

# Seismic response of nuclear fuel assembly Z. Hlaváč<sup>a,\*</sup>, V. Zeman<sup>a</sup>

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#### Abstract

The paper deals with mathematical modelling and computer simulation of the seismic response of fuel assembly components. The seismic response is investigated by numerical integration method in time domain. The seismic excitation is given by two horizontal and one vertical synthetic accelerograms at the level of the pressure vessel seating. Dynamic response of the hexagonal type nuclear fuel assembly is caused by spatial motion of the support plates in the reactor core investigated on the reactor global model. The modal synthesis method with condensation is used for calculation of the fuel assembly component displacements and speeds on the level of the spacer grid cells.

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Keywords: seismic response, nuclear fuel assembly, modal synthesis method, condensation

## 1. Introduction

One of the basic requirements on operation conditions of the nuclear reactor is the feasible seismic response guarantee. Two basic approaches can be applied to seismic response determination. The stochastic approach [5] is based on statistical description of loading process and on the parameters of vibrating system. For the sake of simplicity, it is mostly supposed the stochasticity is solely due to the loading process, while the vibrating system is considered as a deterministic one. The deterministic approach is based on description of the seismic excitation in either analytical or digital form.

The seismic action is most often represented by the response spectrum in displacement, pseudo-velocity or pseudo-acceleration [1] expressed analytically as a function of the eigenfrequency and relative damping of a simple oscillator. The seismic response is calculated by the response spectrum method based on different combination of vibration mode contributions [8]. The specific method of response spectrum method, so called missing mass correction method, includes the high frequency rigid modes into the system response pseudostatically [4]. The seismic action in the digital form is represented by synthetic accelerograms corresponding to given response spectra generally for damping value 5 % for ground spectra and 2 % for floor spectra [1]. Both deterministic approaches require assemblage of the mathematical model of the reactor for frequency area up to about 50 Hz.

An assessment of nuclear fuel assemblies (FA) behaviour at standard and extreme operating conditions belongs to important safety and reliability audit. A significant part of FA assessment plays dynamic deformation and load of FA components especial of fuel rods (FR) and load-bearing skeleton (LS) (see Fig. 1). The beam type FA model used in seismic analyses of WWER type reactors [2] does not enable investigation of seismic deformations and load of FA

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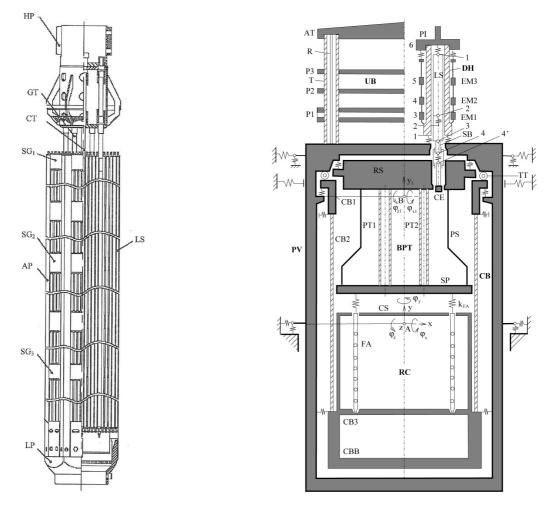


Fig. 1. Fuel assembly

Fig. 2. Reactor

components. The goal of this contribution, in direct sequence at an interpretation of FA modelling, modal analysis and calculation of dynamic response caused by pressure pulsation [12], is a presentation of the newly developed method for seismic analysis of FA components. The seismic displacements, velocities and deformations of the FA components on the level of spacer grids (SG) can be used for their stress analysis.

# 2. The seismic motion of the supporting plates

The original linearized mathematical model of the WWER 1000/320 type reactor intended for seismic response calculation was derived on the basis of computational (physical) model, whose structure is shown in Fig. 2. It was derived using the decomposition method [11]. The reactor was decomposed into eight subsystems [3, 11] — pressure vessel (PV), core barrel (CB) composed from two rigid bodies which are connected by beam-type continuum (CB2), reactor core (RC) formed from 163 FA, block of protection tubes (BPT), upper block (UP), system of 61 control rod drive housing (DH), system of 61 electromagnet blocks (EM) and system of 61 drive assemblies composed from a lifting system mechanism (LS) which ensures a suspension bar (SB) motion with the control elements (CE). The mass and static stiffness of the primary coolant loops between a reactor pressure vesel nozzles and steam generators were ap-

proximately replaced by mass points and springs placed in gravity centers of the nozzles. The components marked by grey in Fig. 2 were reflected as rigid bodies with six degrees of freedom excepted tope part of core barrel (CB1) having only three degrees of freedom with respect to pressure vessel. Other components are modelled as one-dimensional continua of beam types.

The mathematical model of the reactor after discretization of the one-dimensional continuums and a completion of the damping approximated by the proportional damping matrix B and of the seismic excitation has the form [2]

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{B}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = -\boldsymbol{m}_1\ddot{\boldsymbol{u}}_x(t) - \boldsymbol{m}_2\ddot{\boldsymbol{u}}_y(t) - \boldsymbol{m}_3\ddot{\boldsymbol{u}}_z(t), \qquad (1)$$

where components of the vector generalized coordinates q are relative displacements of carried subsystems with respect to supporting subsystems. So, for example, the supporting subsystem for CB, BPT, UB, DH is the pressure vessel (PV). Pressure vessel generalized coordinates are relative displacements with respect to basis. The seismic excitation is expressed by the synthetic accelerograms  $\ddot{u}_l$ , l = x, y, z of the reactor hall as basic in directions of axes x, y, z on the level of point A (see Fig. 2). The first three generalized coordinates of the pressure vessel and the whole reactor are relative translation displacements with respect to basis. That is why the vectors  $m_i$ , i = 1, 2, 3 are the first three columns of the reactor mass matrix M. The same horizontal accelerograms  $\ddot{u}_x(t)$ ,  $\ddot{u}_z(t)$  and one vertical accelerogram  $\ddot{u}_y(t)$ , given by Škoda Nuclear Machinery for NPP Temelín, are presented in Fig. 3 and Fig. 4, along with their power spectral densities.

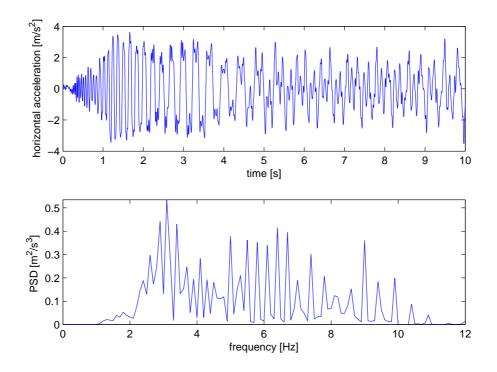
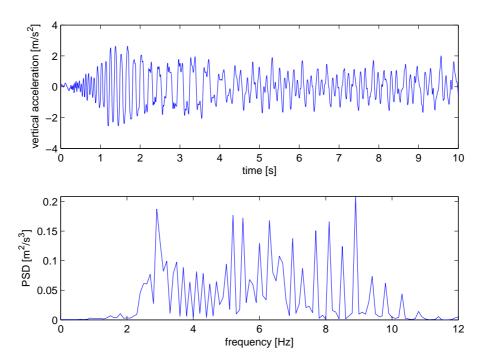


Fig. 3. Horizontal accelerogram

Each fuel assembly (see Fig. 1) is fixed by means of lower tailpiece (LP) into mounting plate in core barrel bottom and by means of head piece (HP) into lower supporting plate of the block of protection tubes. These support plates with pieces can be considered as rigid bodies.



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Fig. 4. Vertical accelerogram

Let use consider the spatial motion of the support plates described in coordinate systems  $x_X, y_X, z_X$  (X = L, U) with origins in plate gravity centres L, U by displacement vectors (see Fig. 5)

$$\boldsymbol{q}_X = [x_X, y_X, z_X, \varphi_{x,X}, \varphi_{y,X}, \varphi_{z,X}]^T, \ X = L, U.$$
(2)

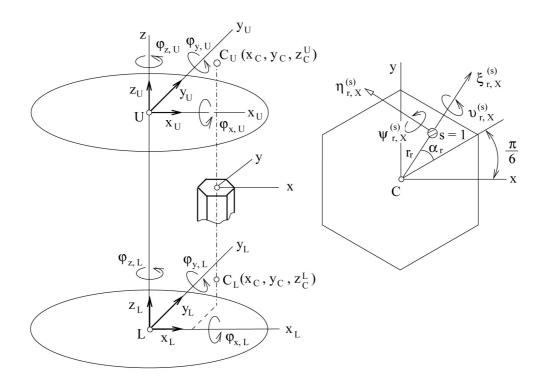


Fig. 5. Spatial motion of the FA support plates

The transformation relations between reactor generalized coordinates and absolute displacements of lower (L) and upper (U) FA supporting plates can be expressed in the global matrix form

$$\boldsymbol{q}_{X} = \boldsymbol{T}_{R,X}\boldsymbol{q} + \boldsymbol{u}(t), \quad \boldsymbol{T}_{R,X} \in R^{6,n_{R}}, X = L, U,$$
(3)

where  $n_R$  is reactor DOF number. The vector  $\boldsymbol{u}(t)$  of basis translational motion, with respect to different reactor and plates coordinate systems (see Fig. 2 and Fig. 5), is

$$\boldsymbol{u}(t) = [u_x(t), -u_z(t), u_y(t), 0, 0, 0]^T.$$
(4)

## 3. Condensed mathematical model of the fuel assembly

In order to model, the hexagonal type FA (Fig. 6) is divided into subsystems-six identical rod segments s = 1, ..., 6, centre tube (CT) and load-bearing skeleton (LS). Each rod segment of the TVSA-T FA (on Fig. 6 drawn in lateral FA cross section and circumscribed by triangles) is composed of 52 fuel rods with fixed bottom ends in lower piece (LP) and 3 guide thimbles (GT) fully restrained in lower and head pieces (HP). The fuel rods and guide thimbles are linked by transverse spacer grids g = 1, ..., 8 of three types (SG1-SG3) inside the segments. All FA components are modelled as one dimensional continuum of beam type with nodal points in the gravity centres of their cross-section on the level of the spacer grids. Mathematical models of six segments s = 1, ..., 6 are identical in consequence of radial  $\xi_{r,g}^{(s)}$  and orthogonal  $\eta_{r,g}^{(s)}$  fuel rods and guide thimbles lateral displacements and bending angles  $\vartheta_{r,g}^{(s)}$ ,  $\psi_{r,g}^{(s)}$  around these lateral displacements on the level of spacer grid g (in the Fig. 5 on the level fixed ends).

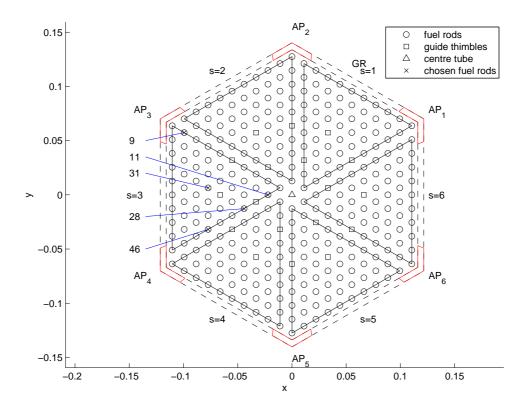


Fig. 6. The FA cross-section

The FA mathematical model was derived in the configuration space [3, 11]

$$\boldsymbol{q} = [\boldsymbol{q}_1^T, \dots, \boldsymbol{q}_s^T, \dots, \boldsymbol{q}_6^T, \boldsymbol{q}_{CT}^T, \boldsymbol{q}_{LS}^T]^T$$
(5)

corresponding to FA decomposition. The vector *s* of generalized coordinates of each subsystem (rod segments, centre tube) losed in kinematically excited nodes fixed into lower and upper supporting plate can be partitioned in the form

$$\boldsymbol{q}_{s} = [(\boldsymbol{q}_{L}^{(s)})^{T}, (\boldsymbol{q}_{F}^{(s)})^{T}, (\boldsymbol{q}_{U}^{(s)})^{T}]^{T}, s = 1, \dots, 6, CT$$
(6)

and the skeleton s = LS fixed in bottom ends only has the form

$$\boldsymbol{q}_{LS} = [(\boldsymbol{q}_{L}^{(LS)})^{T}, (\boldsymbol{q}_{F}^{(LS)})^{T}]^{T}.$$
(7)

The displacements of free system nodes (uncoupled with support plates) are integrated in vectors  $q_F^{(s)} \in \mathbb{R}^{n_s}$ . The conservative mathematical models of the loosed subsystems in the decomposed block form corresponding to partitioned vectors can be written as

$$\begin{bmatrix} \boldsymbol{M}_{L}^{(s)} & \boldsymbol{M}_{L,F}^{(s)} & \boldsymbol{0} \\ \boldsymbol{M}_{F,L}^{(s)} & \boldsymbol{M}_{F}^{(s)} & \boldsymbol{M}_{F,U}^{(s)} \\ \boldsymbol{0} & \boldsymbol{M}_{U,F}^{(s)} & \boldsymbol{M}_{U}^{(s)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_{L}^{(s)} \\ \ddot{\boldsymbol{q}}_{F}^{(s)} \\ \ddot{\boldsymbol{q}}_{U}^{(s)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{L}^{(s)} & \boldsymbol{K}_{L,F}^{(s)} & \boldsymbol{0} \\ \boldsymbol{K}_{F,L}^{(s)} & \boldsymbol{K}_{F}^{(s)} & \boldsymbol{K}_{F,U}^{(s)} \\ \boldsymbol{0} & \boldsymbol{K}_{U,F}^{(s)} & \boldsymbol{K}_{U}^{(s)} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_{L}^{(s)} \\ \boldsymbol{q}_{F}^{(s)} \\ \boldsymbol{q}_{U}^{(s)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{L}^{(s)} \\ \boldsymbol{f}_{C}^{(s)} \\ \boldsymbol{f}_{U}^{(s)} \end{bmatrix}$$
(8)

for the  $s = 1, \ldots, 6, CT$  and for the skeleton as

$$\begin{bmatrix} \boldsymbol{M}_{L}^{(LS)} \ \boldsymbol{M}_{L,F}^{(LS)} \\ \boldsymbol{M}_{F,L}^{(LS)} \ \boldsymbol{M}_{F}^{(LS)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_{L}^{(LS)} \\ \ddot{\boldsymbol{q}}_{F}^{(LS)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{L}^{(LS)} \ \boldsymbol{K}_{L,F}^{(LS)} \\ \boldsymbol{K}_{F,L}^{(LS)} \ \boldsymbol{K}_{F}^{(LS)} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_{L}^{(LS)} \\ \boldsymbol{q}_{F}^{(LS)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{L}^{(LS)} \\ \boldsymbol{f}_{C}^{(LS)} \end{bmatrix}, \quad (9)$$

where letters M(K) correspond to mass (stiffness) submatrices of the subsystems. The force subvectors  $f_C^{(s)}$  express the coupling forces between subsystem *s* and adjacent subsystems transmitted by spacer grids. The second set of equations extracted from (8) and (9) for each subsystem  $s = 1, \ldots, 6, CT, LS$  is

$$\boldsymbol{M}_{F}^{(s)} \ddot{\boldsymbol{q}}_{F}^{(s)} + \boldsymbol{K}_{F}^{(s)} \boldsymbol{q}_{F}^{(s)} = -\boldsymbol{M}_{F,L}^{(s)} \ddot{\boldsymbol{q}}_{L}^{(s)} - \boldsymbol{M}_{F,U}^{(s)} \ddot{\boldsymbol{q}}_{U}^{(s)} - \boldsymbol{K}_{F,L}^{(s)} \boldsymbol{q}_{L}^{(s)} - \boldsymbol{K}_{F,U}^{(s)} \boldsymbol{q}_{U}^{(s)} + \boldsymbol{f}_{C}^{(s)}, \quad (10)$$

where for the skeleton (LS) is  $M_{F,U}^{(LS)} = 0$ ,  $K_{F,U}^{(LS)} = 0$  because the skeleton is fixed only with lower supporting plate.

Displacements and accelerations of the all kinematically excited nodes of the subsystems can be expressed by the displacements and accelerations of the lower (X = L) and upper (X = U) supporting plates as

$$\boldsymbol{q}_{X}^{(s)} = \boldsymbol{T}_{X}^{(s)} \boldsymbol{q}_{X}, \quad \ddot{\boldsymbol{q}}_{X}^{(s)} = \boldsymbol{T}_{X}^{(s)} \ddot{\boldsymbol{q}}_{X}, \quad X = L, U.$$
(11)

The transformation matrices  $T_X^{(s)}$  depend on the FA position in the reactor core.

The global model of the fuel assembly has too large DOF number for calculation of dynamic response excited by support plate motion. Therefore, we assemble the condensed model using the modal synthesis method presented in the paper [10]. Let the modal properties of the conservative models of the mutually uncoupled subsystems with the strengthened end-nodes coupled

with immovable support plates be characterized by spectral  $\Lambda_s$  and modal  $V_s$  matrices of order  $n_s$ , suitable to orthonormality conditions

$$\boldsymbol{V}_{s}^{T}\boldsymbol{M}_{F}^{(s)}\boldsymbol{V}_{s} = \boldsymbol{E}, \quad \boldsymbol{V}_{s}^{T}\boldsymbol{K}_{F}^{(s)}\boldsymbol{V}_{s} = \boldsymbol{\Lambda}_{s}, \quad s = 1, \dots, 6, CT, LS.$$
(12)

The vectors  $q_F^{(s)}$  of dimension  $n_s$ , corresponding to free nodes of subsystems, can be approximately transformed in the form

$$\boldsymbol{q}_{F}^{(s)} = {}^{m}\boldsymbol{V}_{s}\boldsymbol{x}_{s} , \ \boldsymbol{x}_{s} \in R^{m_{s}}, \quad s = 1, \dots, 6, CT, LS,$$
(13)

where  ${}^{m}V_{s} \in R^{n_{s},m_{s}}$  are modal submatrices composed from chosen  $m_{s}$  master eigenvectors of fixed subsystems. The equations (10) can be rewritten using (11) and (13) in the form

$$\ddot{\boldsymbol{x}}_{s}(t) + {}^{m}\boldsymbol{\Lambda}_{s}\boldsymbol{x}_{s}(t) = -{}^{m}\boldsymbol{V}_{s}^{T}\sum_{X=L,U} [\boldsymbol{M}_{F,X}^{(s)}\boldsymbol{T}_{X}^{(s)}\ddot{\boldsymbol{q}}_{X} + \boldsymbol{K}_{F,X}^{(s)}\boldsymbol{T}_{X}^{(s)}\boldsymbol{q}_{X}] + {}^{m}\boldsymbol{V}_{s}^{T}\boldsymbol{f}_{C}^{(s)}, \qquad (14)$$

where spectral submatrices  ${}^{m}\Lambda_{s} \in R^{m_{s},m_{s}}$  correspond to chosen master eigenvectors in matrix  ${}^{m}V_{s}$ . The models (14) of all subsystems can be written in the global configuration space  $\boldsymbol{x} = [\boldsymbol{x}_{s}], s = 1, \ldots, 6, CT, LS$  of dimension  $m = \sum_{s} m_{s} = 6m_{s} + m_{CT} + m_{LS}$ 

$$\ddot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V}) \boldsymbol{x}(t) = -\boldsymbol{V}^T \sum_{X=L,U} \left[ \boldsymbol{M}_X \ddot{\boldsymbol{Q}}_X(t) + \boldsymbol{K}_X \boldsymbol{Q}_X(t) \right], \quad (15)$$

where global vector  $\mathbf{f}_C = [\mathbf{f}_C^{(s)}]$  of coupling forces between subsystems was expressed by means of the stiffness matric  $\mathbf{K}_C$  in the form  $\mathbf{f}_C = -\mathbf{K}_C \mathbf{q}_F$ ,  $\mathbf{q}_F = [\mathbf{q}_F^{(s)}] \in \mathbb{R}^n$ ,  $n = \sum n_s = 6n_s + n_{CT} + n_{LS}$  [3]. In the matrix equation (15), we introduced the block diagonal global matrices

$$\boldsymbol{\Lambda} = \operatorname{diag}[{}^{m}\boldsymbol{\Lambda}_{s}] \in R^{m,m}, \, \boldsymbol{V} = \operatorname{diag}[{}^{m}\boldsymbol{V}_{s}] \in R^{n,m},$$
$$\boldsymbol{M}_{X} = \operatorname{diag}[\boldsymbol{M}_{F,X}^{(s)}\boldsymbol{T}_{X}^{(s)}], \, \boldsymbol{K}_{X} = \operatorname{diag}[\boldsymbol{K}_{F,X}\boldsymbol{T}_{X}^{(s)}] \in R^{n,48}, \, X = L, U$$

and the global vectors

$$\boldsymbol{Q}_X(t) = [\boldsymbol{q}_X^T, \dots, \boldsymbol{q}_X^T]^T, \ \ddot{\boldsymbol{Q}}_X(t) = [\ddot{\boldsymbol{q}}_X^T, \dots, \ddot{\boldsymbol{q}}_X^T]^T \in R^{48}, \ X = L, U$$

describing the kinematical excitation given by FA supporting plates motion. These vectors  $Q_X(t)$ ,  $\ddot{Q}_X(t)$  are assembled, as a result of eight FA subsystems, from eight times repeating support plate displacement and acceleration vectors.

In consequence of slightly damped FA components we consider modal damping of the subsystems characterized in the space of modal coordinates  $\boldsymbol{x}_s$  by diagonal matrices  $\boldsymbol{D}_s = \text{diag}[2D_{\nu}^{(s)}\Omega_{\nu}^{(s)}]$ , where  $D_{\nu}^{(s)}$  are damping factors of natural modes and  $\Omega_{\nu}^{(s)}$  are eigenfrequencies of the mutually uncoupled subsystems. The damping of spacer grids can be approximately expressed by damping matrix  $\boldsymbol{B}_C = \beta \boldsymbol{K}_C$  proportional to stiffness matrix  $\boldsymbol{K}_C$ . The conservative condensed model (15) can be completed in the form

$$\ddot{\boldsymbol{x}}(t) + (\boldsymbol{D} + \beta \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V}) \dot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V}) \boldsymbol{x}(t) = -\boldsymbol{V}^T \sum_{X=L,U} [\boldsymbol{M}_X \ddot{\boldsymbol{Q}}_X(t) + \boldsymbol{K}_X \boldsymbol{Q}_X(t)],$$
(16)

where  $\boldsymbol{D} = \text{diag}[\boldsymbol{D}_s]$ .

## 4. Seismic response of the fuel assembly components

The FA seismic response in modal coordinates x(t) can be investigated by integration of motion equations (16) in time domain transformed into 2m differential equations of the first order

$$\dot{\boldsymbol{z}}(t) = \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{y}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}(t) \\ -\widetilde{\boldsymbol{B}}\boldsymbol{y}(t) - \widetilde{\boldsymbol{K}}\boldsymbol{x}(t) + \widetilde{\boldsymbol{f}}(t) \end{bmatrix},$$
(17)

where corresponding to (16)

$$\widetilde{\boldsymbol{B}} = \boldsymbol{D} + \beta \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V}, \quad \widetilde{\boldsymbol{K}} = \boldsymbol{\Lambda} + \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V}, \quad \widetilde{\boldsymbol{f}}(t) = -\boldsymbol{V}^T \sum_{X=L,U} [\boldsymbol{M}_X \ddot{\boldsymbol{Q}}_X(t) + \boldsymbol{K}_X \boldsymbol{Q}_X(t)].$$

These equations are solved at zero initial conditions  $\boldsymbol{x}(0) = \boldsymbol{0}, \boldsymbol{y}(0) = \dot{\boldsymbol{x}}(0) = \boldsymbol{0}$  using standard software ODE in MATLAB code. We then calculate the displacements and velocities of the free subsystem nodes according to (13)

$$\boldsymbol{q}_{F}^{(s)}(t) = {}^{m}\boldsymbol{V}_{s}\boldsymbol{x}_{s}(t), \quad \dot{\boldsymbol{q}}_{F}^{(s)}(t) = {}^{m}\boldsymbol{V}_{s}\dot{\boldsymbol{x}}_{s}(t)$$
(18)

for selected subsystem  $s = 1, \ldots, 6, CT, LS$ .

The components of the rod segment vectors  $\boldsymbol{q}_{F}^{(s)}$  ( $s = 1, \ldots, 6$ ) defined in (5) are absolute lateral displacements  $\xi_{r,g}^{(s)}$ ,  $\eta_{r,g}^{(s)}$  and bending angles  $\vartheta_{r,g}^{(s)}$ ,  $\psi_{r,g}^{(s)}$  of the fuel rod r cross-section [3] in segment s on the level of spacer grid g (see Fig. 1 and Fig. 5). Corresponding lateral displacements  $\overline{\xi}_{r,g}^{(s)}$ ,  $\overline{\eta}_{r,g}^{(s)}$  of the non-deformed fuel rods can be expressed by means of the lower supporting plate motion defined in (3) in the form

$$\begin{bmatrix} \overline{\xi}_{r,g} \\ \overline{\eta}_{r,g}^{(s)} \end{bmatrix} = \begin{bmatrix} C_r^{(s)} & S_r^{(s)} & 0 & -z_g S_r^{(s)} & z_g C_r^{(s)} & x_C S_r^{(s)} - y_C C_r^{(s)} \\ -S_R^{(s)} & C_r^{(s)} & 0 & -z_g C_r^{(s)} & -z_g C_R^{(s)} & x_C C_r^{(s)} + y_C S_r^{(s)} + r_r \end{bmatrix} \boldsymbol{q}_L, \quad (19)$$

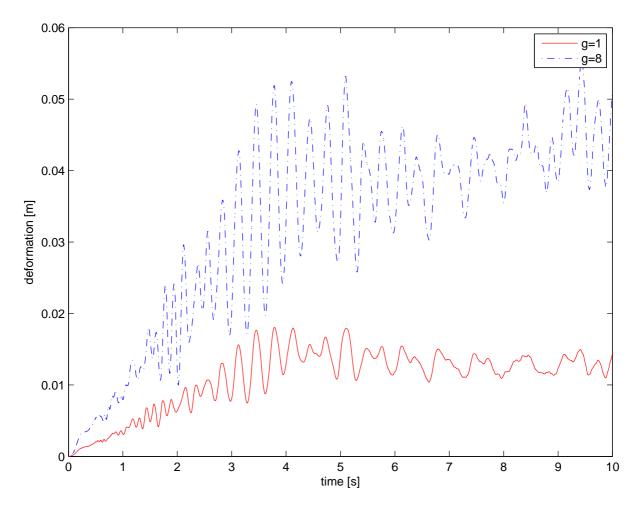
where

$$C_r^{(s)} = \cos\left[\alpha_r + \frac{\pi}{6} + (s-1)\frac{\pi}{3}\right], \quad S_r^{(s)} = \sin\left[\alpha_r + \frac{\pi}{6} + (s-1)\frac{\pi}{3}\right],$$
  
r = 1,...,55; s = 1,...,6,

 $x_C$ ,  $y_C$  are coordinates of the FA centre  $C_L$  in the reactor core,  $z_g$  is vertical coordinate of the spacer grid g in  $x_L$ ,  $y_L$ ,  $z_L$  and  $r_r$ ,  $\alpha_r$  are polar coordinates of the selected fuel rod r in segment s. The fuel rod lateral deformations on the level of spacer grid g are

$$d_{r,g}^{(s)} = \sqrt{(\xi_{r,g}^{(s)} - \overline{\xi}_{r,g}^{(s)})^2 + (\eta_{r,g}^{(s)} - \overline{\eta}_{r,g}^{(s)})^2}, \quad r = 1, \dots, 55; \quad g = 1, \dots, 8; \quad s = 1, \dots, 6.$$
(20)

As an illustration, the time behaviour of lateral deformations of the chosen fuel rod r = 31 in segment s = 3 on the level of the lower (for g = 1) and upper (for g = 8) spacer grid of the FA outside in the WWER 1000 reactor core ( $x_C = 0.59$ ,  $y_C = 1.431$  [m]) is presented in Fig. 7. The condensed FA model (16) with 960 DOF ( $m_s = 150$ ,  $m_{CT} = 20$ ,  $m_{LS} = 40$ ) was used for numerical integration of equations (17).



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Fig. 7. Lateral seismic deformations of fuel rod

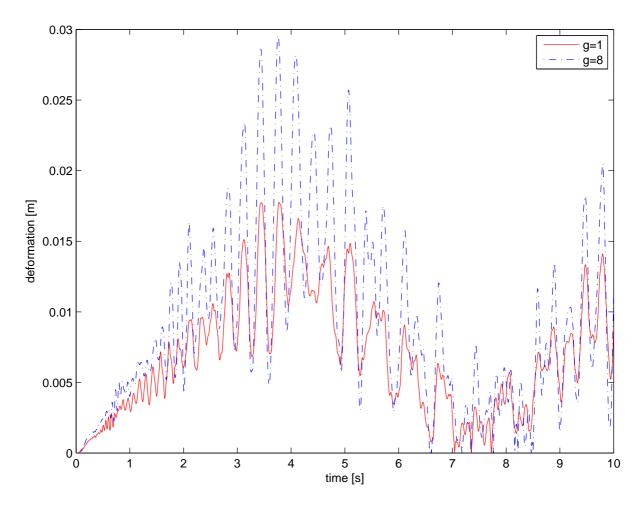
The components of the load-bearing skeleton vector  $\boldsymbol{q}_{LS}$  defined in (5) are absolute lateral displacements  $\xi_{AP,g}^{(s)}$ ,  $\eta_{AP,g}^{(s)}$  of centre of gravity, torsional angles  $\varphi_{AP,g}^{(s)}$  and bending angles  $\vartheta_{AP,g}^{(s)}$ ,  $\psi_{AP,g}^{(s)}$  of the angle pieces  $AP_s$ ,  $s = 1, \ldots, 6$  cross-section [3] on the level of spacer grid g (see Fig. 1 and Fig. 6). Corresponding lateral displacements  $\overline{\xi}_{AP,g}^{(s)}$  in radial direction of the non-deformed angle pieces can be expressed similar as for fuel rods by means of lower supporting plate motion defined in (3) as

$$\overline{\xi}_{AP,g}^{(s)} = [C^{(s)}, S^{(s)}, 0, -z_g S^{(s)}, z_g C^{(s)}, x_C S^{(s)} - y_C C^{(s)}] \boldsymbol{q}_L , \qquad (21)$$

where new  $C^{(s)} = \cos\left[\frac{\pi}{6} + (s-1)\frac{\pi}{3}\right]$ ,  $S^{(s)} = \sin\left[\frac{\pi}{6} + (s-1)\frac{\pi}{3}\right]$ . The other quantity sense is same as in (19). The angle pieces lateral deformations in radial direction on the level of spacer grid g are

$$d_{AP,g}^{(s)} = |\xi_{AP,g}^{(s)} - \overline{\xi}_{AP,g}^{(s)}|, \quad s = 1, \dots; \quad g = 1, \dots, 8.$$
(22)

As an illustration, the time behaviour of lateral deformations of the chosen angle piece s = 3 on the level of spacer grids g = 1, 8 of the identical FA as in case of chosen fuel rod is presented in Fig. 8.



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Fig. 8. Lateral seismic deformations of angle piece

Maximum lateral seismic deformations of fuel rods and angle pieces, in consequence of linking of these component with lower supporting plate, are on the highest level of spacer grids (g = 8). Outer radial deformations of the fuel assembly are determined by radial deformations of the angle pieces. The same condensed mathematical model was applied.

On the basis of lateral deformations on the level of all spacer grids we can relatively easily calculate the maximum stress of fuel assembly components excited by seismic events. The software developed in MATLAB code according to presented method makes possible to study an influence of the fuel assembly and reactor design parameters on seismic deformations of fuel assembly components. The publications dealing with seismic response of fuel assembly components have not been seen yet. The strength criterions of the fuel rods in a general form are presented in [7]. There are many procedures that can be used in the seismic engineering with the aim to mitigate the earthquake impacts [6]. Their description is outside a framework of this paper. The same condensed mathematical model was applied in this work.

## 5. Conclusion

The described method based on mathematical modelling and computer simulation of vibrations in time domain enables to investigate the seismic deformations of all nuclear fuel assembly components. The fuel assembly seismic vibrations are caused by spatial motion of the two

horizontal supporting plates an the reactor core transformed into displacements and accelerations of the kinematically excited nodes of the fuel assembly components — fuel rods, guide thimbles, centre tube and skeleton angle pieces. The seismic motion of the supporting plates is investigated by numerical integration method in the previous analysis stage at the global reactor model whereof fuel assemblies are replaced by one dimensional continuums of beam type. Seismic excitation is described by synthetic accelerograms of the reactor hall translation motion on the level of the reactor pressure vessel seating.

The fuel assembly mathematical model has, in consequence of great number of fuel rods, too large DOF number for calculation of seismic response. Therefore, it is compiled fuel assembly condensed model based on reduction of the subsystems eigenvectors conducive to seismic response by modal synthesis method.

The developed software in MATLAB is conceived in such a way that enables to choose an arbitrary fuel assembly component — fuel rod, guide thimble, centre tube or angle piece of load-bearing skeleton — for calculation its deformation on the level of spacer grids. The presented method was applied for the Russian TVSA-T fuel assembly in the WWER 1000/320 type reactor in NPP Temelín.

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