ADAPTIVE VISUALIZATION FOR INTERACTIVE GEOMETRIC MODELING IN GEOSCIENCE

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ABSTRACT

Many engineering disciplines can profitably use large high-resolution geometric models whose computational requirements exceed current computer hardware capacities. This paper presents an adaptive visualization solution for interactively building such models. Whereas adaptive visualization techniques have conventionally been applied to existing complete models, our work permits adaptive visualization of models while under construction. To achieve this, we use a multiresolution surface representation for both geometric computation and visualization. The paper develops techniques that dynamically and adaptively decimate models, adjusting to changing camera positions. The decimation algorithm preserves intersection curves between surfaces, and applies to models whose surface triangulation is either globally coherent or incoherent. We have embedded the technology presented in this paper into a 3D geoscience geometric modeling application framework that supports many applications.

Keywords: computer graphics, adaptive visualization, multiresolution, geometric modeling, decimation.

1. INTRODUCTION

A geoscience geometric model represents the structure of the subsurface and material properties within. Figure 1 shows examples of cross-sections of 3D subsurface structures. Geometric models representing such structures are usually non-manifold and are

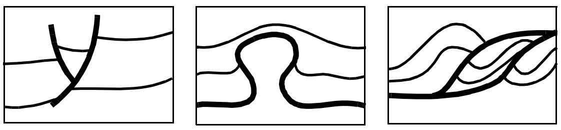


Figure 1: Cross-sections of some 3D subsurface

typically built interactively from surfaces generated from measured data. A large geoscience model may contain many millions of triangles, which cannot be rendered interactively by most graphics hardware. This paper presents an adaptive visualization solution for interactively building large geoscience geometric models.

We apply adaptive visualization in novel ways to interactive geometric modeling. This paper describes the visualization aspect of SIGMA: a Scalable Interactive Geometric Modeling Architecture [8]. The essence of SIGMA is a multiresolution surface representation, which supports geometric modeling and visualization. Reference [8] details the representation and geometric modeling aspect of SIGMA. This paper details the visualization aspect.

The literature on multiresolution surface representations and adaptive visualization is growing rapidly [3],[5],[9],[15]. However the previous work focuses on pre-built geometric models. A user builds a complete geometric model using a geometric modeler. To interactively visualize the model, the user converts the model to a multiresolution surface representation. Figure 2 shows this conventional approach. The previous work has not shown how the multiresolution representation can be updated and how the visualization is affected when the model is changed topologically. This presents problems for interactive geometric modeling where the geometric model goes through topological and geometrical changes. Reconstructing the multiresolution representation is often too expensive.

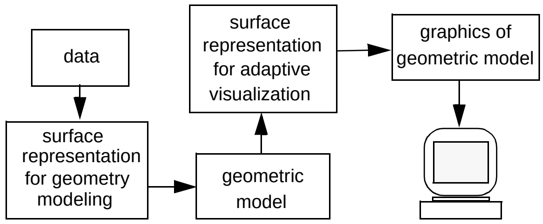


Figure 2: Conventional data flow for adaptive visualization.

Figure 3 shows our approach. SIGMA builds its surface representation and uses this representation for both geometric modeling and visualization. When the model changes (for example, a new surface is inserted), SIGMA incrementally updates the representation, rather than reconstructing it.

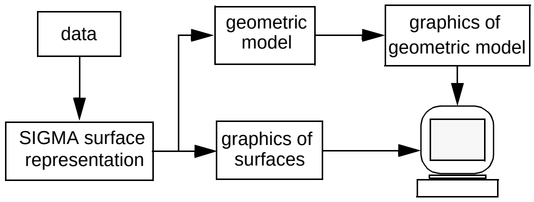


Figure 3: Data flow in SIGMA approach.

The main contributions of this paper include the following. (1) Adaptive visualization is applied to interactive geometric modeling and, in particular, during model construction. (2) Visualization uses the same multiresolution surface representation as used for geometric computations. (3) Visualization is incrementally updated as the model changes. (4) Adaptive visualization applies to coherent and incoherent geometric models, which maybe non-manifold.

1.1. Related Work

Major categories of multiresolution surface representations include: subdivision-based, wavelet-based, and mesh-based. Subdivision surface representations [2],[19],[20] start from a coarser mesh and subdivide elements in the mesh with a fixed scheme to refine the surface. Many wavelet-based multiresolution surface representations have been developed [3],[10], [11],[18]. Subdivision and wavelet techniques can represent surfaces to desired resolutions or smoothness with analytical error analysis. However, they have not been shown to represent and construct non-manifold geometric models, which we need for modeling subsurface structures. see Figure 1. Mesh-based multiresolution and simplification techniques [15], [9], [5] are based on point removal or edge contraction. These representations are flexible and able to represent non-manifold geometries.

The previous work focuses on existing geometric models. Multiresolution editing is possible. But such editing is limited to deforming a surface. We are interested in interactively building geometric models, which are non-manifold.

The following sections provide some background information, describe algorithms for applying adaptive visualization to interactive geoscience geometric modeling, and demonstrate an application.

2. BACKGROUND

A geometric model for geoscience is built from surfaces, such as faults and horizons, which are constructed from acquisition data. The geometric model is built using an Irregular Space Partition [1]. An Irregular Space Partitioning is a boundary representation model that consists of connected manifold components called cells. A 3-cell is a volume bounded by 2-cells or surfaces, which are bounded by 1-cells or curves, which are bounded by 0-cells or vertices. The *macro-topology* of a geometric model is the graph of boundary relationships between cells. This paper assumes a piece-wise linear geometric modeling kernel that computes the boundary representation of a model. We use SHAPES geometric modeling kernel from XOX [17] in our work.

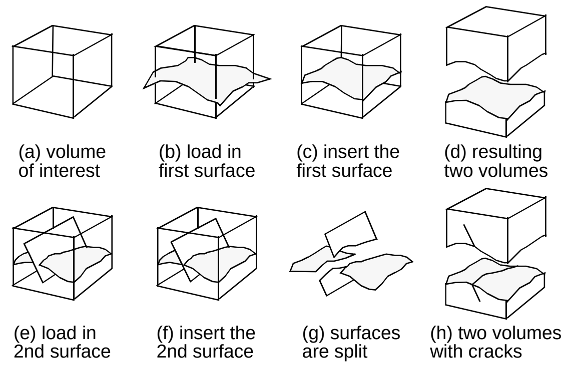


Figure 4: Interactively inserting two surfaces.

The geometric model is built incrementally by inserting surfaces into a volume of interest. The surfaces intersect each other creating new 2-cells and new 3-cells which split the volume of interest into many subvolumes. Figure 4 illustrates an example. Figure 4-(a) shows the volume of interest (VOI). Figure 4-(b) brings in the first surface. We insert the first surface into the VOI. The operation trims the first surface by the boundary of the VOI (Figure 4-(c)) and splits the VOI into two subvolumes (Figure 4-(d)). The second surface is then loaded in (Figure 4-(e)) and inserted into the VOI. This action splits the first and second surface (Figure 4-(g)) and inserts cracks into the two subvolumes (Figure 4-(h)). A typical geoscience geometric model may be built from dozens or hundreds of surfaces. During model building it is important that a user can visualize and interactively edit the model.

3. SIGMA SURFACE REPRESENTATION

The SIGMA surface representation [8] is novel as it extends traditional multiresolution surface techniques to geometric model building. In particular, data structures, typically used in computer graphics, are extended to support Irregular Space Partitioning, and, as shall be shown in this paper, for interactive visualization and building of geometric models.

The SIGMA surface representation is a multiresolution hierarchy implemented as a quadtree [14]. The quadtree is built in the parameter space of structured grid data. The leaves of the quadtree contain a collection of triangles which are the triangulation of the grid cells the quadtree leaf node covered. In this way each triangle of the surface is assigned to a unique quadtree leaf node and a node of the quadtree inherits the triangles and vertices of its descendants. Figure 5 shows an example of a surface, its parameter space, and its quadtree.

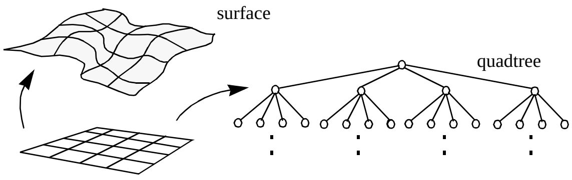


Figure 5: Parameter space and quadtree.

The quadtree aids in model building as it supports triangle refinement. In particular, if a triangle is refined the refining triangles are assigned to the quadtree nodes of the original triangle. The quadtree also aids in computing maximal connected components as many of the quadtree nodes are connected.

Definition: A collection of nodes, *C*, of the tree *T* is a *node front* if every leaf node of *T* has at most one ancestor in *C* (a node is an ancestor of itself). A node front is a *complete node front* if every leaf node of *T* has exactly one ancestor in *C*.

Figure 6 shows an example of a complete node front. For a complete node front each triangle of the surface

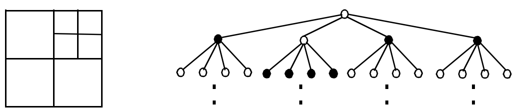


Figure 6: Views of a complete node front.

belongs to a unique node in the node front, on the other hand, a vertex of the surface maybe shared by several nodes. Given a complete node front, the vertices that are shared by three or more nodes, or are on the boundary of the surface and shared by two or more nodes, are called *critical vertices*. Figure 7 shows how the critical vertices approximate the original surface and how they are represented in a quadtree with irregular boundaries. It can be shown, a constrained triangulation in parameter space of the critical vertices of a complete node front describes a decimated non-cracked view of the surface [8].

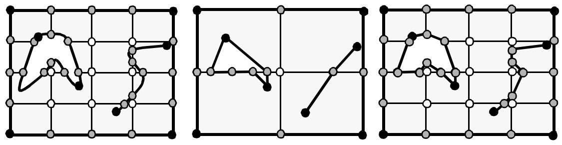


Figure 7: Critical vertices at depths 1 and 2.

Terrain visualization has used the quadtree to achieve interactive rendering performance [6], [13]. However, these techniques have not been applied to visualizing a non-manifold geometric model. We use the quadtree to build and visualize a non-manifold geometric model.

4. ADAPTIVE VISUALIZATION FOR INTERACTIVE GEOMETRIC MODELING

As mentioned earlier, SIGMA is designed for interactive geometric modeling of large geoscience models. We apply adaptive visualization to each step of the modeling process. This section describes our technique and the supporting algorithms. First we introduce the concept of model coherence.

4.1. Model Coherence

A model is *coherent* if the geometry of the cells agrees at all dimensions [17]. With piecewise linear geometry, a coherent model means the 0-cell 0-simplices and 1-cell 1-simplices are faces of the 2-cell 2-simplices.

A model is built by surface insertions (Section 2). Insertion includes two major steps, classification and making coherent. *Classification* is the topological operation that subdivides the point sets of operand geometries and determines the cells. *Making coherent* is the geometric operation that re-tessellates the mesh in the neighborhood of the intersection curve so 1-simplices of the curve are faces of 2-simplices of the mesh. Figure 8 shows an example of classification followed by making coherent. The classification results in two 2-cells A and B. We cannot geometrically separate A and B since triangles straddle both cells. After making coherent, each triangle belongs to a unique 2-cell. Making coherent is expensive, it is advantageous to visualize cells of an incoherent model as this permits viewing after classification.

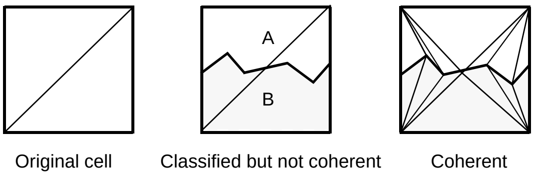


Figure 8: Classification and making coherent.

4.2. The Role of Adaptive Visualization

SIGMA enables adaptive visualization during geometric modeling process. Both the model and individual surfaces can be visualized. To achieve performance, we decimate the surface and the model dynamically according to the viewing parameters. The decimation ensures no cracks are generated and preserves the intersection curves between surfaces in the model. For this purpose, critical vertices (Section 3) are used and the intersection curves are constraints during the model decimation. Figure 9 illustrates the role of adaptive visualization during interactive geometric modeling using SIGMA.

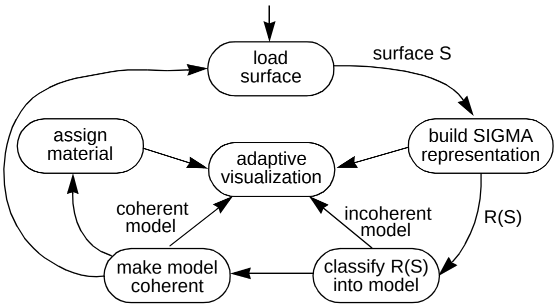


Figure 9: The role of adaptive visualization.

When a user loads a surface, SIGMA builds its surface representation. At this time, the surface is not part of the geometric model. Adaptive visualization algorithm applies to this individual surface. Inserting the surface into the volume of interest makes the surface part of the model. The intersection curves between the surface and the volume of interest constrains the decimation. The geometric computation of the insertion may be time consuming. The visualization algorithm can be applied before the computation completes, that is, visualizing incoherent model.

When the user loads in the next surface, adaptive visualization can apply to the surface and the model independently. In other words, for adaptive visualization, the new surface is not subject to the constraints that surfaces in the model are. After insertion, the independent rendering behavior of this new surface is removed.

SIGMA can render physical material properties together with adaptive visualization of the geometry. Properties are rendered via per vertex coloring or via texture mapping.

4.3. Top Level Algorithm

Adaptive visualization refers to mechanisms that modify the visualization of objects according to given criteria. In this work, we dynamically decimate the geometric model according to camera position to achieve visualization performance.

A geometric model contains many surfaces (2-cells) and their intersections (1-cells). The decimation should be able to respect the macro-topology of the model. If each surface is decimated independently, there may be cracks between intersecting surfaces. To prevent cracking, the decimations are constrained with the same set of 1-cell edges for all the 2-cells intersecting at this 1-cell.

SIGMA adaptive visualization includes three major steps: *vertex and edge selection*, *triangulation*, and *rendering*. The technique applies to individual surfaces, coherent models, and incoherent models. An individual surface can be considered as a model with only one surface.

Selection: The module traverses each quadtree to select vertices and selects edges from each 1-cell in the model.

Triangulation: The module generates a triangle mesh for each surface (2-cell) from selected vertices and edges. The edges are constraints of triangulation.

The vertex and edge selection and the triangulation together are referred to as decimation. Reference [8] describes a general decimation algorithm. This section presents a specific vertex selection algorithm for adaptive visualization. The decimation method in [8] is also extended for incoherent models.

Rendering: The module renders the triangle meshes. The physical material properties assigned to components of the geometric model can also be rendered.

Let *M* be a model, *C* be a set of criteria for vertex and edge selection, *V* be set of selected vertices, and *E* be set of selected edges. The following pseudocode sketches a top level algorithm for this technique.

```
while ((CameraMoved() == TRUE) and
        ((Proj_new(M) - Proj_old(M) > t)) {
    (V,E) := SelectVertexEdge(M, C);
    TM := Triangulate(M, V, E);
    UpdateGraphics(TM);}
```

where the *CameraMoved* function reports when camera has changed position; *Proj* function projects the model to the screen. When the change from the current projection to the previous projection is larger then a pre-specified threshold *t*, the model is re-decimated and graphics is updated. The *Triangulate* function may execute incremental and decremental algorithms instead of re-triangulating [7].

4.4. Vertex and Edge Selection

Vertex selection is performed for each quadtree in a model. The selection algorithm finds a node front of a quadtree via traversing the tree. Critical vertices of the node front form the subset of vertices to be rendered. Two major operations involved during the traversal are: *view frustum culling* and *projection*. View frustum culling determines those parts of the surface that are outside of the view frustum and need not be rendered. Projection determines the decimation resolutions of the surface.

Each quadtree node has a *bounding object* that bounds the triangles of the node. The algorithm traverses the quadtree from its root node. If the bounding object of a node is outside the view frustum, the node need not be visualized; and traversal continues to the next node. Otherwise, the bounding object of the node is projected to the screen. If the projected area is smaller than a pre-specified minimum resolution, the node is in the node front. Otherwise, the traversal needs to resolve the children of the node.

Due to perspective viewing, critical vertices from the node front results in a mesh with varying resolution. The portion of the surface closest to the camera has finer resolution. Further from the camera, the surface has coarser resolution.

For an incoherent model, more than one 2-cell might share the same quadtree. In this case, each quadtree is still traversed once for vertex selection. To simplify edge selection and triangulation, 1-cells in an incoherent model are made coherent. This is not expensive and needs to be done only once.

There are following cases for edge selection:

1. Visualizing an individual surface.
   1.1 The surface has a convex boundary in its parameter space and has no holes.
   1.2 The surface has a non-convex boundary in its parameter space or has holes. The boundary, including the holes, is represented by 1-cells.
2. Visualizing a geometric model that contains 1-cells and 2-cells. Some of the 1-cells represent intersections between 2-cells.

For case 1.1, edge selection is not required. For other cases, edge selection is done by decimating 1-cells.

The vertex and edge selection algorithm can be written in the following pseudocode,

```
SelectVertexEdge(M, C):
    for each quadtree T in M
        V += Traverse(T, C);
    E := DecimateOneCells(M, C);
    return (V,E);
```

where the *Traverse* function traverses quadtree *T* as described earlier. The camera parameters and pre-specified screen resolution are encoded in parameter *C*. The result of *Traverse* is a set of selected vertices. The *DecimateOneCells* function finds all the 1-cells in model *M* and decimates them to obtain edges.

4.5. Triangulation

We use *Delaunay triangulation* [4],[16] to triangulate selected vertices in the parameter space of a surface. When edges are present, constrained Delaunay triangulation is applied to the vertices with edges as constraints. The resulting triangle mesh is mapped to the image space to approximate the original surface. During interaction, the camera moves and many vertices selected are repeated from one position to the next. Incremental and decremental Delaunay triangulation [7] can be used to update the mesh.

The triangulation applies progressively more constraints to the three cases listed in the previous section. For case 1.1 where a surface has a convex boundary and no holes, Delaunay triangulation applies to the selected vertices. For case 1.2 where a surface has non-convex boundary or has holes, constrained Delaunay triangulation is used.

For case 2 where a model is visualized, to preserve intersection curves, constrained Delaunay triangulation is used for each 2-cell in the model. Given a 2-cell, edges from all its 1-cells are constraints to the triangulation. To prevent cracking, for each 1-cell, the same set of edges from the 1-cell constrains all the 2-cells intersecting at this 1-cell. Figure 10 shows an example. Surfaces A and B intersect. The inter-

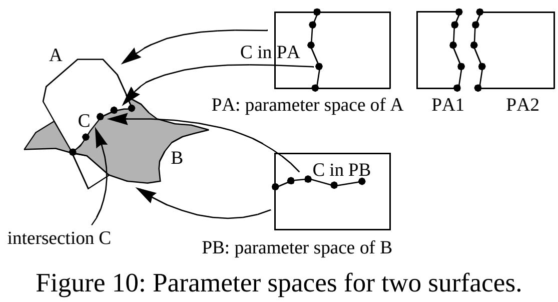


Figure 10: Parameter spaces for two surfaces.

section curve C is a 1-cell. Surface A is split into two 2-cells, A1 and A2. Surface B is still one 2-cell, but has a crack resulting from the intersection. The 1-cell

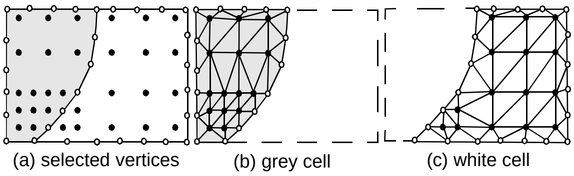
C is shared by three 2-cells, A1, A2, and B. Figure 10 shows the parameter spaces of A, B, A1, and A2. During the edge selection (Section 4.4), C is decimated. In this example, the decimated version of C has four edges. During triangulation, for each 2-cell, B, A1, and A2, we find edges of C in the 2-cell's parameter space, respectively (Figure 10). The edges are part of the triangulation constraints in 2-cells' parameter spaces.

The triangulation algorithm is sketched in the following pseudocode,

```
Triangulate(M, V, E):
    for each 2-cell S in M {
        VP := V.getVerticesInParamSpace(S);
        B := S.getBoundaryCells();
        EB := E.getEdges(B);
        EP := EB.getEdgesInParamSpace(S);
        TM += ConstrainedDelaunay(VP, EP);}
    return (TM);
```

where *getEdgesInParamSpace* function maps the boundary edges EB of S to the parameter space of S. The *ConstrainedDelaunay* function executes in parameter space of S. This algorithm is straightforward for a coherent model since each 2-cell has its own quadtree and a set of selected vertices from the quadtree. The algorithm also works for an incoherent model as explained below.

In an incoherent model, a quadtree may be shared by multiple 2-cells. Therefore, given a 2-cell, some of the selected vertices from the quadtree may not be part of the 2-cell. Figure 11 shows an example. A



A surface is classified into two cells, a grey cell and a white cell.

Figure 11: Triangulation of incoherent 2-cells.

surface is classified into grey cell and white cell, but has not been made coherent. Both the grey cell and the white cell share the same quadtree of the original surface. Figure 11-(a) shows selected vertices from the quadtree and selected edges from 1-cells. Vertices on the white cell are not part of the grey cell, and vice versa. Constrained Delaunay triangulation is applied to each 2-cell separately. Figure 11-(b) and (c) shows the constrained Delaunay triangulations for the grey cell and white cell, respectively. Since each 2-cell has a closed boundary of 1-cells, the vertices outside of the boundary are removed during triangulation. With this approach, one traversal for vertex selection is done for each quadtree. The triangulation runs as many times as the number of 2-cells sharing this quadtree.

4.6. Performance versus Quality

SIGMA is deployed on a wide-range of visualization hardware. It is important a user is given a broad range of rendering options to maintain interactive performance. As has been described in previous sections, SIGMA can reduce triangle counts by using decimation and culling. However, SIGMA permits another group of trade-offs which are based around the visualization of the geometric model. In particular, it is possible to trade-off topological quality for performance, see Figure 12.

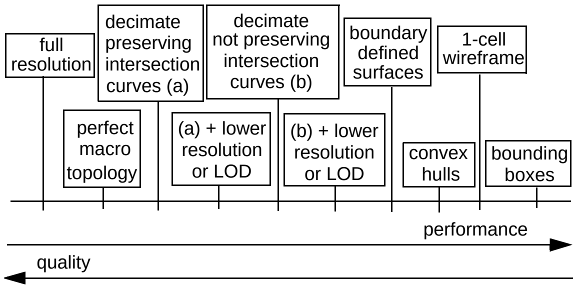


Figure 12: SIGMA provides a range of selections for performance and quality.

For optimum quality, the model is rendered at full resolution. To improve rendering performance the model can be decimated. However, decimating the model may introduce topological artifacts such as cracking and bubbling. Preventing these topological artifacts, especially bubbling, is costly and a user may decide such topological quality is unnecessary. The surfaces can be decimated even further until they have no interior points and are only defined by their boundaries. One can even drop the surfaces altogether and just draw the 1-cell wireframe of the model. Finally, a crude approximation of the model can be given by rendering bounding boxes of the surfaces. These trade-offs can be chosen by users at the application level.

5. APPLICATION

To support interactive geometric modeling in geoscience, we developed the Common Model Builder [12],[1], which is an application-neutral framework. SIGMA is embedded in the Common Model Builder.

Figure 13 shows the adaptive visualization of a synthetic surface, the z value of which is defined by sine functions of x and y values. The surface is adaptively decimated and is rendered with a space varying property. Figure 14 shows a bird's eye view of the surface, the red outlined polyhedron is the view frustum.

Figure 15 to Figure 18 show snapshots from a geoscience geometric modeling process using SIGMA. Surfaces in these figures are rendered with adaptive visualization technique. There are two model display modes: surface mode and volume mode.

In Figure 15, a unconformity surface is inserted into the volume of interest (VOI). This operation split VOI into two volumes. Figure 16 is an explored view of the two volumes. In Figure 17, another unconformity, a horizon surface, and two faults are inserted to the model. Figure 18 shows an explored view of the volumes in the model. In this view, the model is rotated to see some smaller volumes in the back.

6. CONCLUSION

This paper presents an adaptive visualization solution for interactively building large high resolution geometric models. Distinct from existing approaches, adaptive visualization applies during model construction. The same multiresolution surface representation is used for visualization and geometry computations. The representation is incrementally updated, rather than reconstructed, when the model changes. We have developed techniques that adaptively decimate models (coherent and incoherent), according to camera position. The decimation algorithm preserves intersection curves between surfaces in the model. The technology is embedded into a 3D geoscience geometric modeling application framework. Disciplines other than geoscience face similar problems when building large geometric models; SIGMA technology may be applicable. Many optimizations remain to be developed to improve SIGMA, which is the main focus of our future work.

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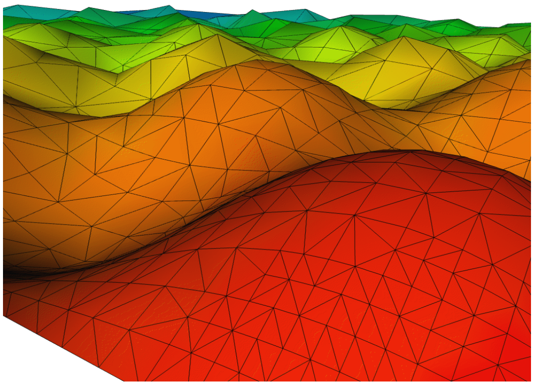

Figure 13: Adaptive Visualization of a surface.

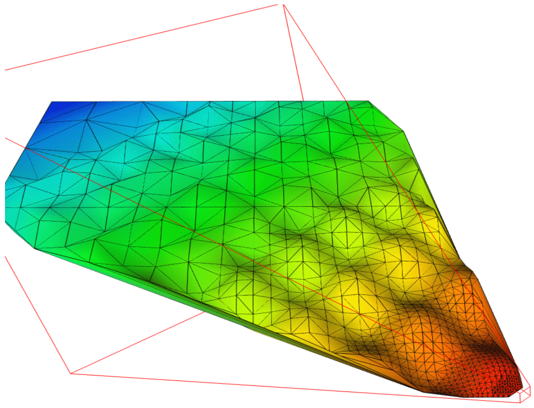

Figure 14: A bird's eye view of the surface in Figure 13.

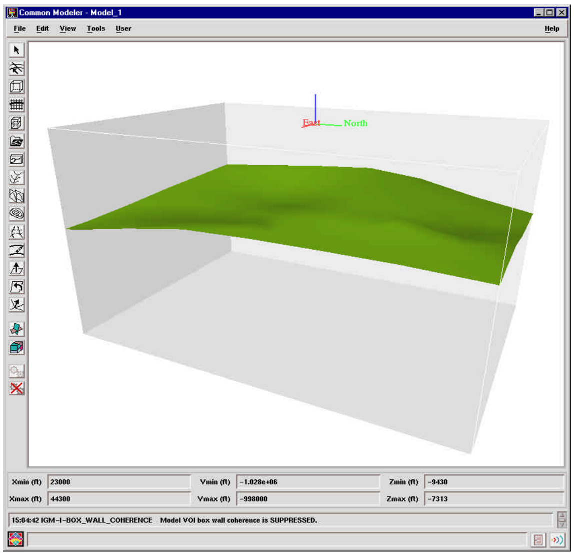

Figure 15: A surface is inserted into the volume of interest.

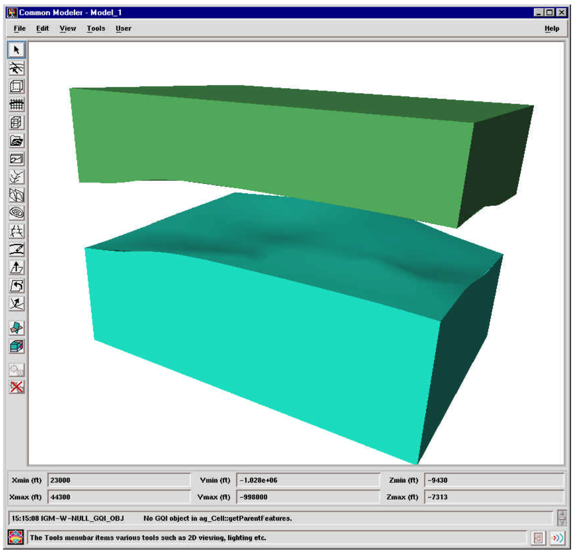

Figure 16: Explored volume view of the model in Figure 15.

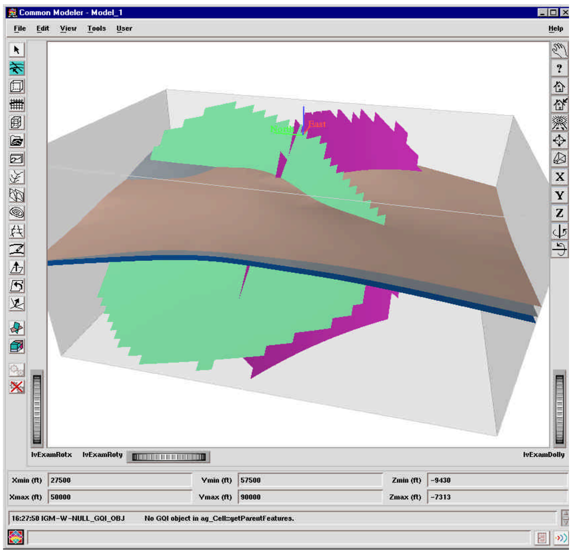

Figure 17: Five surfaces are inserted into the model.

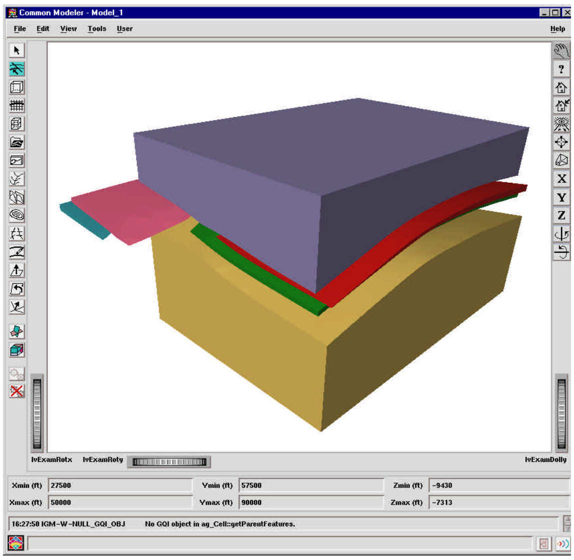

Figure 18: Explored volume view of the model in Figure 17.