}$ is the design parameter vector to determines the electric devices design. Here, to obtain the design basis vectors, eq. (1) is solved for various design parameters $\boldsymbol{u}_i$, where $0 \le i < s$, where s is number of snapshots, which is much smaller than m (m>>s). We then apply principal component analysis to the matrix X defined by

$$\mathrm{X} = \left[\boldsymbol{x}_1 - \boldsymbol{\mu} \quad \boldsymbol{x}_2 - \boldsymbol{\mu} \quad \cdots \quad \boldsymbol{x}_s - \boldsymbol{\mu}\right] \tag{2}$$

to have

$$\mathrm{X} = \mathrm{W}\Sigma\mathrm{V}^\mathrm{t} = \sigma_1 \boldsymbol{w}_1 \boldsymbol{v}_1^t + \sigma_2 \boldsymbol{w}_2 \boldsymbol{v}_2^t + \cdots + \sigma_s \boldsymbol{w}_s \boldsymbol{v}_s^t \tag{3}$$

where σ_i, i=1,2…s are singular values, $\boldsymbol{w}_i$ and $\boldsymbol{v}_i$ are eigenvectors of XX^t and $\mathrm{X}^\mathrm{t}\mathrm{X}$, respectively and $\boldsymbol{\mu}$ is the mean vector of $\boldsymbol{x}_i$. The original unknown vector $\boldsymbol{x}_\mathrm{i}$ can be expressed by the linear combination of reduced vectors $\boldsymbol{y}\in R^r$

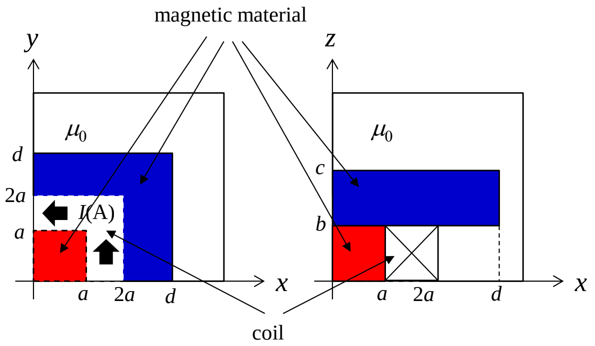
$$\boldsymbol{x} = \mathrm{W}\boldsymbol{y}. \tag{3}$$

III. Adaptive Genetic Algorithm

In the adaptive GA [8], crossover, mutation and selection of probability are varied automatically being dependent on population dispersion. For example, if population dispersion is high, it is suppressed by setting higher crossover of probability while if it is low, crossover probability is made lower. Therefore, this method accelerates convergence of GA processes.

IV. Optimization Results

In this work, we optimize the inductor model shown in Fig. 1 which has four design parameters ($\boldsymbol{u}$ = $[a, b, c, d]^\mathrm{t}$).



Design parameter vector : $\boldsymbol{u} = [a, b, c, d]^t$

Fig. 1, Inductor model for optimization.

TABLE.I, OPTIMIZATION RESULTS

	$f(\boldsymbol{u})$	$L(\boldsymbol{u})$	Computational time
Conventional	0.00321	0.997mH	62944 sec (100 %)
Present	0.00321	0.997mH	22512 sec (35.8%)

The FE mesh has 59317 elements and 187200 variables, while design region has 26460 variables. By applying MOR to the domain surrounding the design domain, the number of unknown variables then is reduced from 160740 to 20. The magnetic material is assumed to have constant permeability whose value is set to μ_r=100.

To test the validity of the present method, the difference between the inductance values $L_{\text{reduction}}$ computed by MOR are L_{original} computed by the conventional FEM is evaluated. The results are shown in Fig.2. We find that the standard deviation in the difference is much less than 1%. The inductor is optimized so that its inductance is L_T=1mH. Hence, the objective function $f(\boldsymbol{u})$ is defined as follows :

$$f(\boldsymbol{u}) = \left| 1 - \frac{L(\boldsymbol{u})}{L_T} \right| \tag{4}$$

where $L(\boldsymbol{u})$ is inductance of the inductor model shown in Fig. 1. The numerical results in the conventional and present optimization methods are summarized in Table I, from which we find that the value of $f(\boldsymbol{u})$ and $L(\boldsymbol{u})$ obtained by conventional method coincide with those obtained by present method and the computational time for optimization in the present method can be shorter than that in the conventional method. The changes in the fitness values for both methods during the optimization processes are shown in Fig. 3, in which we can see no clear differences between them.

V. CONCLUSION

In this paper, we have presented the MOR based on the POD for optimization of electromagnetic devices. It has been shown that the result of optimization with present method is the same as that with conventional method and computational time for optimization is can be effectively shortened. In the full paper, we will apply the present method to other optimization problems, such as inductor with nonlinear material and antenna.

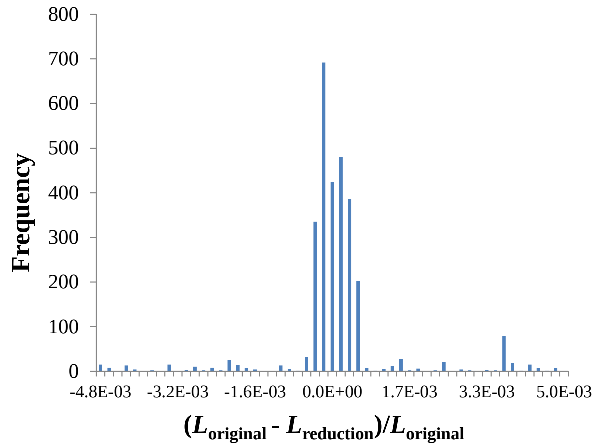


Fig. 2, Relative error distribution in the computed inductance.

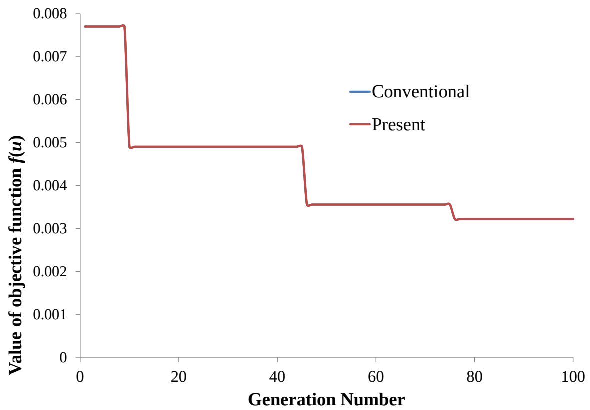


Fig. 3, Changes in the fitness values during optimization.

REFERENCES

[1] P. Krysl, S. Lall, and J.E. Marsden, "Dimensional Model Reduction in Non-Linear Finite Element Dynamics of Solids and Structures," *Int. J. Numer. Meth. Engng*, vol. 51, pp. 479-504, 2001.

[2] D. Schmidthäusler and M. Clemens, "Low-Order Electroquasistatic Field Simulations Based on Proper Orthogonal Decomposition," *IEEE Trans. Magn.*, vol. 48, no. 2, pp, 567-570, Feb. 2012.

[3] Y. Sato and H. Igarashi, "Model Reduction of Three Dimensional Eddy Current Problems Based on the Method of snapshots," *IEEE Trans. Magn*. to be published.

[4] D. Schmidthäusler, S. Schöps and M. Clemens, "Reduction of Linear Subdomain for Non-Linear Electro-Quasistatic Field Simulations," CEFC 2012, Abstract, pp. 369.

[5] Y. Watanabe, K. Watanabe and H. Igarashi, "Optimization meander line antenna considering coupling between non-linear circuit and electromagnetic waves for UHF-band RFID," *IEEE Trans. Magn.*, vol. 47, no.5, pp 1506-1509, May 2011.

[6] B. Mirzaeian, M. Moallem, V. Tahani, and C. Lucas, "Multiobjective optimization method based on a genetic algorithm for switched reluctance motor design," *IEEE Trans. Magn.*, vol. 38, no. 3, pp. 1524–1527, May 2002.

[7] K. Watanabe, F. Campelo, Y. Iijima, K. Kawano, T. Matsuo, T. Mifune,and H. Igarashi, "Optimization of Inductors Using Evolutionary Algorithms and Its Experimental Validation," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 3393–3396, Aug. 2010.

[8] Y. Sato, F. Campelo and H. Igarashi, "Meander Line Antenna Design Using Adaptive Genetic Algorithm," *IEEE Trans. Magn.* to be published.