Evaluation of the linearization of mathematical models of AC magnetic circuits

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Abstract. **Present paper shows that even seemingly acceptable linearization of the mathematical model of harmonically excited nonlinear AC magnetic circuit may lead to appreciable errors.**

Keywords: **magnetic circuits, numerical methods, mathematical models, numerical errors, validity of models.**

I. Introduction

When a mathematical model of AC magnetic circuits of electrical machines and devices is built, its linearization is often done, i.e. the course of permeability $\mu = \mu(H)$ of the magnetic circuit is replaced by a constant value. The error caused by the linearization is intuitively considered as negligible. The solution of a mathematical model with linearization of permeability is much simpler, because harmonically complex representation of the time-variable waveforms is possible. Presented paper shows that primarily when magnetic heavily saturated magnetic circuits are modeled (mentioned devices are normally working under this condition), the linearization of magnetic properties may lead to totally inaccurate results (see also [2]).

II. Formulation of the Problem

To ensure good accuracy and reliability of the solution of an AC magnetic circuit, it is necessary to deal with the validity of the used mathematical model. We focus on a simple magnetic circuit, which consists of a ferrite jacket, solid cylinder of electrical steel and a coil of a thin wire made of copper (skin effect is negligible) with the current density $J(t) = J_0 \sin \omega t$. The magnetic field in the cylinder is calculated using both nonlinear model and by using the linearized model [1]. The results of both calculations are compared based on the values of Joule losses due to eddy currents in the cylinder.

III. Nonlinear Mathematical Model

The equation for the magnetic vector potential $\boldsymbol{A}$ is

$$\mathrm{rot}\frac{1}{\mu}\mathrm{rot}\,\boldsymbol{A} = -\gamma\frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{J}$$

We introduce the definition area and boundary conditions. Joule loses density caused by eddy current in the cylinder are

$$w_{\mathrm{J}}(r,z,t) = \gamma\left\{\mathrm{mod}\left(\frac{\partial A}{\partial t}\right)\right\}^2$$

IV. Linearized Mathematical Model

For the time-varying field in linearized surrounding is

$$\mathrm{rot\,rot}\,\boldsymbol{A} = -\gamma\mu\frac{\partial \boldsymbol{A}}{\partial t} + \mu\boldsymbol{J}$$

For time-harmonic variable magnetic field is

$$\mathrm{rot\,rot}\,\underline{\boldsymbol{A}} + \mathbf{k}^2\underline{\boldsymbol{A}} = \mu\underline{\boldsymbol{J}} \quad \mathbf{k} = -\mathrm{j}\,\omega\mu\gamma \quad \omega = 2\pi f$$

where $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{J}}$ are phasors of vectors $\boldsymbol{A}$, $\boldsymbol{J}$ and μ_s is usually set as the mean permeability $\mu = \mu(H)$.

Joule loses density in the cylinder are

$$w_{\mathrm{J}}(r,z) = \gamma\left\{\mathrm{mod}\left(-\mathrm{j}\omega\underline{A}\right)\right\}^2$$

V. Conclusion

In Fig.1 is a comparison of calculated Joule losses for nonlinear and the linearized model.

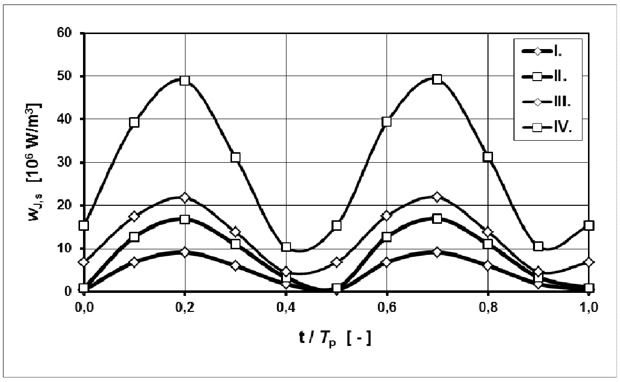

Fig. 1. Nonlinear model (I… $J = 2.10^6$ A/m^2, II… $J = 3.10^6$ A/m^2), linearized model (III… $J = 2.10^6$ A/m^2, IV… $J = 3.10^6$ A/m^2).

Although linearization allows calculating with the use of phasors, which significantly simplifies the numerical solu-tion, it can cause a large error in results. It is therefore reco-mmended to give proper attention to the question of the eligibility of the linearization of the mathematical model and in case of doubt (e.g. for larger values of the flux density), to give priority to the computationally more labo-rious, but more reliable nonlinear mathematical model.

VI. References

[1] Mayer D.: Applied electromagnetics. (In Czech) Edited by Kopp, České Budějovice, 2012, ISBBN 80-7232-5436-1.

[2] Roubicek T.: Bright future mathematical modeling. (In Czech), Vesmir, 91, 2012, No. 9, pp. 484-487.