Connections of parametric sections

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***Abstract* — The effective algorithms of constructing models of generalized parametric sections of the *n*-th order consisting of cascade and parallel connections of elementary LTV sections are presented in this paper. Moreover, the methods of impulse responses determination have been shown.**

Keywords—LTV, parametric section, connection, parallel connection, cascade connection

I. Introduction

In the synthesis of LTI (*linear time invariant*) systems, techniques consisting in the parallel or cascade connection of universal sections are often used. The section connections are used in designing higher order complex filters [3]. In practical applications, the first and second order filters are more often used. The rules of transforming systems composed of elementary SISO (*single input single output*) blocks of LTI class are well known in the control theory [3]. In the continuous-time or discrete-time domain, the algebra of these transformations is based on operations $(\pm, *)$ with impulse responses of elementary sections as kernels of these operations. In the domain of $\mathcal{L}$-transform or $\mathcal{F}$-transform, algebra is based on the classical definition of operations $(\pm, \cdot)$ with arguments being the transfer functions or spectral functions of elementary blocks. In case of LTV (*linear time varying*) sections with the coefficients variable in time, the classical definition of operational transfer functions has no sense [2], [5]. The method of determining differential equations, known from the literature [3], describing systems composed of the connection of LTI sections cannot be used for LTV systems.

In this article, the effective algorithms of constructing the differential equations of generalized *n*-th order substitute consisting of cascade and parallel connections of the first and second order LTV sections has been described. The methods of the determination of substitute system impulse responses have also been shown.

II. Generalized SISO parametric systems

The generalized *n*-th order parametric system (compare fig. 1) is composed of the connection of elementary LTV first and second order sections. In this article two types of substitute systems have been considered. The former, model I, is composed of the parallel connection of elementary sections. The latter, model II, of the cascade connection of elementary sections.

A. Models of elementary LTV 1st and 2nd order sections

The analysis of any parametric section of *n*-th order can be reduced to the analysis of the low-pass LTV section of *n*-th order [5].

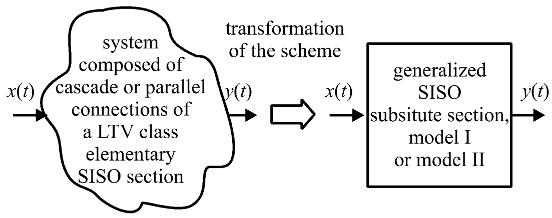

Fig. 1 Transformation of a complex LTV system into an equivalent system

Elementary LTV sections are described by differential equations with coefficients variable in time. The first order sections are described by a parametric differential equation of the first order (1):

$$y'(t) + \omega(t)\, y(t) = x(t),\ i = 1,2,...,n\,, \qquad (1)$$

whereas the second order LTV sections are described by a parametric differential equation of the second order:

$$y''(t) + 2\omega(t)\sigma(t)\, y'(t) + \omega^2(t)\, y(t) = x(t)\,, \qquad (2)$$

where: $\omega(t)$, $\sigma(t)$ – parametric function of $C^{(n-1)}[0,\infty)$ class. In this publication, the non periodic variability of parametric functions has been assumed. These functions can be interpreted as a cut-off (or resonance) angular frequency $\omega(t)$ variable in time or the attenuation ratio $\sigma(t)$ of low-pass filter variable in time. The parametric functions are strictly positive due to the assumed variability of the coefficients. It is significant for the stability of parametric systems [1], [7].

B. Model of generalized section type I

The first of the considered models of a substitute parametric section is a generalized section type I (fig. 2) composed of a parallel connection of the first and second order elementary sections. In the extended version of this paper an algorithm enabling to determine the differential equations of LTV section parallel connection in a closed form has been described. The effective usage of the algorithm for *n*>>2 requires implementation in a symbolical language (i.e. Mathematica).

C. Model of generalized section type II

The cascade connection of elementary LTV sections, shown in fig. 2, is equivalent to the generalized parametric section type II. The order of the differential equation describing the substitute section type II is equal to the sum of orders of all sections included in the connection.

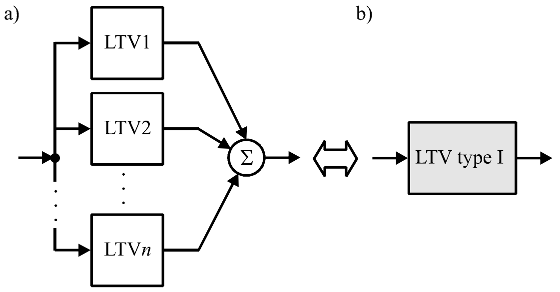

Fig. 2. Parallel connection of LTV sections (a) connection scheme (b) generalized LTV section type I.

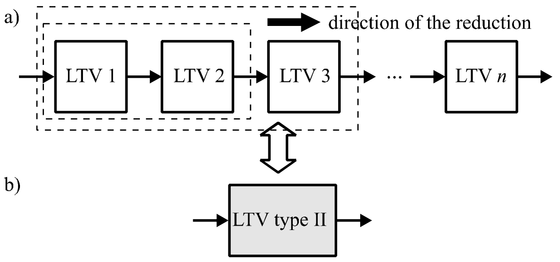

Fig. 3. Cascade connection of LTV sectios (a) connection scheme (b) generalized section of type II

On the contrary to cascade connections of LTI sections, the cascade connection of LTV sections is not commutative [6].

III. Synthesis method of the generalized section

The synthesis of generalized parametric section of I and II type consists in the determination of differential equations describing the systems and their impulse responses.

A. *Synthesis section of I type*

As a result of the considerations, the parallel connection of n sections of the first order and m sections of the second order can be replaced as the connection of $n+2m$ sections of first order. The mathematical model of generalized I type section is the differential equation containing the expressions consisting of the composition of exponential functions, Bessel and hypergeometric functions as well as the integrals of the products of exponential functions, Bessel functions, hypergeometric functions and an input signal [5]. Appropriate derivations and their results are included in the extended version of this article. Assuming that each of sections in the parallel connection is described by a known [5] impulse response $h_i(t,\tau)$, $i=1,2,\ldots,n$, based on the mathematical derivations included in the extended article, one can carry out the formula describing the impulse response of the generalized section of I type:

$$h_z(t,\tau)=\sum_{i=1}^{n} h_i(t,\tau)\ . \qquad (3)$$

The impulse response of substitute sections is the sum of the impulse responses of elementary sections.

B. *Synthesis section of II type*

The connection shown in fig. 3 is not commutative [5]. Therefore, the number of the cascade connection structures of n sections is equal to the number of permutations of n elements (elementary sections). The reduction of the cascade connection can be obtained by the application of the successive method, by repeated blocking of two adjacent elementary sections. Independently of the section cascade connection point and the direction of the reduction, the final reduction result, the substitute section (compare fig. 3), must be described by an identical differential equation.

In the extended version of the article, the issues of the reduction of section connections have been considered for two cases:

– the connection of the first order LTV section with n-th order LTV section,
– the connection of the second order LTV section with n-th order LTV section.

Theoretical considerations and their results as well as the examples of the presented algorithms have been included in the extended version of this article. Assuming that each section in the considered connection is described by an impulse response $h_i(t,\tau)$, it can be proved that for the cascade connection of two sections numbered i, $i+1$, the substitute impulse response is expressed by the formula [4]:

$$h_z(t,\tau)=\int_{\tau}^{t} h_{i+1}(t,\tau_1)h_i(\tau_1,\tau)\mathrm{d}\tau_1 \qquad (4)$$

Using the formula (4) for the connection of two successive sections from fig. 3, the reduction of the cascade connection of n section can be reduced to a single substitute section.

IV. Conclusions

Connections of parametric sections asymptotically approach the substitute sections consisting of LTI systems. It results from the assumptions concerning the parametric functions. In transient states of parameter variability, the connection of non stationary systems allows to modify their properties. The significant property of LTV systems is the fact that the sequence in cascade connection is crucial.

V. References

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