# Space vector PWM for two-phase inverter with three legs in Matlab-Simulink 

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#### Abstract

: This paper deals with design of SVPWM for two-phase inverter in Matlab/Simulink. Nowadays is effort to reduce switching losses and increase efficiency of electrical devices. One of the many possibilities how to deal with this problem is to use inverter control technique called space vector pulse width modulation (SVPWM).


## INTRODUCTION

The single-phase induction motor with two unbalanced winding is widely used at the lower power level, especially in household applications where a three-phase supply is not available. Low power factor and also efficiency represent uncontrolled single phase induction motor. Single phase induction motor can be considered as two phase induction motor with main and auxiliary windings and it is possible to apply vector control of induction motor or any other control technique to increase its efficiency.
Voltage source inverter participates on control scheme and therefore it is necessary to control it. Space vector pulse width modulation gives great results on a field of control techniques. It is possible to switch voltage with variable frequency and magnitude which is great feature of it and it is also necessary for applying vector control. [1-4].

## SVPWM FOR TWO-PHASE INVERTER WITH THREE LEGS

One of the many options for creating two-phase orthogonal voltage system is two-phase three leg inverter (Fig. 1.). The advantage against two-phase two legs is in better utilization of the DC bus. Chyba! Nenalezen zdroj odkazů. shows space vectors in two-phase three leg inverter. Six active voltage (Fig. 2.) vectors and two zero vectors can be turned on. Active voltage vectors divide complex plane into six operating area. Maximal trajectory of the reference vector is $\mathrm{U}_{\mathrm{DC}} / 1.414$. [5-6]

Fig. 1: Two-phase VSI with three legs.



Fig. 2: Space vectors in two-phase VSI with three legs. For performing SVPWM it is necessary to determine length and angle of the reference vector. Length of the reference vector can be determined as avector sum of reference voltages [7-8].

$$
\begin{gather*}
\left|\bar{U}_{\operatorname{Re} f}\right|=\sqrt{U_{\alpha}^{2}+U_{\beta}^{2}}  \tag{1}\\
\theta=\tan ^{-1} \frac{U_{\beta}}{U_{\alpha}}=\omega t=2 \pi f t \tag{2}
\end{gather*}
$$

From the angle of the reference vector can be derived location of the reference vector. Fig. 3. shows simple algorithm of the sector determination where the reference voltage vector is located.


Fig. 3: Sector determine algorithm.
The switching operation is performed in one sample period $T_{S}$. It means that it is necessary to spend this time to turn on adjacent active voltage vectors to the reference vector. The first adjacent vector $\bar{V}_{1}$ must be turned on in time $\mathrm{T}_{1}$. The second adjacent vector $\bar{V}_{2}$ must be turned on in time $\mathrm{T}_{2}$ [9.].

$$
\begin{equation*}
T_{S} \bar{U}_{\operatorname{Re} f}=T_{1} \bar{V}_{1}+T_{2} \bar{V}_{2} \tag{3}
\end{equation*}
$$

Equation 3 is correct only if the length of the reference vector is equal to its maximal length. If the length of the reference vector is less than its maximal length, then the sum of times $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ is less than sample period $\mathrm{T}_{\mathrm{S}}$. In that case it is necessary to calculate difference between the sample period $\mathrm{T}_{\mathrm{S}}$ and sum of $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$. Let's call this difference T0 [5.].

$$
\begin{equation*}
T_{0}=T_{S}-\left(T_{1}+T_{2}\right) \tag{4}
\end{equation*}
$$

Voltage vectors $\bar{V}_{1}, \bar{V}_{2}$ and $\bar{V}_{\operatorname{Re} f}$ can be written as follows.

$$
\begin{gather*}
\bar{V}_{1}=U_{1} e^{j \theta_{1}}=  \tag{5}\\
U_{1}\left(\cos \theta_{1}+j \sin \theta_{1}\right) \\
\bar{V}_{2}=U_{2} e^{j \theta_{2}}=  \tag{6}\\
U_{2}\left(\cos \theta_{2}+j \sin \theta_{2}\right) \\
\bar{V}_{\operatorname{Re} f}=U_{\operatorname{Re} f} e^{j \theta_{\mathrm{Ref}}}=  \tag{7}\\
U_{\operatorname{Re} f}\left(\cos \theta_{\operatorname{Re} f}+j \sin \theta_{\operatorname{Re} f}\right)
\end{gather*}
$$

Zero voltage vectors have no role in creating of the reference voltage vector, thus:

$$
\begin{equation*}
T_{S} \bar{U}_{\operatorname{Re} f}=T_{1} \bar{V}_{1}+T_{2} \bar{V}_{2} \tag{8}
\end{equation*}
$$

Substituting equations (5. to 7.) in equation (8.) we get:
$T_{S}\left(U_{\operatorname{Re} f} \cos \theta_{\operatorname{Re} f}+j U_{\operatorname{Re} f} \sin \theta_{\operatorname{Re} f}\right)=$
$T_{1} U_{1} \cos \theta_{1}+j T_{1} U_{1} \sin \theta_{1}+$
$T_{2} U_{2} \cos \theta_{2}+j T_{2} U_{2} \sin \theta_{2}$
After separating real and imaginary parts and writing in matrix notation we get:
$T_{S}\left[\begin{array}{l}U_{\operatorname{Re} f} \cos \theta_{\operatorname{Re} f} \\ U_{\operatorname{Re} f} \sin \theta_{\operatorname{Re} f}\end{array}\right]=\left[\begin{array}{ll}U_{1} \cos \theta_{1} & U_{2} \cos \theta_{2} \\ U_{1} \sin \theta_{1} & U_{2} \sin \theta_{2}\end{array}\right]\left[\begin{array}{l}T_{1} \\ T_{2}\end{array}\right]$
From equation (10.) we can separate times $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$

$$
\left[\begin{array}{l}
T_{1}  \tag{11}\\
T_{2}
\end{array}\right]=T_{S}\left[\begin{array}{ll}
U_{1} \cos \theta_{1} & U_{2} \cos \theta_{2} \\
U_{1} \sin \theta_{1} & U_{2} \sin \theta_{2}
\end{array}\right]^{-1}\left[\begin{array}{l}
U_{\mathrm{Re} f} \cos \theta_{\mathrm{Re} f} \\
U_{\mathrm{Re} f} \sin \theta_{\mathrm{Re} f}
\end{array}\right]
$$

Therefore:

$$
\begin{align*}
& T_{1}=T_{S} \frac{U_{\operatorname{Re} f}}{U_{1}} \frac{\sin \left(\theta_{2}-\theta_{\operatorname{Re} f}\right)}{\sin \left(\theta_{2}-\theta_{1}\right)}  \tag{12}\\
& T_{2}=T_{S} \frac{U_{\operatorname{Re} f}}{U_{2}} \frac{\sin \left(\theta_{\operatorname{Re} f}-\theta_{1}\right)}{\sin \left(\theta_{2}-\theta_{1}\right)} \tag{13}
\end{align*}
$$

A value of $U_{1}, U_{2}, \theta_{1}$ and $\theta_{2}$ depends on operating area where reference vector is located.
Tab. 1: Parameters definition for active voltage vectors

| Sector | $\overline{\boldsymbol{V}}_{\mathbf{1}}$ | $\overline{\boldsymbol{V}}_{\mathbf{2}}$ | $\boldsymbol{U}_{\mathbf{1}}$ | $\boldsymbol{U}_{\mathbf{2}}$ | $\boldsymbol{\theta}_{\mathbf{1}}$ | $\boldsymbol{\theta}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bar{V}_{1}(1,0,0)$ | $\bar{V}_{2}(1,0,1)$ | 1 | $\sqrt{2}$ | $0^{\circ}$ | $45^{\circ}$ |
| 2 | $\bar{V}_{3}(0,0,1)$ | $\bar{V}_{2}(1,0,1)$ | 1 | $\sqrt{2}$ | $90^{\circ}$ | $45^{\circ}$ |
| 3 | $\bar{V}_{3}(0,0,1)$ | $\bar{V}_{4}(0,1,1)$ | 1 | 1 | $90^{\circ}$ | $180^{\circ}$ |
| 4 | $\bar{V}_{5}(0,1,0)$ | $\bar{V}_{4}(0,1,1)$ | $\sqrt{2}$ | 1 | $225^{\circ}$ | $180^{\circ}$ |
| 5 | $\bar{V}_{5}(0,1,0)$ | $\bar{V}_{6}(1,1,0)$ | $\sqrt{2}$ | 1 | $225^{\circ}$ | $270^{\circ}$ |
| 6 | $\bar{V}_{1}(1,0,0)$ | $\bar{V}_{6}(1,1,0)$ | 1 | 1 | $0^{\circ}$ | $280^{\circ}$ |

The Fig. 4. shows times $T_{1}, T_{2}$ and $T_{0}$ for active and zero vectors in one period.


Fig. 4: Times $T_{1}, T_{2} T_{0}$ as a function of time in one period.
If times $T_{1}, T_{2}$ and $T_{0}$ are known, active voltage vectors can be assigned to them and it is possible to
generate switching signals. There are many ways how to generate switching signals. In this case it is symmetrical switching sequence. For first sector it looks like:
Tab. 2: Parameters definition for active voltage vectors

| Secto <br> r | $\frac{T_{0}}{2}$ | $T_{1}$ | $T_{2}$ | $T_{0}$ | $T_{2}$ | $T_{1}$ | $\frac{T_{0}}{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{0}=(0,0,0)$ |
| 2 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{0}=(0,0,0)$ |
| 3 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{0}=(0,0,0) \mid$ |
| 4 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{0}=(0,0,0)$ |
| 5 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{0}=(0,0,0)$ |
| 6 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{0}=(0,0,0)$ |

Because of effort to reduction switching losses there is necessity to optimize switching sequence. Chyba! Nenalezen zdroj odkazů. shows the possibility how to realize the switching sequence for two-phase three leg inverter. As we can see, active vectors were chosen with consideration of switching states of

Tab. 3: Parameters definition for active voltage vectors

|  |  | Sector 1 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |


|  |  | Sector 3 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |

$$
\begin{aligned}
& \bar{V}_{0}(0,0,0)-\bar{V}_{1}(1,0,0)-\bar{V}_{2}(1,0,1)-\bar{V}_{7}(1,1,1) \\
& -\bar{V}_{2}(1,0,1)-\bar{V}_{1}(1,0,0)-\bar{V}_{0}(0,0,0)
\end{aligned}
$$

switches. There is always only one switch which changes its switching state.It is possible to derive switching pattern from Chyba! Nenalezen zdroj odkazů.. It is necessary to derive it only for upper switches, because lower switches switch complementary to upper.




From Chyba! Nenalezen zdroj odkazů. it is possible to derive duty cycles for upper switches of two phase three leg inverter. Lower switches switch complementary to upper, thus it is not necessary to determine duty cycles to lower switches of two-phase three leg inverter.


Fig. 5: Duty cycles for upper switches for one period.
Fig. 5. shows duty cycles for switches in one period. The blue curve shows duty cycle for switch S1. The red one shows duty cycle for switch $S_{5}$ and green curve shows duty cycle for switch $S_{3}$. Switch $S_{3}$ switches middle leg of inverter. The common point of TPIM is connected to this leg.
The simulation was performed with the simulation step of $1 \mathrm{e}-6$ and the sample time $\mathrm{T}_{\mathrm{S}}=0,0002 \mathrm{~s}$. Fig. 6. shows the stator current at start-up and in steady state. As a load of the inverter, asymmetrical two phase induction motor (TPIM) was used.
Tab. 4: Parameters of the TPIM

| $\begin{aligned} & \text { Rsd }= \\ & 61.3 \Omega \end{aligned}$ | $\begin{aligned} & \text { Rsq }= \\ & 68.8 \Omega \end{aligned}$ | Rrd $87.25 \Omega$ | $\begin{aligned} & \text { Rrq }=109.95 \\ & \Omega \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Lsd }= \\ & 1.154 \mathrm{H} \end{aligned}$ | $\begin{aligned} & \mathrm{Lsq}= \\ & 1.645 \mathrm{H} \end{aligned}$ | $\begin{aligned} & \text { Lrd }= \\ & 1.174 \mathrm{H} \end{aligned}$ | $\operatorname{Lrq}=1.665$ |
| $\begin{aligned} & \hline \text { Msrd }= \\ & 1.12 \mathrm{H} \end{aligned}$ | $\begin{aligned} & \text { Msrq }= \\ & 1.6 \mathrm{H} \end{aligned}$ | $\begin{aligned} & \mathrm{TN}=0.15 \\ & \mathrm{Nm} \end{aligned}$ | $\mathrm{np}=1$ |
| $\begin{aligned} & \mathrm{UN}=230 \\ & \mathrm{~V} \end{aligned}$ | $\mathrm{f}=50 \mathrm{~Hz}$ | $\begin{aligned} & \mathrm{nN}=2850 \\ & \mathrm{rpm} \end{aligned}$ | $\begin{aligned} & J=25.10-5 \\ & \text { Kg.m2 } \end{aligned}$ |


|  |  | Sector 6 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |



Fig. 6: Stator current at start-up and in steady state.
The simulink tool powergui offers option of Fast Fourier Transformation (FFT). With this tool it is possible to make FFT analysis of the stator currents and it can also compute THD (Total Harmonic Distortion) of the currents. For example stator current in phase $\beta$ has THD equal to value $6,33 \%$ (Fig. 7.).


Fig. 7: FFT analysis of stator current.

## CONCLUSION

The two-phase space vector PWM for the three leg inverter topologies is proposed in this paper. Principles of the SVPWM for three leg inverter with respect to switching reduction are also described. Switching reduction increase efficiency and this could be the main reason for using of the SPWM.

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