Stochastic models of lumped elements

Janusz Walczak, Seweryn Mazurkiewicz, Dariusz Grabowski
Faculty of Electrical Engineering
Silesian University of Technology
Gliwice, Poland
Email: janusz.walczak@polsl.pl, seweryn.mazurkiewicz@polsl.pl, dariusz.grabowski@polsl.pl

***Abstract*—Equations expressing the first and the second order moments for stochastic models of $RLCM$ as well as $R(t), L(t), C(t), M(t)$ elements have been derived in the paper. The analysis has been carried out for two cases. The first approach assumes that a stochastic current process describing a given excitation source and a random variable describing an element parameter are independent. The second one takes into account the relation between the current process and the random variable which are not independent in this case. The results have been illustrated by examples.**

I. Introduction

Many works, including monograph [1], have been devoted to analysis of stochastic phenomena in electrical and electronic circuits. They often deal with determination of probabilistic characteristics for stochastic processes observed in systems. Works in the field of stochastic system analysis can be divided into two main topic groups. The first one concerns deterministic systems in which some stochastic signal sources are present [2]. The analysis of such systems is usually carried out by means of stochastic differential or integral equations [2], [3]. The second one concerns systems in which sources as well as basic elements require probabilistic description [4], [5]. In this case problems under consideration include model construction either for elements whose lumped parameters R, L, M, C are random variables or for time-varying elements described by functions $R(t), L(t), M(t), C(t)$ which are deterministic functions or stochastic processes [6]. Such models can be built among others with the aid of stochastic moments [7]. This paper deals with determination of the first and the second order moments for the voltage stochastic process observed in the case of random elements R, L, M, C and $R(t), L(t), M(t), C(t)$ assuming that the current stochastic process moments are given. It is a continuation of the previous works devoted to determination of stochastic process moments for deterministic elements R and C supplied by stochastic current source [8], [9] and for nonlinear inertiales elements described by random polynomials [10].

II. Resistor stochastic models

Stochastic current and voltage processes in the case of a resistor whose the resistance is a random variable are related by:

$$U(t) = R \cdot I(t), \tag{1}$$

where:

R – random variable with given distribution,
$U(t)$ – voltage stochastic process of the resistor,
$I(t)$ – current stochastic process of the resistor.

The following two cases can be considered:

- the current process and the random variable R are statistically independent,
- the current process and the random variable R are not statistically independent.

In the first case, assuming that the resistor current is known and applying expected value operator [11] to equation (1) results in a closed form formulae expressing the first and the second moments of the voltage across the resistor:

$$m_U(t) = \mathrm{E}[U(t)] = \mathrm{E}[R] \cdot \mathrm{E}[I(t)] = m_R \cdot m_I(t), \tag{2}$$

$$\sigma_U^2(t) = \mathrm{E}[R^2] \cdot R_I(t,t) - m_U^2(t), \tag{3}$$

$$R_U(t_1,t_2) = \mathrm{E}[U(t_1)U(t_2))] = \mathrm{E}[R^2] \cdot R_I(t_1,t_2), \tag{4}$$

where:

$m_U(t)$ – expected value of the process $U(t)$,
$m_I(t)$ – expected value of the process $I(t)$,
m_R – expected value of the variable R,
$\sigma_U^2(t)$ – variance of the process $U(t)$,
$R_U(t_1,t_2)$ – autocorrelation function of the process $U(t)$,
$R_I(t_1,t_2)$ – autocorrelation function of the process $I(t)$,
$\mathrm{E}[R^2]$ – second moment of the random variable R.

The analysis of equations (2), (3), (4) leads to a conclusion that if the given current process and the random variable representing the resistance R are independent, then the complete description of the resistor can be made with the aid of the first and the second moments of the current process as well as the random variable R.

In the second case, equation (2) is not valid and the following relation can be written:

$$m_U(t) = \mathrm{E}[U(t)] = \mathrm{E}[R \cdot I(t)]. \tag{5}$$

There are few methods to expand formula (5) [12]. The simplest one consists in application of the expected value definition:

$$\mathrm{E}[g(X(t_1),Y(t_2))] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{XY}(x,y,t_1,t_2)\,dxdy, \tag{6}$$

where:

$f_{XY}(x, y, t_1, t_2)$ – joint probability density function of the random variables $X(t_1)$ and $Y(t_2)$ defined by the stochastic processes $X(t)$ and $Y(t)$ for moments t_1 and t_2, respectively,

$g(x, y)$ – deterministic function of two variables.

Equation (6) can be applied to express moments of the voltage stochastic process across the resistor in the the case of statistical dependence between the resistance R and the current stochastic process:

$$m_U(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ri f_{RI}(r, i, t)\, drdi, \tag{7}$$

$$\sigma_U^2(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r^2 i^2 f_{RI}(r, i, t)\, drdi - m_U^2(t), \tag{8}$$

$$R_U(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r^2 i_1 i_2 f_{RI}(r, i_1, i_2, t_1, t_2)\, drdi_1 di_2, \tag{9}$$

where:

$f_{RI}(r, i, t)$ – joint probability density function of the random variable R and the stochastic processes $I(t)$ for a moment t.

In this case, moments of the input current process and the random variable describing the element parameter are not sufficient to express moments of the output voltage process. Joint probability density functions must be given.

Models of a time-varying resistor have been also presented in the paper. Such a resistor is described by the following equation:

$$U(t) = R(t) \cdot I(t). \tag{10}$$

Three special cases have been considered - the function $R(t)$ has been assumed to be:

- a deterministic function,
- a stochastic process which is independent of the current processes $I(t)$,
- a stochastic process which is not independent of the current processes $I(t)$.

In the first case, the first and the second moments of the output voltage process are expressed by:

$$m_U(t) = R(t) \cdot m_I(t), \tag{11}$$

$$\sigma_U^2(t) = R^2(t)\left(R_I(t, t) - m_I^2(t)\right), \tag{12}$$

$$R_U(t_1, t_2) = R(t_1)R(t_2)R_I(t_1, t_2). \tag{13}$$

The relations between moments of the processes $U(t)$ and $I(t)$ for the other cases are more complex and they will be presented in the full paper.

III. Dynamic element stochastic models

Similarly to the previous section, relations between the first and the second moments of the current and voltage stochastic processes for elements L, C, M and $L(t), C(t), M(t)$ have been given in the paper. The following special cases have been considered:

- parameters L, C, M are random variables independent or dependent of the input current process $I(t)$,
- parameters $L(t), C(t), M(t)$ are deterministic functions,
- parameters $L(t), C(t), M(t)$ are stochastic processes independent or dependent of the input current process $I(t)$.

The theoretical results have been illustrated by examples.

IV. Conclusions

Methods which enable calculation of expected values, variances and correlation functions for processes observed in the case of random elements R, L, C, M as well as $R(t), L(t), C(t), M(t)$ have been described in the full paper. If the random variable describing the element parameter and the input stochastic process are independent, then the output stochastic process moments can be determined only on the base of the moments of the input process and the parameter random variable. Otherwise, the joint probability density functions must be known.

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