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Springback analysis of thermoplastic composite plates

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Abstract

Residual stresses, which are set in the fiber reinforced composites during the laminate curing in a closed form, lead to dimensional changes of composites after extracting from the form and cooling. One of these dimensional changes is called "springback" of the angle sections. Other dimensional changes are warpage of flat sections of composite or displacement of single layers of composite for example. In our case four different lay-ups were analysed (three symmetrical and one unsymmetrical). An analytical model which covers temperature changes, chemical shrinkage during curing and moisture change was used. Also a FEM analysis was done for predicting the springback, and both calculations were compared with the measured data from manufacturer.

Keywords: springback, thermoplastic composite, analytical calculation, FEM

1. Introduction

Incorporating thermoplastic matrices (PEEK etc.) in carbon fibre-reinforced composites results considerably in higher toughness and impact resistance than have traditional thermoset based composites. In addition, the thermoplastic materials have significant advantages during fabrication and allow applying optimized metal working technology (stamping). However, the high temperature at which the thermoplastic composite must be processed does suggest an increased significance of thermally induced stresses and distortions in a product finished. Therefore it is desirable to be able to predict distortions accurately, reducing the trial and error time when producing a new component [5].

Change in composite dimensions is related to many parameters as: part angles, thicknesses, lay-ups, flange length, but also tool materials, tool surface or cure cycles [1]. When the composite L (or U) section is extracted from the form that was cooled to the room temperature, the change in the angle of the part (e.g. Fig.1) can be observed.

Tool angles have to be modified to affect this problem. The tool design is based on either the "rules of thumb", from the past experience, or on the trial-and-error. For the angular parts, the compensation is normally between 1 and $2{,}5^{\circ}$. The most common problem found, using a standard factor, is that the springback may vary with the lay-up, material, cure temperature, etc. Therefore, what worked once does not necessarily work next time. The main cause of springback was the mismatch in thermal expansion along and across the fibers in a laminate. But there are also other causes which will be discussed in the next analysis [4].

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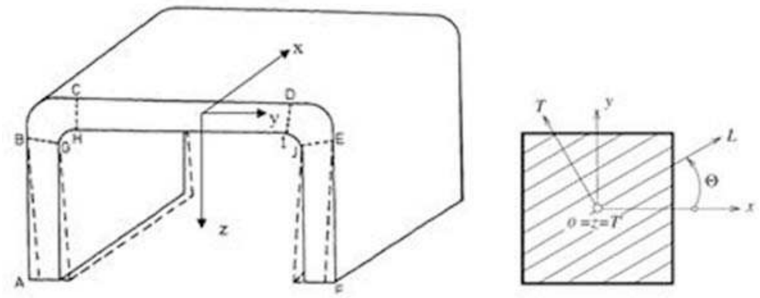

Fig. 1. Distortion of moulded U-section [3]

2. Analytical calculation

Analytical solution of the springback of the layered composite plate is based on the derivation of the springback for the unidirectional composite plate. The springback (SB) is defined as a difference between the final angle of the product and the tool angle divided by the tool angle

$$SB = \frac{\gamma' - \gamma}{\gamma} \tag{1}$$

when the angles can be computed from the lengths of arcs, thickness and their changes due the temperature, moisture and matrix shrinkage during cure (e.g. Fig. 2).

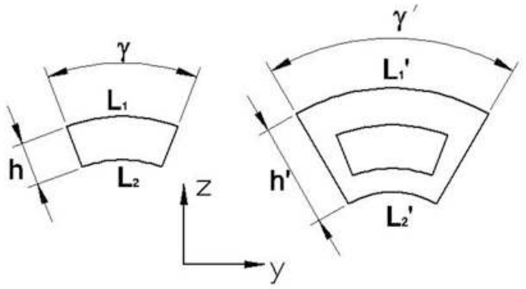

Fig. 2. To the derivation of springback [2]

Total change in the angle can be written as

$$\Delta\gamma = \Delta\gamma_t + \Delta\gamma_h + \Delta\gamma_c = \gamma\frac{\varepsilon_y^t - \varepsilon_z^t}{1+\varepsilon_z^t} + \gamma\frac{\varepsilon_y^h - \varepsilon_z^h}{1+\varepsilon_z^h} + \gamma\frac{\varepsilon_y^c - \varepsilon_z^c}{1+\varepsilon_z^c}, \tag{2}$$

where $\Delta\gamma_t$ is temperature part of the angle change, $\Delta\gamma_h$ is change in the angle due to hygroscopic effect and $\Delta\gamma_c$ is change in the angle due to shrinkage effect during the cure cycle. Strains (from temperature, moisture absorption and shrinkage) marked with y index stand for longitudinal directions, coefficients marked with z stand for transversal (trough thickness) directions. Temperature and moisture absorption effects are notoriously known so in next paragraph will be described just the resin shrinkage effect during curing.

This structural effect of volumetric change of matrix due to the crystallization will be important in composites with thermoplastic matrices that change their phase from amorphous to crystalline one during heating and curing. During curing and crystallization, the semi-crystalline matrices shrink due to crowding of the mass (the crystals have higher density then the amorphous phase). The shrinkage of semi-crystalline matrix due to this effect can be significantly higher than that due to the temperature change. It is evident that amorphous polymers haven't got the effect of volumetric change due to the recrystallization, so the influence of temperature change is crucial [4].

Let's have a laminated composite plate with $i = 1, \ldots, N$ layers with thickness H ($H = \sum_i^N h_i$). Springback derivation for the layered composite plate stands on derivation of the thickness change due to temperature, moisture and shrinkage effect. Detailed derivation is presented in [2] or [4]. Final equations of the thickness change are

$$\begin{aligned} \Delta H^t &= \sum_{k=1}^{N} \left\{ [S_{13}\; S_{23}\; S_{36}]\,[T_q]_k \left((z_k - z_{k-1})\,[\bar{Q}]_k \left(\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} - \Delta T \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \right) + \right.\right. \\ &\qquad \left.\left. \frac{z_k^2 - z_{k-1}^2}{2}\,[\bar{Q}]_k \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) + (z_k - z_{k-1})\,(\Delta T(\alpha_3)_k) \right\}, \end{aligned} \tag{3}$$

$$\begin{aligned} \Delta H^h &= \sum_{k=1}^{N} \left\{ [S_{13}\; S_{23}\; S_{36}]\,[T_q]_k \left((z_k - z_{k-1})\,[\bar{Q}]_k \left(\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} - \Delta c \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k \right) + \right.\right. \\ &\qquad \left.\left. \frac{z_k^2 - z_{k-1}^2}{2}\,[\bar{Q}]_k \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) + (z_k - z_{k-1})\,(\Delta c(\beta_3)_k) \right\}, \end{aligned} \tag{4}$$

$$\begin{aligned} \Delta H^c &= \sum_{k=1}^{N} \left\{ [S_{13}\; S_{23}\; S_{36}]\,[T_q]_k \left((z_k - z_{k-1})\,[\bar{Q}]_k \left(\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} - \begin{bmatrix} \Phi_x \\ \Phi_y \\ \Phi_{xy} \end{bmatrix}_k \right) + \right.\right. \\ &\qquad \left.\left. \frac{z_k^2 - z_{k-1}^2}{2}\,[\bar{Q}]_k \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) + (z_k - z_{k-1})\,(\Phi_3)_k \right\}, \end{aligned} \tag{5}$$

where N is a number of layers, S_{ij} are elements of the compliance matrix, T_q is a transformation matrix, z is a coordinate of the layer, $\bar{Q}$ is 2D plain stress matrix in x, y coordinate system (see Fig. 1) , ΔT is a change in temperature, Δc change in moisture, α_{ij} is a coefficient of thermal expansion, β_{ij} is a coefficient of moisture absorption and φ_{ij} is a coefficient of chemical shrinkage during recrystallization.

The total change in the laminate thickness can be computed as

$$\Delta H^{thc} = \Delta H^t + \Delta H^h + \Delta H^c. \tag{6}$$

Generalized strain in z direction can be computed from Eq. (6) as

$$\varepsilon_z^{thc} = \frac{\Delta H^{thc}}{H}. \tag{7}$$

According to the classical lamination theory (CLT, e.g. [5]), the strain and curvatures of the middle plane of laminated composite plate, loaded with unit forces and moments, can be obtain. Accordingly, it can be stated that strain and curvatures, arised from the temperature, moisture and resin shrinkage changes, will produce mechanical forces and moments. According to CLT

$$\left\{\begin{array}{c} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{array}\right\} = \left[\begin{array}{cccccc} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & & & \\ D_{16} & D_{26} & D_{66} & & & \end{array}\right] \left(\left\{\begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{array}\right\} - \left\{\begin{array}{c} N_x^{thc} \\ N_y^{thc} \\ N_{xy}^{thc} \\ M_x^{thc} \\ M_y^{thc} \\ M_{xy}^{thc} \end{array}\right\} \right), \tag{8}$$

where A_{ij}, B_{ij} and D_{ij} are the generally known elements of membrane stiffness, bending-extension coupling stiffness and bending stiffness matrices, and quantities N_i^{thc}, M_i^{thc} are defined by equations

$$N_i^{thc} = \int Q_{ij}\varepsilon_j^{thc}\,\mathrm{d}z, \tag{9}$$

$$M_i^{thc} = \int Q_{ij}\varepsilon_j^{thc}z\,\mathrm{d}z, \tag{10}$$

with the fact that integration boundaries are from $-H/2$ to $H/2$. N_i^{thc} and M_i^{thc} have the same dimension as N_i and $M_i$and they are called the resultants of the thermohygrocrystallic unit internal forces and moments. Q is 2D plain stress matrix in L, T coordinate system (see Fig. 1). The plane strain $\varepsilon_i^{0,thc}$ and relative change in curvature κ_i^{thc} will arise due to N_i^{thc} and M_i^{thc} at absence of N_i and M_i and they can be calculated by

$$\left\{\begin{array}{c} \varepsilon_x^{0,thc} \\ \varepsilon_y^{0,thc} \\ \gamma_{xy}^{0,thc} \\ \kappa_x^{thc} \\ \kappa_y^{thc} \\ \kappa_{xy}^{thc} \end{array}\right\} = \left[\begin{array}{cccccc} a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \\ a_{12} & a_{22} & a_{26} & b_{12} & b_{22} & b_{26} \\ a_{16} & a_{26} & a_{66} & b_{16} & b_{26} & b_{66} \\ b_{11} & b_{12} & b_{16} & d_{11} & d_{12} & d_{16} \\ b_{12} & b_{22} & b_{26} & d_{21} & d_{22} & d_{26} \\ b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66} \end{array}\right] \left\{\begin{array}{c} N_x^{thc} \\ N_y^{thc} \\ N_{xy}^{thc} \\ M_x^{thc} \\ M_y^{thc} \\ M_{xy}^{thc} \end{array}\right\}, \tag{11}$$

where a_{ij}, b_{ij} and d_{ij} are elements of inverse matrix to the whole A, B, B, D matrix. If the lay-up of the composite is symmetric (according to CLT there is no connection between normal forces and moments or between axial strains and curvatures) then the springforward of the complete composite can be calculated by Eq. (2) which was derived for one layer.

For the unsymmetrical lay-up, an additional term must be included. Eq. (2) rewritten to the springback form will look [2]

$$SB = \frac{\Delta\gamma}{\gamma^{thc}} = R_y\kappa_y^{thc} + \frac{\varepsilon_y^{thc} - \varepsilon_z^{thc}}{1 + \varepsilon_z^{thc}}, \tag{12}$$

where κ_y^{thc} is the change in curvature and R_y is the radius which affected the straight part of the plate.

When the cylindrical shell of the diameter $D = 2R_y$ is analyzed, this springback effect must be included in the thermal force-strain relationship of laminated plates. This is accomplished by making following replacement in Eq. (11) [2]

$$\kappa_y^{thc} \Rightarrow \kappa_y^{thc} + \frac{1}{R_y}\left(\varepsilon_y^{thc} - \varepsilon_z^{thc}\right). \tag{13}$$

In the case of double curved shells (with two main curvatures radiuses R_x, R_y) we have to make also this replacement in Eq. (11) [2]

$$\kappa_x^{thc} \Rightarrow \kappa_x^{thc} + \frac{1}{R_x}\left(\varepsilon_x^{thc} - \varepsilon_z^{thc}\right). \tag{14}$$

3. Calculation for the given plates

The main goal of this work is to make a tool, to predict springback angle for given laminated plate with single or double curvature with defined number of layers made from defined combination of fibre and matrix. The springback angle is necessary to predict because of manufacturing more precious parts and also for good correction of the compression mould. The manufacturer of the composite plates (Letov Letecká Výroba, s. r. o.) has provided some measured data from the manufacturing process. The data was for the layered composite plates made from carbon fibre and PPS matrix with $V_f = 49\,\%$.

Lay up's and radiuses in y direction for the single curvature plates are:

- 30 layers $[(0{,}90)/(\pm45)]_7/(0{,}90)$ $R_y = 5$ mm
- 32 layers $[[(0{,}90)/(\pm45)]_4]_s$ $R_y = 5$ mm
- 36 layers $[[(0{,}90)/(\pm45)]_4/(0{,}90)]_s$ $R_y = 6$ mm
- 40 layers $[(0{,}90)/[(0{,}90)/(\pm45)]_4/(0{,}90)]_s$ $R_y = 7$ mm

Lay up's and radiuses in x and y directions for the double curvature plates are:

- 30 layers $[(0{,}90)/(\pm45)]_7/(0{,}90)$ $R_x = 2\,811{,}5$ mm, $R_y = 5$ mm
- 32 layers $[[(0{,}90)/(\pm45)]_4]_s$ $R_x = 2\,811$ mm, $R_y = 5$ mm
- 36 layers $[[(0{,}90)/(\pm45)]_4/(0{,}90)]_s$ $R_x = 2\,810{,}5$ mm, $R_y = 6$ mm
- 40 layers $[(0{,}90)/[(0{,}90)/(\pm45)]_4/(0{,}90)]_s$ $R_x = 2\,810$ mm, $R_y = 7$ mm

Single and double curvature plates can be seen in Fig. 3.

Fig. 3. Investigated plates with single and double curvature [4]

Eqs. (7) and (11) were used for computing coefficients of thermal expansion and recrystallization for the analyzed composite plates. Computed coefficients can be seen in Table 1. The hygroscopic effect is not considered ($\Delta c = 0$).

The comparison between the data calculated by the analytical model, the data from the manufacturer and the data from numerical simulation are shown in Figs. 6 and 7.

Table 1. Coefficients of thermal expansion and recrystallization for the analyzed lay-ups

Lay-up	α_x [K^{-1}]	α_y [K^{-1}]	α_z [K^{-1}]	Φ_x [−]	Φ_y [−]	Φ_z [−]
$[(0{,}90)/(\pm 45)]_7/(0{,}90)$	4,82e^{-6}	4,82e^{-6}	3,91e^{-5}	1,38e^{-3}	1,38e^{-3}	1,23e^{-2}
$[[(0{,}90)/(\pm 45)]_4]_s$	4,98e^{-6}	4,98e^{-6}	3,88e^{-5}	1,43e^{-3}	1,43e^{-3}	1,22e^{-2}
$[[(0{,}90)/(\pm 45)]_4/(0{,}90)]_s$	4,70e^{-6}	4,70e^{-6}	3,93e^{-5}	1,34e^{-3}	1,34e^{-3}	1,24e^{-2}
$[(0{,}90)/[(0{,}90)/(\pm 45)]_4/(0{,}90)]_s$	4,48e^{-6}	4,48e^{-6}	3,98e^{-5}	1,27e^{-3}	1,27e^{-3}	1,25e^{-2}

4. Numerical calculation

A finite element model was created and analyzed for thermo-elastic deformations using ABAQUS solver. For the calculation of plates with the single curvature was created model of the one quarter of the plate, for the plates with the double curvature was done model of one half. The model was solved with the hexahedron incompatible mode elements and two types of material properties. The first was defined by the orthotropic linear elastic properties and the thermal expansion material coefficients for the unidirectional lamina, the second one by the linear elastic material properties and the orthotropic thermal expansion for the whole composite. For the case of the first material model the local orientation of material axis in element has to be done with the method "OFFSET TO NODES" because of the change in material orientation due to changing radius. The theoretical material constants were computed by using CLT. The chemical shrinkage during the curing process was modeled by using "fake" coefficients of thermal expansion which represented the values of the chemical shrinkage. When the change in the temperature was 1°, we obtained deformations according to the chemical shrinkage effect. The model of the single curvature plate can be seen in Fig. 4 and the double curvature plate can be seen in Fig. 5.

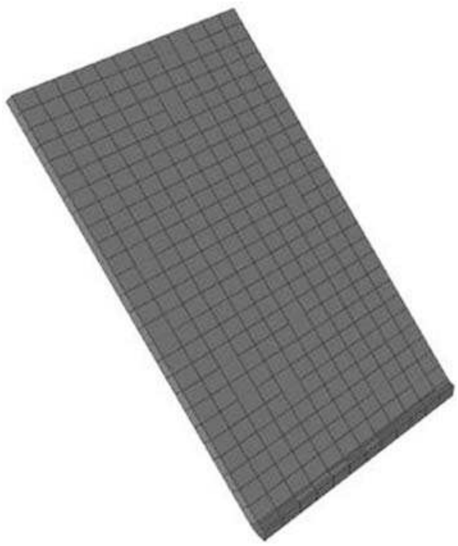

Fig. 4. FE model of single curvature plate

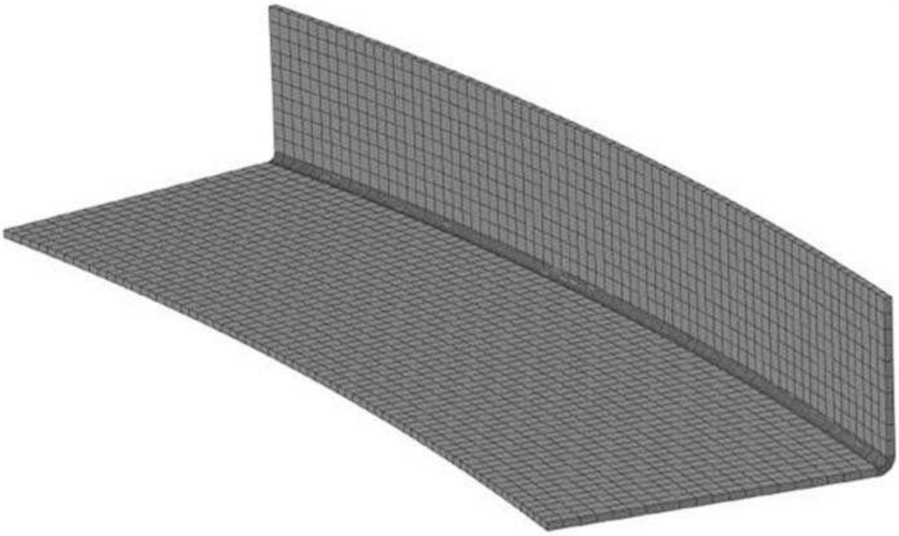

Fig. 5. FE model of double curvature plate

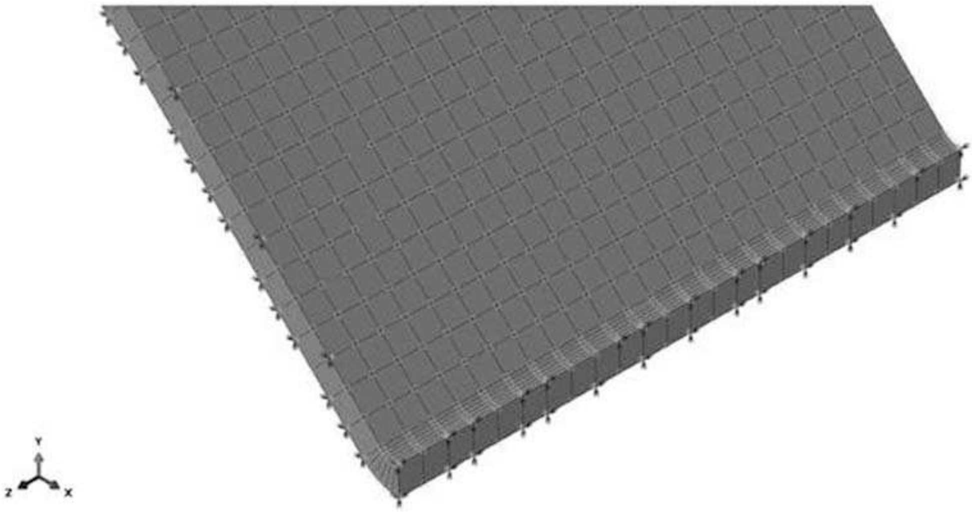

Fig. 6. Boundary conditions for single curvature part

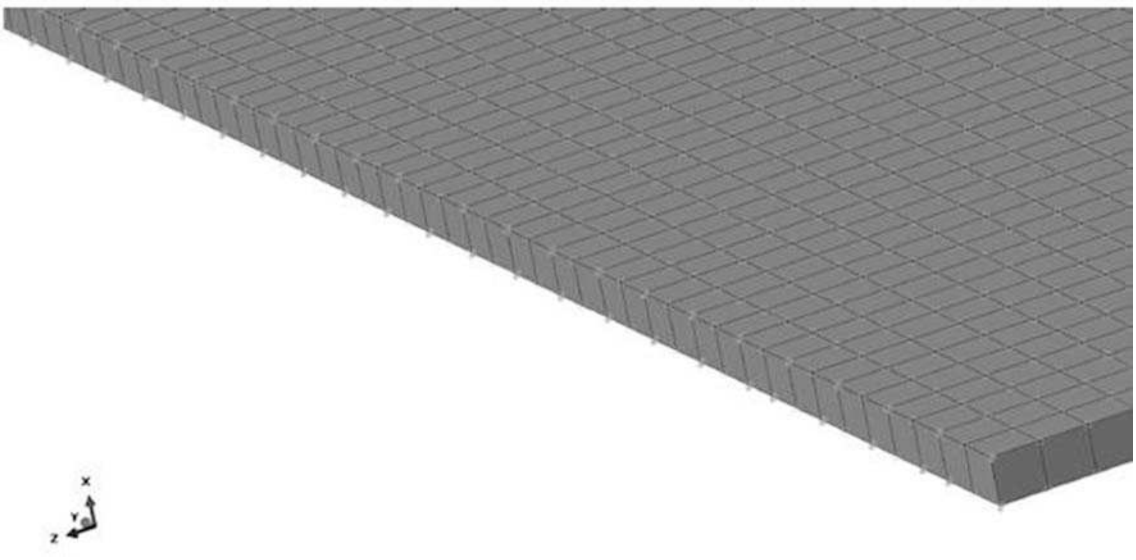

Fig. 7. Boundary conditions for double curvature part

Boundary conditions for the case of single curvature part were symmetrical and displacement in y direction was forbidden to highlighted node in Fig. 6. For the case of double curvature part, boundary conditions are shown in Fig. 7. Displacements in x and z are forbidden for marked area and also displacement in y direction was forbidden to highlighted node. Elements which were used in mesh of both parts were C3D8I, which are improved versions of C3D8 elements (there is no shear locking and volumetric locking is much reduced). These elements have an additional degree of freedom that enhances the ability to model a displacement gradient through the element (these elements act like quadratic elements but the computational cost is lower). It means that this element can be used in all instances, in which linear elements are subjected to bending (our case) and it allows the use of just one element in through thickness direction. A drawback to these elements are their sensitivity to element distortions which end up making the elements too stiff [6].

5. Results comparison, conclusions and future work

The comparison of the results computed by analytical and numerical solution with the data measured by manufacturer is shown in Figs. 8 and 9. Both figures show good agreement between measured data and springback angle predicted by analytical solution. Numerical model which used thermomechanical data computed by CLT shows the same values of springback angle for all types of the symmetrical lay-up. Numerical model which used thermomechanical data for the whole composite shows the same trend as the analytical results — springback angle for the symmetrical lay-up grows with the number of the layers (the third column in each data set — the growth is not very clear because of the chosen scale).

The analytical solution was put into the Matlab code. This program allows calculation of the springback angle for several combinations of fiber and matrix which can be chosen by user. User can also choose between the calculation of composite properties by using CLT or by input of properties directly. User also set up the lay-up of whole composite plate, radiuses of the

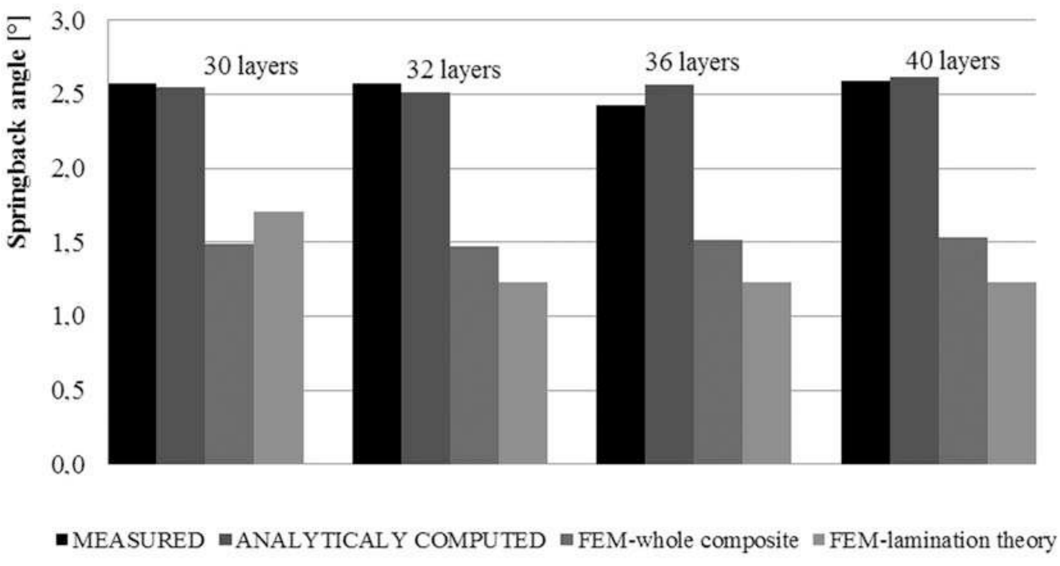

Fig. 8. Comparison of the analytical, numerical and experimental results for every type of investigated lay-up — plates with the single curvature

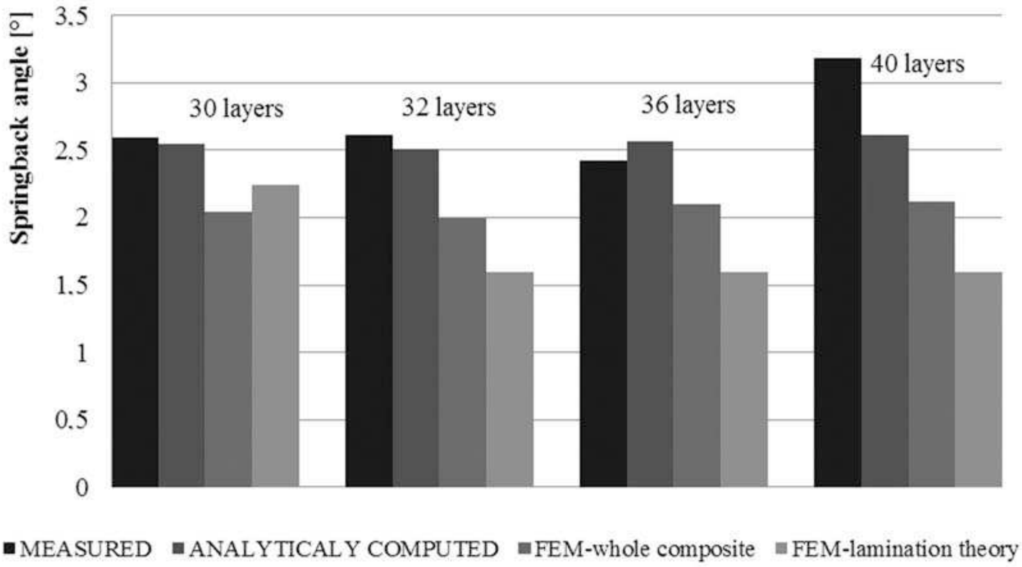

Fig. 9. Comparison of the analytical, numerical and experimental results for every type of investigated lay-up — plates with the double curvature

single and double curvature plates and the part angle of the plate. The analytical solution is faster than the numerical and the program doesn't need any special knowledge of software as FE solution does.

The future work in this area will focus on improving the FE results and their comparison with analytical and experimental (using other types of elements, mesh density, etc.). The material model which deals with the undulation of the fibers will be implemented into current Matlab code for comparison with the straight fibers model (because the reinforcement of the composite is 5H satin fabric). Also the influence of different types of the thermoplastic matrices on the springback angle will be focused in the future.

Acknowledgements

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