Gradient method of learning for stochastic kinetic model of neuron

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***Abstract*—In this paper we are focusing on the kinetic extension [4] of classic model of Hodgkin and Huxley [2]. We are showing the descent gradient method used in the learning process of neuron, which is described with stochastic kinetic model. In comparison with [1] we use only 3 weights instead of 9: g_{Na}, g_K and g_L. We show that this model behaves equally accurate as the model of Hodgkin and Huxley with slighter system description.**

I. Introduction

We will deal with the stochastic kinetic model of neuron, in which processes that takes place on the membrane are described with kinetic Markov schemes. The equation that describes the dynamics of potential on the membrane can be written in the form [4]:

$$\begin{aligned} C\frac{dV}{dt} &= I - g_{Na}\left[m_3h_0\right]\left(V - V_{Na}\right) \\ &\quad - g_K\left[n_4\right]\left(V - V_K\right) - g_L\left(V - V_L\right) \end{aligned} \tag{1}$$

where I represents the input current, g_{Na}, g_K, g_L - the ion conductances and V_{Na}, V_K, V_L - the reverse potentials, for sodium, potassium and chloride ions respectively. Markov kinetic schemes for sodium and potassium ions can be drawn in the form [4]:

$$\begin{array}{ccccccc} m_0h_0 & \underset{\beta_m}{\overset{3\alpha_m}{\rightleftharpoons}} & m_1h_0 & \underset{2\beta_m}{\overset{2\alpha_m}{\rightleftharpoons}} & m_2h_0 & \underset{3\beta_m}{\overset{\alpha_m}{\rightleftharpoons}} & m_3h_0 \\ \alpha_h \Big\uparrow\Big\downarrow \beta_h & & \alpha_h \Big\uparrow\Big\downarrow \beta_h & & \alpha_h \Big\uparrow\Big\downarrow \beta_h & & \alpha_h \Big\uparrow\Big\downarrow \beta_h \\ m_0h_1 & \underset{\beta_m}{\overset{3\alpha_m}{\rightleftharpoons}} & m_1h_1 & \underset{2\beta_m}{\overset{2\alpha_m}{\rightleftharpoons}} & m_2h_1 & \underset{3\beta_m}{\overset{\alpha_m}{\rightleftharpoons}} & m_3h_1 \end{array} \tag{2}$$

$$n_0 \underset{\beta_n}{\overset{4\alpha_n}{\rightleftharpoons}} n_1 \underset{2\beta_n}{\overset{3\alpha_n}{\rightleftharpoons}} n_2 \underset{3\beta_n}{\overset{2\alpha_n}{\rightleftharpoons}} n_3 \underset{4\beta_n}{\overset{\alpha_n}{\rightleftharpoons}} n_4 \tag{3}$$

The membrane is built from channels which consist from small gates, that control the movement of ions between interior and exterior of the neuron. In the classic Hodgkin-Huxley approach it is assumed that gates can be in on of two states, namely permissive or non-permissive. In the kinetic model we assume that sodium gates can be in one of eight states (eq. (2)) and potassium gates can be in one of five states (eq. (3)), where only one state ($[m_3h_0]$ and $[n_4]$) is permissive while the rest of the states remain non-permissive. For kinetic formalism see [3].

In the stochastic approach, we assume that number of gates that are at the moment changing the state is taken from the binomial distribution, which under certain assumptions can be approximated with the normal distribution [5].

We are using the descent gradient method of learning in the stochastic kinetic model. This learning method allows to adjust the weights (in our case parameters of the model g_{Na}, g_K, g_L), so the course of the potential could fit the given pattern potential. The task of the neuron is to learn the pattern potential in the adaptively way for given current input.

In the next section we provide a short description of the descent gradient method, while in section III we show some experimental results. We end this paper with short conclusion in section IV.

II. Descent gradient method

In the process of learning of neuron we used the descent gradient method. Scheme to be proceed can be written in the form of algorithm:

1) Define the problem - minimization of error function E
2) Designate initial values of weights $w_1, w_2, \ldots, w_n$ (in our case g_{Na}, g_K, g_L)
3) Designate the value of gradient of error function:
$$\nabla E = \left[\frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, \ldots, \frac{\partial E}{\partial w_n}\right] \tag{4}$$
4) Actualize the values of weights according to rule
$$w_i \rightarrow w_i - \eta \nabla E\left(w_i\right) \tag{5}$$
(where η is the learning rate) until the condition of the end is met
$$e = \left|V - V^*\right| < \epsilon \tag{6}$$

In the case of our problem we can write the minimized function as:

$$E = \frac{1}{T}\int_0^t \frac{1}{2}\left(V\left(t\right) - V^*\left(t\right)\right)^2 dt \tag{7}$$

where V is the potential obtained from the model of neuron, while V^* is the pattern potential. In comparison with [1], where nine weights were assumed, in our system we consider only three weights, namely g_{Na}, g_K and g_L rewritten in the form [1]:

$$\begin{aligned} w_1 &= g_{Na} = \bar{g}_{Na} e^{\tilde{g}_{Na}} \\ w_2 &= g_K = \bar{g}_K e^{\tilde{g}_K} \\ w_3 &= g_L = \bar{g}_L e^{\tilde{g}_L} \end{aligned} \tag{8}$$

where $\bar{w}$ represents the default value of the parameter, while $\tilde{w}$ - the adopted weight.

We are considering the system of the form [1]:

$$\dot{V} = F\left(V, g_{Na}, g_K, g_L\right) \tag{9}$$

where

$$\begin{aligned} F = \frac{1}{C} &\left(I - g_{Na}\left[m_3 h_0\right]\left(V - V_{Na}\right)\right. \\ &\left. - g_K\left[n_4\right]\left(V - V_K\right) - g_L\left(V - V_L\right)\right) \end{aligned} \tag{10}$$

If we want to know the influence of particular parameters of the system on equation (9), then we can shape a differential equation, for each parameter:

$$\dot{Y}_i = \frac{\partial F}{\partial V} \cdot Y_i + \frac{\partial F}{\partial w_i} \tag{11}$$

where

$$Y_i = \frac{\partial V}{\partial w_i} \tag{12}$$

hence the differential equations system:

$$\begin{cases} \dot{Y}_1 = G \cdot Y_1 + \left[m_3 h_0\right]\left(V - V_{Na}\right) \\ \dot{Y}_2 = G \cdot Y_2 + \left[n_4\right]\left(V - V_K\right) \\ \dot{Y}_3 = G \cdot Y_3 + \left(V - V_L\right) \end{cases} \tag{13}$$

where

$$G = \frac{\partial F}{\partial V} = -\frac{1}{C}\left(g_{Na}\left[m_3 h_0\right] + g_K\left[n_4\right] + g_L\right) \tag{14}$$

Actualization of weights is done with the following scheme [1]:

$$T_a \dot{\Delta} \tilde{w}_i = -\Delta \tilde{w}_i + \frac{1}{T}\left(V\left(t\right) - V^*\left(t\right)\right) \frac{\partial V\left(t\right)}{\partial w_i} \cdot \frac{\partial w_i}{\partial \tilde{w}_i} \tag{15}$$

where

$$\tilde{w}_i = -\eta \tilde{w}_i \tag{16}$$

and $T_a = T$, $\eta = 0.01$.

III. Experimental results

In Fig. 1 we are showing three potential waveforms. The first one is the pattern potential obtained from the Hodgkin-Huxley model with the assumption of input current $I = 40\,[nA]$. The second and the third waveforms were obtained in the learning process of neuron modelled by Hodgkin and Huxley and from the stochastic kinetic model, respectively.

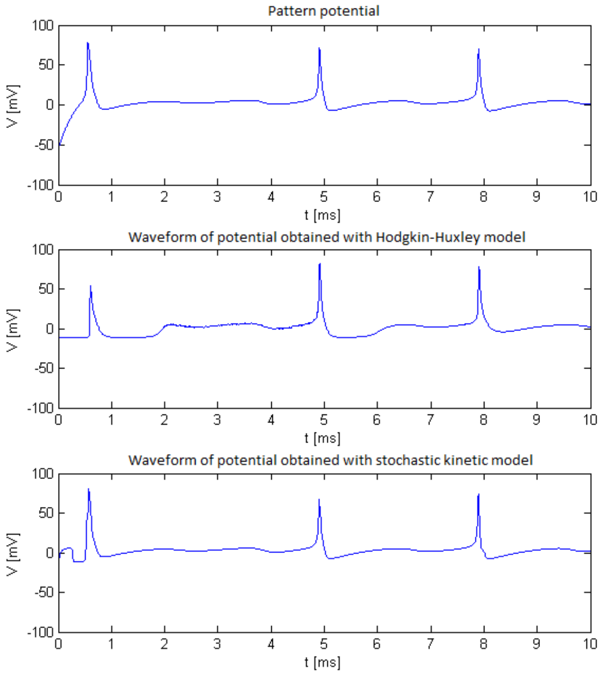

Fig. 1

IV. Conclusion

As we can observe, results from the Hodgkin-Huxley model and from the stochastic kinetic model are equally accurate. This means that we can easily replace learning in Hodgkin-Huxley model with learning in the stochastic kinetic model. This will give us the implementation benefits related with the reduction of calculations resulting from the formulation of the model and from the reduction of number of weights in the learning process.

References

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