

# A New Algorithm for Pyramidal Clipping of Line Segments in $E^3$

Vaclav Skala<sup>1</sup>, Duc Huy Bui  
Department of Informatics and Computer Science<sup>2</sup>  
University of West Bohemia  
Univerzitni 22, Box 314  
306 14 Plzen  
Czech Republic

{skala | bui} @kiv.zcu.cz

<http://iason.zcu.cz/~skala> | [~bui](http://iason.zcu.cz/~bui)

## Abstract

A new algorithm for clipping a line segment against a pyramid in  $E^3$  is presented. This algorithm avoids computation of intersection points which are not end-points of the output line segment. It also allows solving all cases more effectively. The performance of this algorithm is shown to be consistently better than existing algorithms, including the Cohen-Sutherland, Liang-Barsky and Cyrus-Beck algorithms.

**Keywords:** Line Clipping, Computer Graphics, Algorithm Complexity, Geometric Algorithms, Algorithm Complexity Analysis.

## 1. Introduction

Let us assume that two points  $A(x_A, y_A, z_A)$  and  $B(x_B, y_B, z_B)$  are given and we wish to compute the intersection of the line segment  $AB$  with the unitary clipping pyramid, defined as the set of all points  $(x, y, z)$  such that  $-z \leq x \leq z$  and  $-z \leq y \leq z$  ( $z \geq 0$ ). The intersection is either empty or a line segment whose end-points we must compute.

Many algorithms for clipping a line or a line segment in  $E^3$  have been published, see standard textbooks (Cyrus and Beck 1978; Liang and Barsky 1983-1984; Foley, van Dam, Feiner and Hughes 1990; Skala 1996; Skala 1997) for main references. For a long time the Cohen-Sutherland algorithm (CS) and its extensions to  $E^3$  (Foley, van Dam,

Feiner and Hughes 1990) were the only line segment clipping algorithms found in most textbooks. Later, the Liang-Barsky (LB) and Cyrus-Beck (CB) algorithms were proposed and are based on the line parametric representation.

Before describing the proposed algorithms for line segment clipping, it is necessary to unify some definitions.

## 2. Definitions

The planes  $x = -z$ ,  $x = z$ ,  $y = -z$  and  $y = z$  are called the right, left, bottom and top boundaries of the unitary pyramid, respectively. We will say that:

- a point or a line segment is visible, if it lies entirely inside the given pyramid,
- a point or a line segment is invisible, if it lies entirely outside the given pyramid,
- a line segment is partially visible, if it lies partly inside the given pyramid and partly outside.

If a line segment is invisible, then no part of the line segment appears in the output, and the line segment is said to be rejected by the clipping algorithm.

The boundaries of the pyramid divide the Cartesian positive half-space ( $z \geq 0$ ) into 9 regions. Regions which are bounded by only two boundaries are called the corner regions and regions which are bounded by three boundaries are called the edge regions, see Figure 1.

## 3. Proposed Pyramidal clipping algorithm

<sup>1</sup> Affiliated with the Multimedia Technology Research Centre, University of Bath, U.K.

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The proposed Pyramidal clipping algorithm (PC) uses a similar approach as Nicholl-Lee-Nicholl algorithm (Nicholl, Lee and Nicholl 1987) that was derived for  $E^2$  case only. Given a line segment  $AB$ , for simplicity we assume that both points  $A$  and  $B$  are in the positive half-space. The end-point  $A$  therefore can lie inside of the pyramid, in an edge region or in a corner region. For each of these cases, we can divide the positive half-space into certain number of sub-regions, see Figures 2-4. These sub-regions are bounded by the pyramid's boundaries and planes determined by point  $A$  and one edge of pyramid. These planes will be denoted  $\rho_1, \rho_2, \rho_3, \rho_4$ , clockwise from the top-left edge, see Figures 2-4. All other cases can be obtained from one of these cases in Figures 2-4 by rotating the scene around the  $z$  axis.

With the above definitions the algorithm can be described by the following basic steps:

- characterize the location of point  $A$  among the 9 regions,
- characterize the location of point  $B$  among the appropriate sub-regions,
- compute the intersection points according to above characterization.

The main advantage of this approach is that we can determine which pyramid boundaries are intersected and therefore avoid unnecessary computation of invalid intersection points.

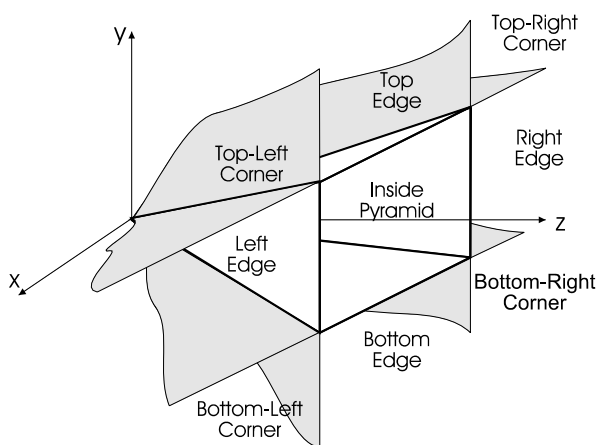


Figure 1: Subdivision of the positive half-space into regions.

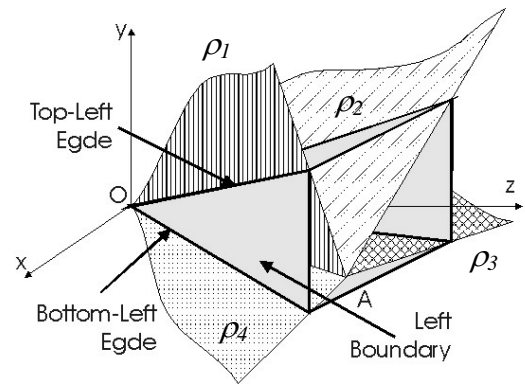


Figure 2: Sub-regions for the case when point  $A$  lies inside the pyramid.

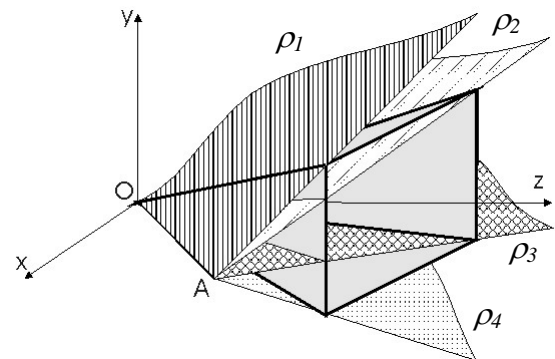


Figure 3: Sub-regions for the case when point  $A$  is in left edge region.

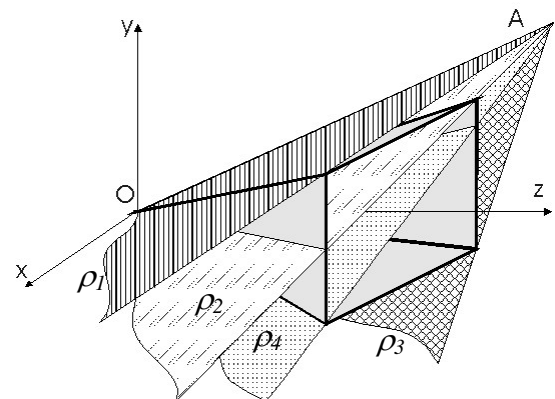


Figure 4: Sub-regions for the case when point  $A$  is in top-right corner region.

The proposed PC algorithm will be explained in more detail by the top-down approach. At the beginning we must determine whether the end-point  $A$  of the given line segment is beyond the right boundary, beyond the left boundary or

between those two boundaries. The main procedure, in Pascal-like code is:

```

procedure Clip(  $x_A, y_A, z_A, x_B, y_B, z_B$ : real);
begin
  if  $x_A < -z_A$  then case_I
    {point A is beyond right boundary}
  else if  $x_A \leq z_A$  then case_II
    {point A is between 2 boundaries}
  else case_III
    {point A is beyond left boundary}
end;

```

We will use the EXIT command in the following parts of the algorithm to denote the end of the procedure and to avoid “else if” sequences unnecessary for the algorithm explanation.

### *I. Point A is beyond the right boundary.*

If point  $B$  is also beyond the right boundary, it is not necessary to further characterize point  $A$ , because the line segment is invisible. Therefore, we must check whether point  $B$  is beyond the right boundary before proceeding on. After that, we must test whether point  $A$  lies either in the corner region or in the edge region. This section of algorithm can be implemented as follows:

```

begin {case_I}
  if  $x_B < -z_B$  then
    EXIT; {Line segment is rejected}
  if  $y_A > z_A$  then case_I_1
    {point A is in the top-right
     corner region}
  else if  $y_A \geq -z_A$  then case_I_2
    {point A is in the right edge
     region}
  else case_I_3
    {point A is in the bottom-right
     corner region}
end {case_I};

```

#### *I.1. Point A is in the top-right corner region and point B is not beyond the right boundary.*

If point  $B$  is above the top boundary, the line segment is invisible and no further computation is needed. Therefore, we need to

check this condition first, and then characterize point  $B$  so that we can distinguish between the case when point  $B$  is beyond the left boundary and the case when point  $B$  is inside of the pyramid or in the bottom edge region, see Figure 4. The following pseudo-code shows how it can be implemented:

```

begin {case_I_1}
  if  $y_B > z_B$  then
    EXIT; {Line segment is rejected}
  if  $x_B > z_B$  then case_I_1_a
    {point B is in the left edge or in the
     bottom-left corner region}
  else case_I_1_b
    {point B is inside of the pyramid or
     in the bottom edge region}
end {case_I_1};

```

#### *I.1.a) Point A is in the top-right corner region and point B is in the left edge region or in the bottom-left corner region.*

If point  $B$  is above the plane  $\rho_1$ , the line segment is rejected, see Figure 4. Therefore, we must check this condition first, and then distinguish the case when point  $B$  is in the left edge region and the case when point  $B$  is in the bottom-left corner region. In the case of the left edge region, one intersection point lies on the pyramid’s left boundary. The location of point  $B$  against the plane  $\rho_2$  will specify, that the second intersection point lies on the top or on the right boundary. By this way only the appropriate intersection point is computed. In the case of bottom-left corner region, the comparison of point  $B$  against the plane  $\rho_3$  is performed first to eliminate the case when the line segment is rejected. After that, we compare the position of point  $B$  against the plane  $\rho_4$  to determine location of the first intersection point (on the bottom or on the left boundary). At the end, a similar comparison with the plane  $\rho_2$  is performed to determine the second intersection point. An implementation can be as follows:

```

begin {case_I_1_a}
   $\Delta x := x_B - x_A$ ;  $\Delta y := y_B - y_A$ ;  $\Delta z := z_B - z_A$ ;
  if  $((x_A - z_A) * (\Delta z - \Delta y) > (y_A - z_A) * (\Delta z - \Delta x))$ 
    then EXIT; {Line segment is rejected}
  {first intersection point computation}
  if  $y_B > -z_B$  then {B is in the left edge region}
     $t_1 := (x_A - z_A) / (\Delta z - \Delta x)$ 
  else {B is in the bottom left corner region}
    begin
      if  $((x_A + z_A) * (\Delta z + \Delta y) > (y_A + z_A) * (\Delta z + \Delta x))$ 
        then EXIT; {Line segment is rejected}
      if  $((z_A - x_A) * (\Delta y + \Delta z) > (z_A + y_A) * (\Delta z - \Delta x))$ 
        then {intersection with left boundary}
           $t_1 := (x_A - z_A) / (\Delta z - \Delta x)$ 
        else {intersection with bottom boundary}
           $t_1 := -(y_A + z_A) / (\Delta z + \Delta y)$ 
        end;
      {second intersection point computation}
      if  $((x_A + z_A) * (\Delta z - \Delta y) > (z_A - y_A) * (\Delta z + \Delta x))$ 
        then {intersection with top boundary}
           $t_2 := (y_A - z_A) / (\Delta z - \Delta y)$ 
        else {intersection with right boundary}
           $t_2 := -(x_A + z_A) / (\Delta z + \Delta x)$ 
        end {case_I_1_a};

```

***1.1.b) Point A is in the top-right corner region and point B is inside of the pyramid or in the bottom edge region.***

In the case when point *B* is inside of the pyramid, the position of point *B* against the plane  $\rho_2$  specify whether the intersection point lies either on the top or on the right boundary. In the case when point *B* is in the bottom edge region, the comparison of point *B* against the plane  $\rho_3$  must be performed first to eliminate the situation when the line segment is rejected. If the line segment *AB* is not rejected by the clipping algorithm then one intersection point lies on the bottom boundary. The other intersection point lies on the top or on the right boundary according to the position of point *B* against the plane  $\rho_2$ . This section can be implemented as follows:

```

begin {case_I_1_b}
   $\Delta x := x_B - x_A$ ;  $\Delta y := y_B - y_A$ ;  $\Delta z := z_B - z_A$ ;
  {first intersection point computation}
  if  $y_B < -z_B$  then {B in bottom edge region}

```

```

  begin
    if
       $((x_A + z_A) * (\Delta z + \Delta y) > (y_A + z_A) * (\Delta z + \Delta x))$ 
        then EXIT; {Line segment is rejected}
       $t_1 := -(y_A + z_A) / (\Delta z + \Delta y)$ 
    end
    else {B is inside the pyramid}
       $t_1 := 1$ ;
      {second intersection point computation}
      if  $((x_A + z_A) * (\Delta z - \Delta y) > (z_A - y_A) * (\Delta z + \Delta x))$ 
        then {intersection with top boundary}
           $t_2 := (y_A - z_A) / (\Delta z - \Delta y)$ 
        else {intersection with right boundary}
           $t_2 := -(x_A + z_A) / (\Delta z + \Delta x)$ 
        end {case_I_1_b};

```

***1.2. Point A is in the right edge region and point B is not beyond the right boundary.***

We need to distinguish the cases, when point *B* is bellow the bottom boundary (point *B* is in bottom-left corner region or in the bottom edge region), or above the top boundary (point *B* is in top-left corner region or in the top edge region) or between top and bottom boundaries. An implementation can be as follows:

```

begin {case_I_2}
  if  $y_B < -z_B$  then case_I_2_a
    {point B is in the bottom-left corner
    or in the bottom edge region}
  else if  $y_B \leq z_B$  then case_I_2_b
    {point B is in the left edge region or
    inside of the pyramid}
  else case_I_2_c
    {point B is in the top-left corner
    or in the top edge region}
end {case_I_2}

```

***1.2.a) Point A is in the right edge region and point B is in the bottom-left corner or in the bottom edge region.***

The location of point *B* against the plane  $\rho_3$  helps us to eliminate the case when the line segment is rejected. If point *B* is in the bottom edge region then the intersection points are on the right and the bottom boundaries. If point *B* is in the bottom-left corner region then one intersection point lies on the right boundary

and the second intersection point's location (either on the left or on the bottom boundary) is determined by the location of point  $B$  against the plane  $\rho_A$ .

```

begin {case_I_2_a}
  if  $((x_A+z_A)*(\Delta z+\Delta y) > (y_A+z_A)*(\Delta z+\Delta x))$ 
    then EXIT; {Line segment is rejected}
  {first intersection point computation}
  if  $x_B > z_B$  then { $B$  in the bottom-left corner}
    if  $((z_A-x_A)*(\Delta z+\Delta y) > (z_A+y_A)*(\Delta z-\Delta x))$ 
      then {intersection with left boundary}
         $t_1 := (x_A-z_A)/(\Delta z-\Delta x)$ 
      else {intersection with bottom boundary}
         $t_1 := -(y_A+z_A)/(\Delta z+\Delta y)$ 
    else { $B$  in bottom edge region}
      {intersection with bottom boundary}
       $t_1 := -(y_A+z_A)/(\Delta z+\Delta y)$ ;
  {second intersection is on right boundary}
   $t_2 := -(x_A+z_A)/(\Delta z+\Delta x)$ 
end {case_I_2_a};

```

**I.2.b) Point  $A$  is in the right edge region and point  $B$  is inside of the pyramid or in the left edge region.**

In this case, one intersection point is on the right boundary and the second one (if point  $B$  is in the left edge region) is on the left boundary, see following pseudo-code:

```

begin {case_I_2_b}
  {first intersection is on right boundary}
   $t_1 := -(x_A+z_A)/(\Delta z+\Delta x)$ ;
  {second intersection point computation}
  if  $x_B > z_B$  then {point  $B$  in left edge region}
     $t_2 := (x_A-z_A)/(\Delta z-\Delta x)$ 
  else  $t_2 := 1$ 
end {case_I_2_b};

```

**I.2.c) Point  $A$  is in the right edge region and point  $B$  is in the top-left corner or in the top edge region:** similar to case\_I\_2\_a.

**I.3. Point  $A$  is in the bottom-right corner region and point  $B$  is not beyond the right boundary.**

The case is similar to case\_I\_1.

**II. Point  $A$  is between the left and the right boundaries.**

In this case, we need to characterize the location of point  $A$  to specify that, if point  $A$  lies inside of the pyramid or in an edge region. The following pseudo-code shows how it can be done:

```

begin {case_II}
  if  $y_A > z_A$  then case_II_1
    { $A$  is in the top edge region}
  else if  $y_A < -z_A$  then case_II_2
    { $A$  is in the bottom edge region}
  else case_II_3 { $A$  is inside the pyramid}
end {case_II};

```

We need to consider only the case when point  $A$  is inside the pyramid (case\_II\_3). The cases, when point  $A$  is in the top (case\_II\_1) or bottom edge region (case\_II\_2), are similar to the case when point  $A$  is in the right edge region (case\_I\_2).

**II.3. Point  $A$  is inside the pyramid.**

If point  $B$  lies in an edge region then the boundary, on which the intersection point lies, is determined, see the Figure 2, and the appropriate intersection point is computed. If point  $B$  lies in a corner region then a comparison of the location of point  $B$  with an appropriate plane  $\rho_i$  is necessary before the appropriate intersection point is computed. An implementation can be illustrated as follows:

```

begin {case_II_3}
  if  $x_B < -z_B$  then
    if  $y_B > z_B$  then case_II_3_a
      { $B$  is in the top-right corner region}
    else if  $y_B \geq -z_B$  then case_II_3_b
      { $B$  is in the right edge region}
    else case_II_3_c
      { $B$  is in the bottom-right corner region}
  else if  $x_B > z_B$  then
    if  $y_B > z_B$  then case_II_3_d
      { $B$  is in the top-left corner region}
    else if  $y_B < -z_B$  then case_II_3_e
      { $B$  is in the bottom-left corner region}
    else case_II_3_f
      { $B$  is in the left edge region}
  else
    if  $y_B > z_B$  then case_II_3_g

```

```

        {B is in the top edge region}
    else if  $y_B < -z_B$  then case_II_3_h
        {B is in the bottom edge region}
    else case_II_3_i
        {B is inside pyramid, the whole line
        segment is visible}
    end {case_II_3};

```

$$y := t * \Delta y + y_A;$$

$$z := t * \Delta z + z_A$$

It can be seen that all possible cases were solved and the complete algorithm can be got by substitution all procedures by appropriate codes.

### II.3.a) Point A is inside of the pyramid and point B is in top-right corner region.

The comparison of point B with the plane  $\rho_2$  specifies which boundary (top or right) to be used to compute the intersection point. An implementation can be as follows:

```

begin {case_II_3_a}
    if  $((x_A + z_A) * (\Delta z - \Delta y) > (z_A - y_A) * (\Delta z + \Delta x))$ 
    then {intersection with top boundary}
         $t_1 := (y_A - z_A) / (\Delta z - \Delta y)$ 
    else {intersection with right boundary}
         $t_1 := -(x_A + z_A) / (\Delta z + \Delta x);$ 
     $t_2 := 0;$ 
end {case_II_3_a};

```

The cases case\_II\_3\_c, case\_II\_3\_d and case\_II\_3\_e are similar to case\_II\_3\_a.

### II.3.b) Point A is inside of the pyramid and point B is in right edge region.

The appropriate intersection point (the intersection point on the right boundary) is calculated. The following pseudo-code shows how it can be implemented:

```

begin {case_II_3_b}
     $t_1 := -(x_A + z_A) / (\Delta z + \Delta x);$ 
     $t_2 := 0$ 
end {case_II_3_b};

```

The cases case\_II\_3\_f, case\_II\_3\_g and case\_II\_3\_h can be solved similarly.

### III. Point A is beyond the left boundary.

This case can be solved similarly to the case when point A is beyond the right boundary (case\_I).

Finally, we can easily compute the end-points of the output line segment from the parameter value  $t$  as follow:

$$x := t * \Delta x + x_A;$$

## 4. Experimental results

To be able to compare the CS, LB and CB algorithms with the proposed PC algorithm and evaluate the efficiency of the PC algorithm, we introduce three coefficients of efficiency as:

$$v_1 = \frac{T_{CS}}{T_{PC}}, v_2 = \frac{T_{LB}}{T_{PC}}, v_3 = \frac{T_{CB}}{T_{PC}}$$

where:  $T_{CS}$ ,  $T_{LB}$ ,  $T_{CB}$  and  $T_{PC}$  denote the time consumed by the CS algorithm, LB algorithm, CB algorithm and the proposed PC algorithm, respectively.

For experimental verification,  $80 \cdot 10^6$  different line segments were randomly generated for each of the 21 cases shown in Figures 5 and 6. The tests were performed on the PC Intergraph Pentium-II, 400MHz, 512MB RAM, 256 KB CACHE. The obtained results are presented in Table 1, which shows that the proposed algorithm is just as fast as the CS algorithm when the line segment lies inside the pyramid, and is significantly faster in all other cases. It can be seen that the speed-up varies from 1.17 to 2.08 approximately for these cases. Table 1 also shows that the PC algorithm is significantly faster than LB and CB algorithm for the most cases.

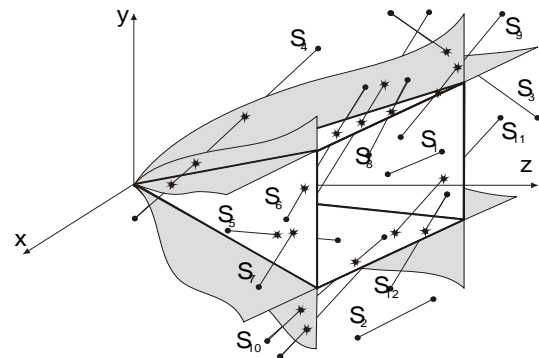


Figure 5: Generated line segments.

