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Tangent Modulus in Numerical Integration of Constitutive Relations and its Influence on Convergence of N-R Method

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Abstract

For the numerical solution of elasto-plastic problems with use of Newton-Raphson method in global equilibrium equation it is necessary to determine the tangent modulus in each integration point. To reach the parabolic convergence of Newton-Raphson method it is convenient to use so called algorithmic tangent modulus which is consistent with used integration scheme. For more simple models for example Chaboche combined hardening model it is possible to determine it in analytical way. In case of more robust macroscopic models it is in many cases necessary to use the approximation approach. This possibility is presented in this contribution for radial return method on Chaboche model. An example solved in software Ansys corresponds to line contact problem with assumption of Coulomb's friction. The study shows at the end that the number of iteration of N-R method is higher in case of continuum tangent modulus and many times higher with use of modified N-R method, initial stiffness method.

Keywords: FEM, implicit stress integration, consistent tangent modulus, Newton-Raphson method

1. Introduction

Behavior of ductile materials over yield stress under cyclic loading is in many cases complicated. An accumulation of plastic deformation — ratcheting — can be observed in case of force controlled loading with non-zero mean stress. Its simulation can be problematical [1]. For sufficient accurate description of stress-strain behavior is then necessary to use a robust plasticity model. However numerical integration of constitution laws is not trivial. The aim of the contribution is to show alternatives in the solution of such problem and to mark subsequent implementation of chosen plasticity model into the finite element software.

2. Newton-Raphson method and its modification

2.1. Solution of Global Equilibrium Equations

At the beginning can be reminded that by assuming of deformation variant of FEM is after finite element discretization obtained the system of equilibrium equations in the nodes

$$[K]\{u\} = \{F^a\}, \tag{1}$$

where $[K]$ is the global stiffness matrix, $\{u\}$ vector of unknown nodal parameters and $\{F^a\}$ vector of applied forces. In case of elasto-plastic problem the matrix $[K]$ depends on unknown

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nodal displacements or its derivations indeed and becomes the system of nonlinear algebraic equations [12].

$$[K(\{u\})]\{u\} = \{F^a\}. \tag{2}$$

Equation (2) is mostly solved in the iterative way by Newton-Raphson method [9] or by its modification [14]. One step of Newton-Raphson method is then described by equation

$$[K_i^T]\{\Delta u_i\} = \{F^a\} - \{F_i^{nr}\}, \tag{3}$$

where $[K_i^T]$ is tangent stiffness matrix, $\{F_i^{nr}\}$ is the value of load vector in i-th iteration corresponding to equivalent vector of inner forces and $\{\Delta u_i\}$ is the unknown nodal parameters vector increment which determines the vector $\{u\}$ in following iteration, i.e.

$$\{u_{i+1}\} = \{u_i\} + \{\Delta u_i\}. \tag{4}$$

It is necessary to note that in every iteration the $[K_i^T]$ and $\{F_i^{nr}\}$ are solved from unknown nodal parameters $\{u_i\}$. For more details see [2].

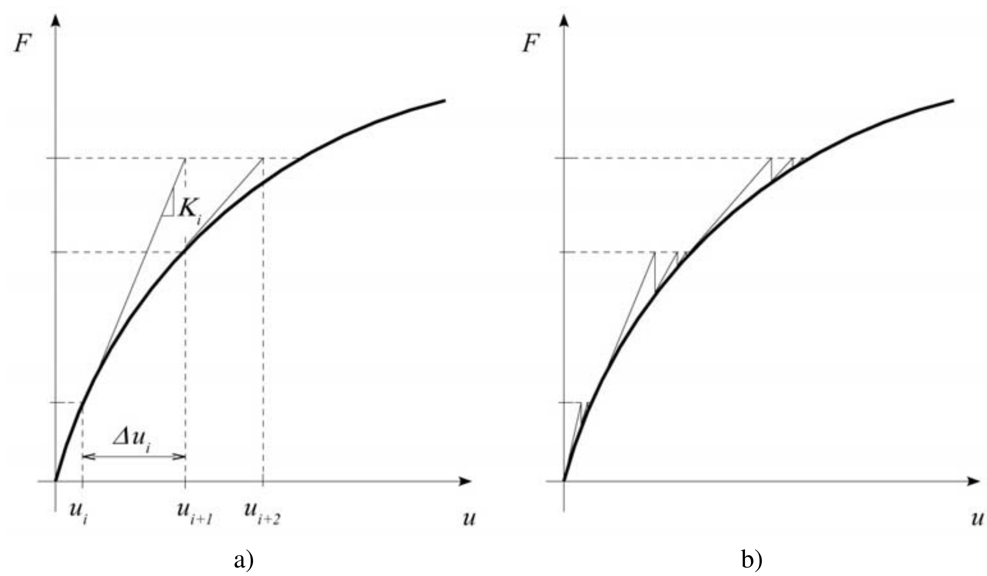

Fig. 1. Newton-Raphson method for one degree of freedom: a) basic variant, b) increment variant (full N-R)

In case of elasto-plastic problems the nonlinearity in equation (2) depends on loading history and it is necessary to use the iteration substeps. This type of Newton-Raphson method is called full Newton-Raphson method. In such case the resulting loading $\{F^a\}$ is divided in certain substeps and in each of them is applied the Newton-Raphson procedure, so

$$[K_{n,i}^T]\{\Delta u_i\} = \{F_n^a\} - \{F_{n,i}^{nr}\}, \tag{5}$$

where n marks the n-th step of solution and i is the i-th iteration inside of n-th step. The difference between both variants of Newton-Raphson method is evident from Fig. 1 where there are marked for one-dimensional problem.

Except described full Newton-Raphson method its modifications can be also used which eliminate the necessity of tangent stiffness matrix computation in every iteration substep. For example only the tangent stiffness matrix from first iteration $[K_{n,1}^T]$ can be used (Fig. 1b).

2.2. Elasto-plastic Stiffness Matrix

To guarantee the convergence of Newton-Raphson method it is necessary to determine properly the tangent stiffness matrix $[K_i^T]$ in every iteration step. It can be obtained in the "classical" way by assembly from stiffness matrixes of all elements

$$[K_i^T] = \sum_{j=1}^{NELEM} [K_e^T] \tag{6}$$

by help of so called code numbers based on indexes of unknown vector parameters $\{u\}$. The stiffness matrix of each element is then defined as

$$[K_e^T] = \int_{\Omega_e} [B]^T [D^{ep}][B] \, \mathrm{d}\Omega, \tag{7}$$

where $[D^{ep}] = \frac{\{d\sigma\}}{\{d\epsilon\}}$ is so called elasto-plastic stiffness matrix, $[B]$ is transformation matrix which depends on nodal coordinates and on derivations of approximation functions and Ω_e represents the sub-region allocated by element. The quadratic convergence of Newton-Raphson method is ensured by use of consistent elasto-plastic stiffness matrix with integration scheme used for determination of stresses in integration points, so called consistent tangent modulus (also called algorithmic tangent operator) [3]. If the non-discretized constitutive relations are used for determination of elasto-plastic matrix, the expression continuum tangent modulus (CTM) is then usually used. In paper [4] it is shown on Armstrong-Frederick nonlinear kinematic model that for small time step the Newton-Raphson method converges in the same rate both for CTM and consistent tangent modulus. The authors found also out that use of CTM for small step has not an influence of the accuracy of Newton-Raphson method.

3. Implicit stress integration

Within the solution of elasto-plastic FEM problem it is necessary to integrate in each iteration step chosen constitutive relations to obtain updated stress values [1]. Because of effectiveness and stability the implicit methods are mostly used, especially the radial return method, firstly mentioned by Wilkins [5]. Although it is usual to use the tensor notation in topical literature, the matrix notation will be used in this paper with respect to programming of described methods. The stress vector and total strain vector will be assumed as: $\{\sigma\} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}\}^T$, $\{\epsilon\} = \{\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}\}^T$, respectively.

3.1. Description of Assumed Cyclic Plasticity Model

Constitutive equations for the mechanical behavior of materials developed with internal variable concept are the most expanded technique at the last two decades [13]. In this concept, the present state of the material depends on the present values only of both observable variables and a set of internal state variables. When time or strain rate influence on the inelastic behavior can be neglected, time-independent plasticity is considered. In this paper the mixed hardening rule sometimes called Chaboche model is used. The rate-independent material's behavior model consists of von Mises yield criterion

$$f = \frac{3}{2}(\{s\} - \{a\})^T [M_1](\{s\} - \{a\}) - Y^2 = 0, \tag{8}$$

the associated plastic flow rule

$$\{d\epsilon_p\} = \sqrt{\frac{3}{2}}\, dp[M_1]\left\{\frac{\partial f}{\partial\{\sigma\}}\right\}, \tag{9}$$

the additive rule

$$\{\epsilon\} = \{\epsilon^e\} + \{\epsilon^p\}, \tag{10}$$

with Hook's law assumption for elastic strain

$$\{\sigma\} = [D^e]\{\epsilon^e\} = [D^e](\{\epsilon\} - \{\epsilon^p\}), \tag{11}$$

the nonlinear kinematic hardening rule proposed by Chaboche [6]

$$\{a\} = \sum_{i=1}^{M}\{a^{(i)}\}, \{da^{(i)}\} = \frac{2}{3}C_i[M_2]\{d\epsilon^p\} - \gamma_i\{a^{(i)}\}\, dp \tag{12}$$

and this nonlinear isotropic hardening rule

$$Y = \sigma_Y + R, \tag{13a}$$

$$dR = B(R_\infty - R)\, dp, \tag{13b}$$

where $\{s\}$ is the deviatoric part of stress vector $\{\sigma\}$, $\{a\}$ is the deviatoric part of back-stress $\{\alpha\}$, Y is the radius of the yield surface, σ_Y is the initial size of the yield surface, R is the isotropic variable, $\{\epsilon_p\}$ is the plastic strain vector, $[D^e]$ is the elastic stiffness matrix and dp is the equivalent plastic strain increment $dp = \sqrt{\frac{2}{3}\{d\epsilon_p\}^T[M_2]\{d\epsilon_p\}}$.

Auxiliary matrixes $[M_1]$ and $[M_2]$ have non-zero elements only on their diagonals $[M_1] = \mathrm{diag}\,[1,1,1,2,2,2]$, $[M_2] = \mathrm{diag}\,[1,1,1,1/2,1/2,1/2]$. Three kinematic parts ($M = 3$) in equation (12) will be assumed in this study. The model contains, except elastic parameters, nine material parameters, more precisely $\sigma_Y, B, R_\infty, C_1, \gamma_1, C_2, \gamma_2, C_3, \gamma_3$. The initial value of isotropic variable is taken as zero $R_0 = 0$.

3.2. *Euler Explicit Discretization*

Equations (8)–(13) can be discretized by Euler's backward scheme [10]. Let's assume an interval from the state n to state $n+1$

$$\begin{aligned}
\{\epsilon_{n+1}\} &= \{\epsilon^e_{n+1}\} + \{\epsilon^p_{n+1}\}, && (14)\\
\{\epsilon^p_{n+1}\} &= \{\epsilon^p_n\} + \{\Delta\epsilon^p_{n+1}\}, && (15)\\
\{\sigma_{n+1}\} &= [D^e](\{\epsilon_{n+1}\} - \{\epsilon^p_{n+1}\}), && (16)\\
f_{n+1} &= \frac{3}{2}(\{s_{n+1}\} - \{a_{n+1}\})^T[M_1](\{s_{n+1}\} - \{a_{n+1}\}) - Y^2_{n+1} = 0, && (17)\\
\{\Delta\epsilon^p_{n+1}\} &= \sqrt{\frac{3}{2}}\Delta p_{n+1}[M_1]\{n_{n+1}\}, && (18)\\
\{n_{n+1}\} &= \sqrt{\frac{3}{2}}\frac{\{s_{n+1}\} - \{a_{n+1}\}}{Y_{n+1}}, && (19)\\
\{a_{n+1}\} &= \sum_{i=1}^{M}\{a^{(i)}_{n+1}\}, && (20)\\
\{a^{(i)}_{n+1}\} &= \{a^{(i)}_n\} + \frac{2}{3}C_i[M_2]\{\Delta\epsilon^p_{n+1}\} - \gamma_i\{a^{(i)}_{n+1}\}\Delta p_{n+1}, && (21)
\end{aligned}$$

where indexes n and $n+1$ mark values in the time n a $n+1$, the symbol Δ marks the increment of the value between n and $n+1$. It can be also written

$$\{n_{n+1}\}^T[M_1]\{n_{n+1}\} = 1, \quad \text{if} \quad f_{n+1} = 0. \tag{22}$$

Assuming the combined hardening the isotropic part is to discretize. After integration of second term in (13) and after performing of discretization can be written

$$Y_{n+1} = \sigma_Y + R_\infty(1 - e^{-B \cdot p_{n+1}}), \tag{23}$$

where

$$p_{n+1} = p_n + \Delta p_{n+1}. \tag{24}$$

3.3. Application of Radial Return Method

For integration of constitutive equations the radial return method will be used now, more accurately the implicit algorithm using successive substitution method proposed by Kobayashi and Ohno [7].

It is necessary to determine a vector $\{\sigma_{n+1}\}$ to satisfy the discretized constitutive equation (14)–(24) for all known values in time n and value $\{\Delta\epsilon_{n+1}\}$ — strain-controlled algorithm.

Radial return method is classical two-step method consisting of elastic predictor and plastic corrector [5]. Elastic predictor is elastic testing stress vector

$$\{\sigma^*_{n+1}\} = [D^e](\{\epsilon_{n+1}\} - \{\epsilon^p_n\}) \tag{25}$$

and the yield criterion is then verified by testing plasticity function

$$f^*_{n+1} = \frac{3}{2}(\{s^*_{n+1}\} - \{a_n\})^T[M_1](\{s^*_{n+1}\} - \{a_n\}) - Y_n^2, \tag{26}$$

where $\{s^*_{n+1}\}$ is deviator of $\{\sigma^*_{n+1}\}$. In case of $f^*_{n+1} < 0$ the plastic deformation increment will be zero and $\{\sigma_{n+1}\} = \{\sigma^*_{n+1}\}$. However if $f^*_{n+1} \geq 0$ the condition $f_{n+1} = 0$ has to be fulfilled. After using (25) and (15) can be equation (16) written as

$$\{\sigma_{n+1}\} = \{\sigma^*_{n+1}\} - [D^e]\{\Delta\epsilon^p_{n+1}\}. \tag{27}$$

The second term at the right side, so $[D^e]\{\Delta\epsilon^p_{n+1}\}$ is called plastic corrector. If only deviator part of the equation is used, material stiffness matrix $[D^e]$ is assumed as symmetric and plastic incompressibility is supposed, then it can be written with use of (20)

$$\{s_{n+1}\} - \{a_{n+1}\} = \{s^*_{n+1}\} - 2G[M_2]\{\Delta\epsilon^p_{n+1}\} - \sum_{i=1}^{M}\{a^{(i)}_{n+1}\}. \tag{28}$$

In this relation the $\{a^{(i)}_{n+1}\}$ is given by equation (21) which can be written as

$$\{a^{(i)}_{n+1}\} = \theta^{(i)}_{n+1}(\{a^{(i)}_n\} + \frac{2}{3}C_i[M_2]\{\Delta\epsilon^p_{n+1}\}), \tag{29}$$

where

$$\theta^{(i)}_{n+1} = \frac{1}{1 + \gamma_i \Delta p_{n+1}} \tag{30}$$

fulfills the condition $0 < \theta^{(i)}_{n+1} \le 1$. Parameter $\theta^{(i)}_{n+1}$ was used in paper [7] for assuming of general kinematic rule. Now, if (29) is used in (28) assuming (18) and (19) following term will be obtained

$$\{s_{n+1}\} - \{a_{n+1}\} = \frac{Y_{n+1}(\{s^*_{n+1}\} - \sum_{i=1}^{M} \theta^{(i)}_{n+1}\{a^{(i)}_n\})}{Y_{n+1} + (3G + \sum_{i=1}^{M} C_i\theta^{(i)}_{n+1})\Delta p_{n+1}}. \tag{31}$$

Replacing of yield criterion (26) by (31) the required accumulated plastic deformation increment can be obtained in the form

$$\Delta p_{n+1} = \frac{\sqrt{\frac{3}{2}\left(\{s^*_{n+1}\} - \{a_n\}\right)^T [M_1](\{s^*_{n+1}\} - \{a_n\})} - Y_{n+1}}{3G + \sum_{i=1}^{M} C_i\theta^{(i)}_{n+1}}. \tag{32}$$

The obtained equation is non-linear scalar equation because the quantity $\theta^{(i)}_{n+1}$ and Y_{n+1} are the functions of Δp_{n+1}. For a simple example of kinematic hardening without assumption of isotropic hardening, when $Y_{n+1} = Y_n = \sigma_Y$ the equation (32) can be solved directly, because $\theta^{(i)}_{n+1} = 1$ for all of i. In other cases the solution can be found for example by iteration algorithm with successive substitution [7]. The flowchart of such method is marked in Fig. 2. Algorithm is based on following steps:

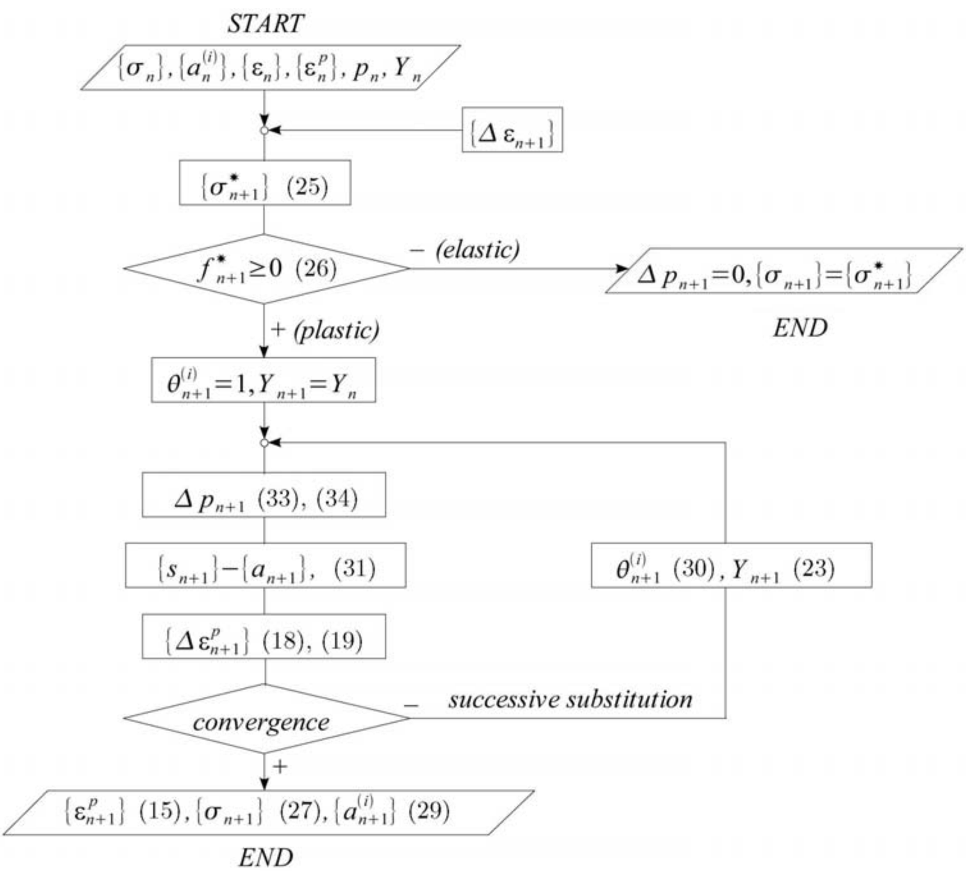

Fig. 2. Flowchart for constitutive relations integration

1. From quantities known in timestep n and from chosen $\{\Delta\epsilon_{n+1}\}$ the elastic testing stress vector $\{\sigma^*_{n+1}\}$ and using of testing plasticity function f^*_{n+1} it is decided if is the loading active or passive (see above).

2. Values $\theta^{(i)}_{n+1}$ and Y_{n+1} are chosen as $\theta^{(i)}_{n+1} = 1$, $Y_{n+1} = Y_n$.

3. From (32) Δp_{n+1} is calculated.

4. From Δp_{n+1} using (31), (19) the $\{\Delta\epsilon^p_{n+1}\}$ is calculated from (18) and $\{a^{(i)}_{n+1}\}$ from (29).

5. Convergence check using criterion (33) is done

$$\left|1 - \frac{\Delta p_{n+1}(k-1)}{\Delta p_{n+1}(k)}\right| < 10^{-4}, \tag{33}$$

   where k marks the k-th iteration. If this condition is not fulfilled the actualization of $\theta^{(i)}_{n+1}$ and Y_{n+1} is done which the Δp_{n+1} in following iteration can be calculated from. Steps 3–5 are repeated until (33) is fulfilled. The algorithm run can be accelerated [7] if after each third iteration the Aitken's Δ^2 process is calculated

$$\Delta p = \Delta p_{n+1}(k) - \frac{[\Delta p_{n+1}(k) - \Delta p_{n+1}(k-1)]^2}{\Delta p_{n+1}(k) - 2\Delta p_{n+1}(k-1) + \Delta p_{n+1}(k-2)}. \tag{34}$$

If the results is bigger then zero, the value $\Delta p_{n+1} = \Delta p$ is taken. The convergence proof of presented stress integration method with successive substitution can be found also in [7].

4. Tangent Modulus

The requirement of determination of tangent modulus in each integration point was explained in article 2.2. Therefore only tangent modulus for Chaboche model will be defined in this chapter.

4.1. Consistent Tangent Modulus (ATO)

For determination of algorithmic tangent operator — ATO can be used the paper [7] and so can be written

$$[D^{ATO}] = \frac{\{d\Delta\sigma_{n+1}\}}{\{d\Delta\epsilon_{n+1}\}} = [D^e] - 4G^2[M_2][L_{n+1}]^{-1}[I_d], \tag{35}$$

where

$$\begin{aligned}[L_{n+1}] \quad = \quad & \left(G + \frac{2Y_{n+1}}{3\Delta p_{n+1}}\right)[I] + [M_1]\sum_{i=1}^{M}[H^{(i)}_{n+1}] + \\ & + \frac{2}{3}\left(\left(\frac{dY}{dp}\right)_{n+1} - \frac{Y_{n+1}}{\Delta p_{n+1}}\right)[M_1]\{n_{n+1}\}\{n_{n+1}\}^T\end{aligned} \tag{36}$$

and deviatoric operator

$$[I_d] = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}. \tag{37}$$

For Chaboche model according to (12) and (13) can be written

$$[H_{n+1}^{(i)}] = \frac{\{d\Delta a_{n+1}\}}{\{d\Delta\epsilon_{n+1}^p\}} = \frac{2}{3}C_i\theta_{n+1}^{(i)}\left([M_2] - \{m_{n+1}^{(i)}\}\{n_{n+1}\}^T\right), \tag{38}$$

$$\left(\frac{dY}{dp}\right)_{n+1} = B(R_\infty - R_{n+1}) \tag{39}$$

and the consistent modulus can be obtained in explicit way.

4.2. Continuum tangent modulus (CTM)

Until the year 1985 when Simo and Taylor published their theory about requirement of consistent tangent modulus [3], the CTM was frequently used. The CTM can written in general form as

$$[D^{CTM}] = \frac{\{d\sigma\}}{\{d\epsilon\}} = [D^e] - 6G^2\frac{\{n\}\{n\}^T}{3G+h}, \tag{40}$$

where h is the plastic modulus. For Chaboche model can be in analytical way determined — see [1]

$$h = \sum_{i=1}^{M} C_i - \sqrt{\frac{3}{2}}\{n\}^T[M_1]\sum_{i=1}^{M}\gamma_i\{a^{(i)}\} + \frac{\partial Y}{\partial p}. \tag{41}$$

4.3. Numerical Computation of Consistent Tangent Modulus (NTM)

Let's go back to analytical determination of ATO. For expression of matrix $[L_{n+1}]$ according to (36) it is necessary to obtain for chosen kinematic rule $[H_{n+1}^{(i)}] = \frac{\{d\Delta a_{n+1}\}}{\{d\Delta\epsilon_{n+1}^p\}}$. For general kinematic rule the increment of certain kinematic parts can be determined by derivation of (29), so

$$\{d\Delta a_{n+1}^{(i)}\} = \frac{2}{3}\theta_{n+1}^{(i)}C_i[M_2]\{d\Delta\epsilon_{n+1}^p\} + \{a_{n+1}^{(i)}\}\frac{d\theta_{n+1}^{(i)}}{\theta_{n+1}^{(i)}}. \tag{42}$$

For differential approach it is suitable to rewrite (29) into

$$\{d\Delta a_{n+1}^{(i)}\} = \frac{2}{3}\theta_{n+1}^{(i)}C_i[M_2]\{d\Delta\epsilon_{n+1}^p\} + \frac{\{a_{n+1}^{(i)}\}}{\theta_{n+1}^{(i)}}\left\{\frac{\partial\theta_{n+1}^{(i)}}{\{\partial\Delta\epsilon_{n+1}^p\}}\right\}^T\{d\Delta\epsilon_{n+1}^p\} \tag{43}$$

and apply standard forward difference scheme to approximate the derivatives

$$\frac{\partial\theta_{n+1}^{(i)}}{\partial(\Delta\epsilon_{n+1}^p)_j} = \frac{\theta_{n+1}^{(i)}(\{\Delta\epsilon_{n+1}^p\} + h_T\{e_j\}) - \theta_{n+1}^{(i)}}{h_T}, \tag{44}$$

where j marks the component of vector and h_T optimal stepsize. It is obvious that the choose of stepsize will strongly influence the accuracy of differential approach. The reader is forwarded to [8] according to the comprehension of the contribution.

5. Numerical Study

According to following application of the tangent modulus determination procedure in case of more complicated constitutive relations for numerical simulations of the wheel/rail system the unique example is a cylinder loaded on its surface by normal pressure according to the Hertze

$p(x) = p_0\sqrt{1-(x/a)^2}$ and by shear stress assumed proportional to normal pressure (Fig. 3) i.e. with assumption of Coulomb's friction $\tau(x) = f \cdot p$, where f is friction coefficient. The diameter of the cylinder $d = 85$ mm, maximal pressure 800 MPa, axis $a = 0.35$ mm and coefficient of friction $f = 0.2$ were assumed in this task. The aim of the computation was to determine the stress distribution in the cylinder within one substep of NR method. Material parameters used in this numerical experiment are mentioned in Tab. 1.

Table 1. Material parameters of Chaboche model

material parameters
elastic constants: $E = 190\,000$ MPa, $\mu = 0.3$
$\sigma_Y = 235$ MPa, $B = 1$, $R_\infty = 20$ MPa, $C_1 = 67\,800$ MPa, $\gamma_1 = 694$, $C_2 = 20\,763$ MPa, $\gamma_2 = 136$, $C_3 = 2\,670$ MPa, $\gamma_3 = 0$.

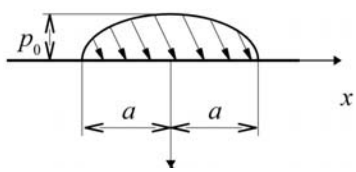

Fig. 3. Pressure applied to the surface

Described case was solved stepwise with consistent tangent modulus (ATO), continuum tangent modulus (CTM), elastic stiffness matrix (ESM) and using the tangent matrix from first iteration (ITM) so with use of modified Newton-Raphson method described in article 2.1. Results are shown in Fig. 4 left. It is obvious that influence of tangent modulus on the solution time is significant. For higher loading then in this study are the differences even more significant [1].

Consequently the influence of stepsize on the convergence and the calculation accuracy in case of numerical tangent modulus (NTM) was examined. The results with use of NTM were compared with solution using ATO, because in given case the analytical solution is not known.

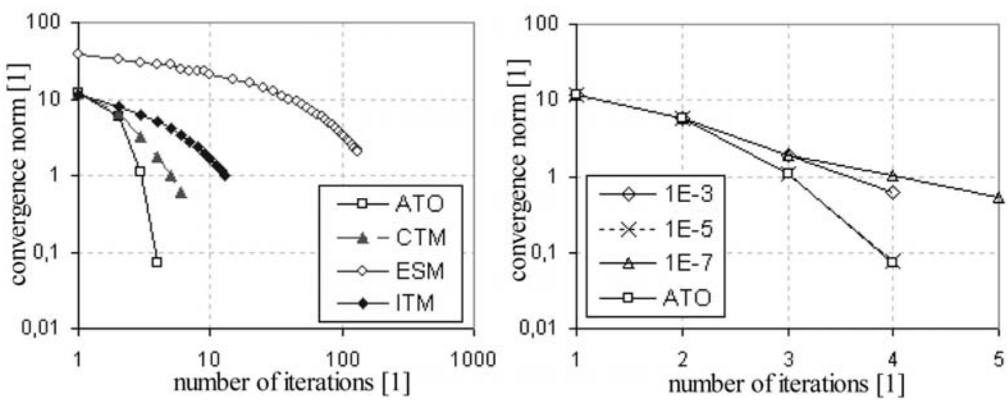

Fig. 4. Convergence of NR method for particular tangent modulus (left) and the influence of chosen stepsize on the corvergence of NR method within the numerical calculation of tangent modulus (NTM)

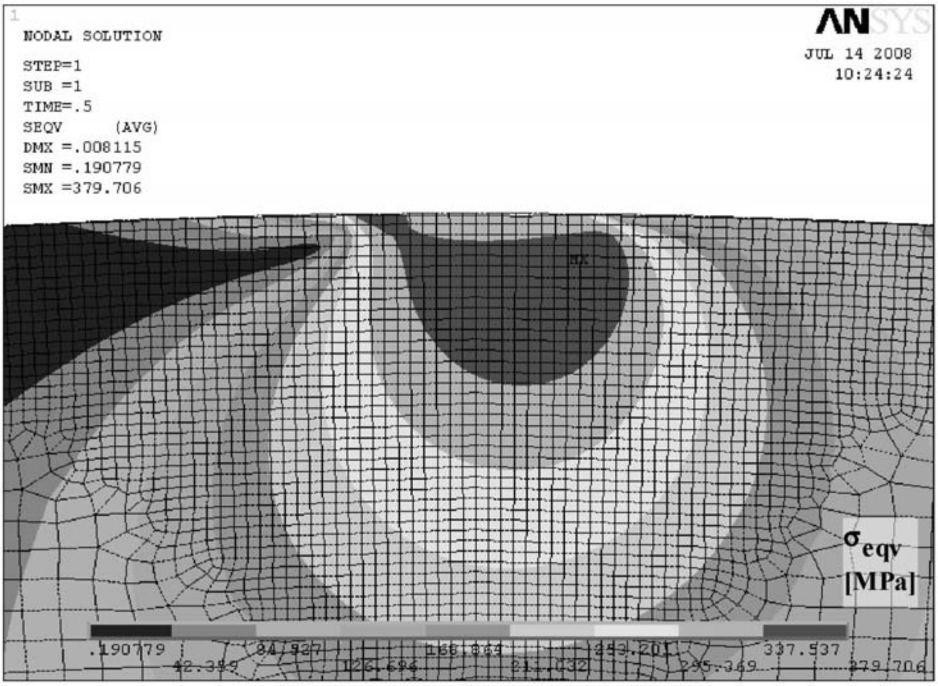

Fig. 5. Contours of equivalent stresses from computation with numerical tangent modulus (NTM, $h_T = 1e-4$)

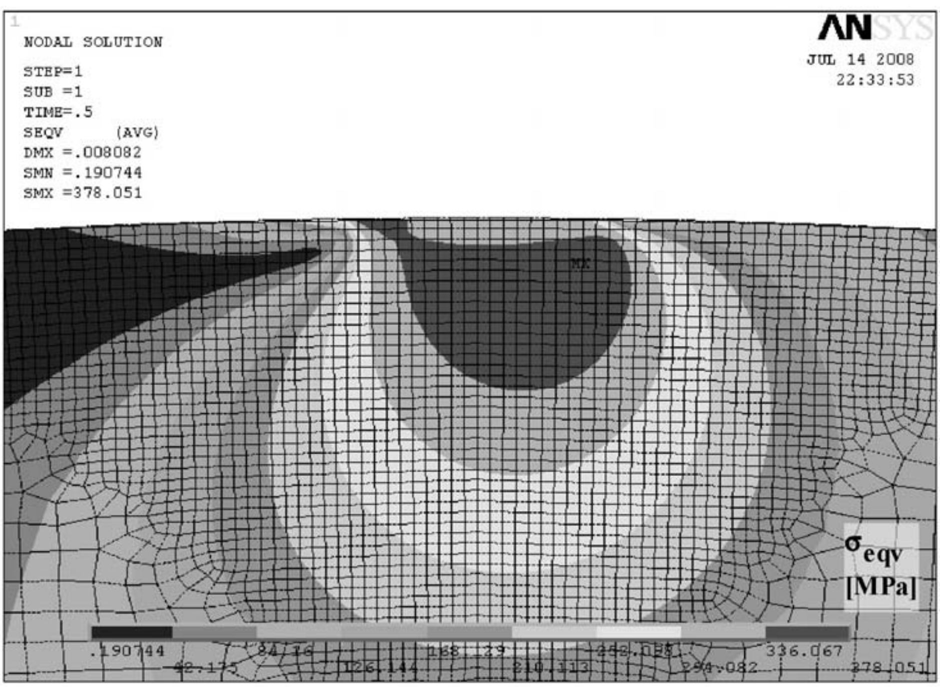

Fig. 6. Contours of equivalent stresses from computation with continuum tangent modulus (CTM)

Table 2. Some results of performed numerical experiment

Convergence Norm in Iteration	ATO	NTM				CTM
		1,00E-03	1,00E-04	1,00E-05	1,00E-07	
1	11,99	11,99	11,99	11,99	11,99	11,99
2	5,919	5,86	5,918	5,918	5,91	6,655
3	1,086	1,254	1,077	1,086	1,92	3,193
4	0,07306	0,08644	0,07288	0,07793	1,026	1,706
5					0,5318	0,9762
6						0,5905
$\Delta\varepsilon_{EQV}$	0,004996	0,004987	0,004995	0,004995	0,004992	0,004822
%error	-	0,180144	0,020016	0,020016	0,080064	3,482786

From Tab. 2 it is obvious that used method is very stable and effective. For the interval of $h_T\langle 1^{-3}; 1^{-6}\rangle$ was the solution achieved within 4 iterations, the same as in case of consistent tangent modulus. From the practical point of view the optimal value of stepsize is between 1^{-4} and 1^{-5} when the relative error of maximal plastic deformation increment was lowest — ca 0.02 percent.

The study also confirmed that CTM gives sufficiently accurate results in cases of low equivalent plastic strain increment. It can be shown on the value of maximum von Mises equivalent stress σ_{eqv}. The value of maximum equivalent stress from computation using ATO was 379.711 MPa, from computation by NTM ($h_T = 1e - 4$) then 379.706 MPa (Fig. 5) and 378.051 MPa using CTM (Fig. 6). However it is recommended by authors to use ATO or NTM in the most of cases, because of faster convergence and CTM to use for example in the case of debugging of source code, when a new plasticity model has to be tested.

6. Conclusion

In the contribution it is presented the new approximation expression of ATM using the classical differential approach. The methodology can be used in case of plasticity models where it is not possible to obtain the tangent modulus in analytical way. The advantage is obtaining of parabolic convergence of N-R method preserving the accuracy of calculation. Presented numerical experiment was performed in software ANSYS with use of user subroutine USERPL.F which serves for implementation of own constitutive relations into the software ANSYS [11]. Described procedure of numerical stress integration in case of Chaboche model can be programmed according to this paper and after linking and compiling of subroutine USERPL.F can be used for solution of described method for implementation of cyclic plasticity model which was developed for better description of stress-strain behavior of steels within author's dissertation work [1].

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