Analysis of linear stochastic dynamic systems of the n–th order of the method of moments

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***Abstract* In the article, the linear dynamic random model of n–th order described by the random state equation, is considered. The method for determining the probabilistic characteristics of stochastic process which is the solution of that equation, is shown. These characteristics, such as average values and correlation functions, are determined by auxiliary solutions–deterministic systems of ordinary differential equations.**

***Keywords* moments of stochastic processes, n–th order random system.**

I. Introduction

Random dynamical systems are models of many systems occurring in electrical engineering and electronics [1]. They are usually described by the stochastic differential equations [3].

In the article is described the method of determining the average value and the correlation function of the stochastic processes that are solutions of linear random differential equations of n–th order. The presented results are a generalization of previous works [1], [3] concerning to this subject.

II. The formalization of the problem

A random dynamical system described by the state equation is given:

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{F}(t),\ \mathbf{X}(0) = \mathbf{X}_0, \tag{1}$$

where:

$\mathbf{A}, \mathbf{B}$ – deterministic matrices, $\mathbf{A} \in \mathbf{R}^{n\times n}, \mathbf{B} \in \mathbf{R}^{n\times m}$,
$\mathbf{X}_0$ – random vector of initial conditions,
$\mathbf{F}(t)$ – random vector of stochastic processes – excitations satisfying the mean-square Lipschitz condition [2],
$\mathbf{X}(t)$ – random vector which is the solution of (in mean-square sense) the equation (1).

To solve the equation (1) the method of moments is used [2].

III. Method of moments

Using the expected value operator $\mathrm{E}[\cdot]$ to the both sides of to equation (1) a deterministic system of differential equations in respect to the vector $\boldsymbol{\mu}_X(t)$ is obtained, which is the expected value of the process $\mathbf{X}(t)$:

$$\frac{d\boldsymbol{\mu}_X(t)}{dt} = \mathbf{A}\boldsymbol{\mu}_X(t) + \mathbf{B}\boldsymbol{\mu}_F(t),\ \boldsymbol{\mu}_X(0) = \boldsymbol{\mu}_{X0}, \tag{2}$$

where:

$$\boldsymbol{\mu}_F(t) = \mathrm{E}[\mathbf{F}(t)].$$

The system is solved by classical methods of mathematical analysis.

Performing the operations:

- bilateral transposition of the equation (1) and substitution $t = t_2$,
- multiply both sides of the equation by the process $\boldsymbol{\mu}_F(t_1)$,
- use the expected value operator to the result of the previous steep,

the deterministic equation system is obtained:

$$\begin{aligned} \frac{\partial \mathbf{R}_{FX}(t_1,t_2)}{\partial t_2} &= \mathbf{R}_{FX}(t_1,t_2)\mathbf{A}^{\mathrm{T}} + \mathbf{R}_F(t_1,t_2)\mathbf{B}^{\mathrm{T}} \\ \mathbf{R}_{FX}(t_1,0) &= \boldsymbol{\mu}_F(t_1)\boldsymbol{\mu}_{X_0}^{\mathrm{T}}, \end{aligned} \tag{3}$$

respect to the vector cross-correlation function of processes $\mathbf{X}(t)$, $\mathbf{F}(t)$. In this system, the variable t_1 is treated as a parameter. For a fixed value of the variable t_1, this system is solved by classical methods.

Using the described procedure, in a similar way one can get a deterministic system of ordinary differential equations in respect to the vector function $\mathrm{R}_X(t_1,t_2)$ which is the autocorrelation function of the process $\mathbf{X}(t)$:

$$\frac{\partial \mathbf{R}_X(t_1,t_2)}{\partial t_2} = \mathbf{R}_X(t_1,t_2)\mathbf{A}^{\mathrm{T}} + \mathbf{R}_{XF}(t_1,t_2)\mathbf{B}^{\mathrm{T}} \tag{4}$$

A method for constructing the initial condition for the system (4) as well as concrete examples of the determination of moments based on the equations (2), (3), (4) will be presented at the conference.

IV. Conclusion

The proposed method helps to avoid the problem of solving stochastic differential equation (1) in respect to unknown process $\mathbf{X}(t)$. Instead of that, one can solve the deterministic systems of differential equations in respect to moments of this process, which is much easier.

References

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