Influence of BDF order on FEM computation

Marcin Sowa

Faculty of Electrical Engineering, Silesian University of Technology, Akademicka Street 10, Gliwice, Poland, e-mail: marcin.sowa@polsl.pl

Abstract **The influence of the time derivative formula and time adaptive predictor-corrector scheme on the solution accuracy and computation time is studied. An exemplary electromagnetic field problem is solved by the finite element method. Backward differentiation (BDF) formulae of higher order are applied.**

Keywords **backward differentiation formula, finite element method, predictor-corrector.**

I. INTRODUCTION

The paper presents an initial study of certain aspects that are applied when solving electromagnetic field problems with the finite element method. The influence of the time derivative approximation and the time-step adaptation on the computation time and the accuracy of the result is studied.

II. PROBLEM FORMULATION

An exemplary problem of the electromagnetic field is considered. The field distribution is described by the magnetic vector potential component differential equation:

$$\frac{\partial^2 A}{\partial x^2} - \mu\gamma \frac{\partial A}{\partial t} = 0. \qquad (1)$$

Neumann boundary conditions on $x = x_1$ and $x = x_2$ are imposed in the form of time-periodic functions:

$$\frac{\partial A}{\partial x} = a\sin(2\pi f_1 t) + 0.1a\sin(2\pi 9 f_1 t). \qquad (2)$$

III. TIME DERIVATIVE FORMULAE

For the approximation of the time derivative the BDF formulae [1] are used (**B**ackward **D**ifferentiation **F**ormula):

$$\left.\frac{\partial A}{\partial t}\right|_{t=t_m} = \left.\left(\sum_{i=0}^{n} \frac{\partial \ell_{m-(n-i)}}{\partial t} A_{m-(n-i)}\right)\right|_{t=t_m} \qquad (3)$$

where n denotes the order of the BDF (n = 1 means the backward Euler scheme), t_m is the time value for the current step, A_m is the component value for the current time step. ℓ_j is the Lagrange polynomial which is 1 at t_j and 0 at every other included time step.

An adaptive time-step technique is applied. The error is estimated by means of a predictor-corrector scheme where for a chosen n the predictor step uses the solution for $n - 1$ and the solution for n is applied in the corrector step. A relative maximum error e is calculated by means of differences between both solutions. If the error fits a chosen threshold $e_{\text{tol}} \in [e_{\min}, e_{\max}]$ then the next time step is calculated. If $e < e_{\min}$ then the time step value Δt is doubled. If $e > e_{\max}$ then Δt is divided by two and the calculations are repeated for the current time step.

IV. ANALYSIS

The main aspect that is studied is the choice of an efficient algorithm in the sense that computation requires less time and the solution accuracy is either improved or at least kept within a relatively close range.

An average relative error e_{r} has been computed for both frequencies (denoted later on by $e_{\text{r}1}$ and $e_{\text{r}9}$ respectively for f_1 and $9f_1$):

$$e_{\text{r}} = \overline{\left|\underline{A}_{i\,\text{num}} - \underline{A}_{i\,\text{an}}\right| / \left|\underline{A}_{i\,\text{an}}\right|} \cdot 100\%, \qquad (4)$$

where $\underline{A}_{i\,\text{num}}$ are the complex values obtained by applying an FFT algorithm on the numerical solution. $\underline{A}_{i\,\text{an}}$ are the values of the analytical solution. The error values and computation times are presented in Tables 1 and 2 respectively.

TABLE I
RELATIVE ERROR VALUES FOR VARIOUS n AND e_{tol} THRESHOLD

Relative error [%]			
$e_{\text{tol}} \in [0.1, 1]$	$n = 2$	$n = 3$	$n = 4$
$e_{\text{r}1}$	$9.62 \cdot 10^{-2}$	$7.61 \cdot 10^{-2}$	$9.62 \cdot 10^{-2}$
$e_{\text{r}9}$	4.76	7.37	6.96
$e_{\text{tol}} \in [0.01, 1]$	$n = 2$	$n = 3$	$n = 4$
$e_{\text{r}1}$	$2.15 \cdot 10^{-2}$	$2.79 \cdot 10^{-2}$	$2.29 \cdot 10^{-2}$
$e_{\text{r}9}$	1.36	3.19	1.90
$e_{\text{tol}} \in [0.01, 0.1]$	$n = 2$	$n = 3$	$n = 4$
$e_{\text{r}1}$	$1.69 \cdot 10^{-2}$	$7.85 \cdot 10^{-3}$	$1.75 \cdot 10^{-2}$
$e_{\text{r}9}$	$8.90 \cdot 10^{-1}$	1.04	1.35

TABLE II
COMPUTATION TIME FOR VARIOUS n AND e_{tol} THRESHOLD

Computation time [s]			
$e_{\text{tol}} \in [0.1, 1]$	$n = 2$	$n = 3$	$n = 4$
	121.53	112.06	94.84
$e_{\text{tol}} \in [0.01, 1]$	$n = 2$	$n = 3$	$n = 4$
	177.53	134.00	128.11
$e_{\text{tol}} \in [0.01, 0.1]$	$n = 2$	$n = 3$	$n = 4$
	215.46	182.88	161.30

V. CONCLUSION

The study shows that satisfactory accuracies can be obtained when using predictor-corrector schemes that apply backward differentiation formulae. An increase in the BDF order has allowed to compute the result in a shorter period of time while keeping a relatively close error value.

VI. REFERENCES

[1] Ovando-Martínez R. B. B., Arjona-López M. A., Hernández Flores C.: "A Finite-Element Variable Time-Stepping Algorithm for Solving the Electromagnetic Diffusion Equation", IEEE Transactions on Magnetics, Vol. 48, No. 2, February 2012.