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Stress wave propagation in thin long-fiber carbon/epoxy composite panel. Numerical and experimental solutions.

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Abstract

The article deals with experimental and numerical analysis of stress wave propagation in a thin long fiber carbon/epoxy composite material. Experiments were performed on in-plane loaded square composite panels with dimensions $501\,\mathrm{mm} \times 501\,\mathrm{mm} \times 2.2\,\mathrm{mm}$. The panels have several fiber orientations (0°, 30°, 60° and 90° measured from the loaded edge). They were loaded by in-plane impact of steel sphere. The impact area was on the edge, exactly $150\,\mathrm{mm}$ from top left corners corner of the panels. The loading force was approximated by a time dependent function. Its shape was obtained from three dimensional contact analysis, which was performed on smaller area of panel. The function was used in further plane stress analysis of the whole panels. The comparison of the numerical and experimental results was executed. An attempt at determination of velocity of propagation of Rayleigh waves on the loaded edge was performed and the results are discussed in the paper. Further directions of the research are proposed.

Keywords: FRP composite, carbon, epoxy, orthotropic, Stress wave, Rayleigh waves, FEM, numerical, experimental, impact, plane stress, in-plane loading

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1. Introduction

The problem concerned with wave propagation in a long-fiber reinforced composite is discussed in the paper. The considered composite material is epoxy resin (matrix) reinforced by long carbon fibers, which are systematically arranged in the matrix in specified direction. Since the fibers are systematically orientated, a composite of this kind has strong directional properties, thus macroscopically for sufficiently long wavelength it can be regarded as a homogeneous anisotropic material.

The propagation of elastic waves in anisotropic media differs in many aspects from that customarily attributed to elastic waves in isotropic media. This fact is evident from results presented in e.g. [8], [9], [7] or [5]. For a given direction of waves propagation represented by a wave vector there will be generally three phase velocities [1], the three corresponding displacement (polarization) vectors will be mutually orthogonal but contrary to the isotropic

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case the displacements are neither truly longitudinal nor truly transverse in character. As the mechanical or material behavior of the solid becomes more complicated, the description of non-stationary wave propagation starts to be analytically intractable and, consequently, such problems are often modeled by means of discretization techniques such as finite elements (FE) or finite differences. It is very useful to supplement the theoretical analysis by experimentally obtained results.

The main aim of the paper is to study the wave propagation in thin orthotropic laminate panel loaded in-plane by stress pulse. Experimental solution utilizes for noncontact measurements a laser vibrometer. Experiments were performed on in-plane loaded square composite panels with dimensions $501\,\mathrm{mm} \times 501\,\mathrm{mm} \times 2.2\,\mathrm{mm}$. Panels have several fiber orientations. They were loaded by in-plane impact of steel sphere (diameter $4\,\mathrm{mm}$). The impact area was on the edge, exactly $150\,\mathrm{mm}$ from to left corners of the panels.

Theoretical solution is based on FE approach. The loading force was approximated by a time dependent function. Its shape was obtained from three dimensional contact analysis, that was performed on smaller area of the panel with fiber direction $0°$. The function was used in further plane stress analysis of the whole panels with all of the fiber orientation mentioned above. This was performed this way with the aim to investigate and show wether the shape of loading force depends on the fiber orientation. The comparison of the numerical and experimental results was performed and is discussed in the paper.

2. Problem formulation

Let us consider a thin unidirectional fiber-reinforced composite panel. It is also assumed that fiber diameters and thickness of the composite panel are small compared to the shortest wavelength taken into account. Hence one can consider the material as orthotropic in the state of plane stress. The principal directions of orthotropy often do not coincide with coordinate directions that are geometrically natural to the solution of the problem. Therefore it is assumed that body axes x, y form a nonzero angle α with principal material axes L, T as may be seen in fig. 1. Third axis z is identical with material axis T' and constitutes axis of rotation of principal material axes L, T from body axes x, y (fig. 1).

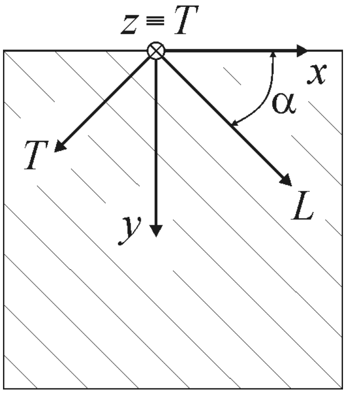

Fig. 1. Orthotropic panel.

The stress-strain relation for plane stress in principal material axes L, T can be written in the form [4]

$$\begin{pmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{pmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{pmatrix}, \tag{1}$$

where matrix elements C_{ij} depend on Young's moduli and Poisson's ratios (for explicit expressions see e.g. [4]), σ_L, σ_T and τ_{LT} are normal stresses and shear stress in material axes coordinate system and ε_L, ε_T and γ_{LT} strains and shear strain also in material axes coordinate system. Stress-strain relation in x, y coordinate system can be written in the form

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \cdot \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}, \tag{2}$$

where matrix elements Q_{ij} depend on elements C_{ij} and fiber direction angle α (for further details see e.g. [4]). The displacement equations of motion in the absence of body forces are [9]

$$Q_{11}\frac{\partial^2 u}{\partial x^2} + 2Q_{16}\frac{\partial^2 u}{\partial x \partial y} + Q_{66}\frac{\partial^2 u}{\partial y^2} + Q_{16}\frac{\partial^2 v}{\partial x^2} + (Q_{12}+Q_{66})\frac{\partial^2 v}{\partial x \partial y} + Q_{26}\frac{\partial^2 v}{\partial y^2} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{3}$$

$$Q_{11}\frac{\partial^2 v}{\partial y^2} + 2Q_{16}\frac{\partial^2 v}{\partial x \partial y} + Q_{66}\frac{\partial^2 v}{\partial x^2} + Q_{16}\frac{\partial^2 u}{\partial x^2} + (Q_{12}+Q_{66})\frac{\partial^2 u}{\partial x \partial y} + Q_{26}\frac{\partial^2 u}{\partial y^2} = \rho\frac{\partial^2 v}{\partial t^2}. \tag{4}$$

where t stands for time, u and v are displacements in the axes x and y respectively, ρ is material density.

The material consists of carbon fibers and epoxy matrix as was proposed above. It is usual, that material constants gained from manufacturer are not accurate [6], therefore they must be modified. The modified material constants were obtained by comparison with one set of experimental data. Both sets of material constants are shown in the tab. 1.

In the case of plane stress and for a given direction of wave propagation there will be two phase and at least two group velocities. Values of the phase velocities of quasi-longitudinal and quasi-transverse waves, which propagate in the direction parallel to the loaded edge of the panel with fiber direction 0° are shown in the 1. These values can be obtained from explicit relations

$$c_L = \sqrt{\frac{E_L}{\rho(1-\nu_{LT}^2\frac{E_T}{E_L})}}, \tag{5}$$

$$c_T = \sqrt{\frac{G_{LT}}{\rho}}. \tag{6}$$

The fig.2 and fig.3 show phase and group velocity curves for both sets of mechanical constants of unbounded orthotropic material.

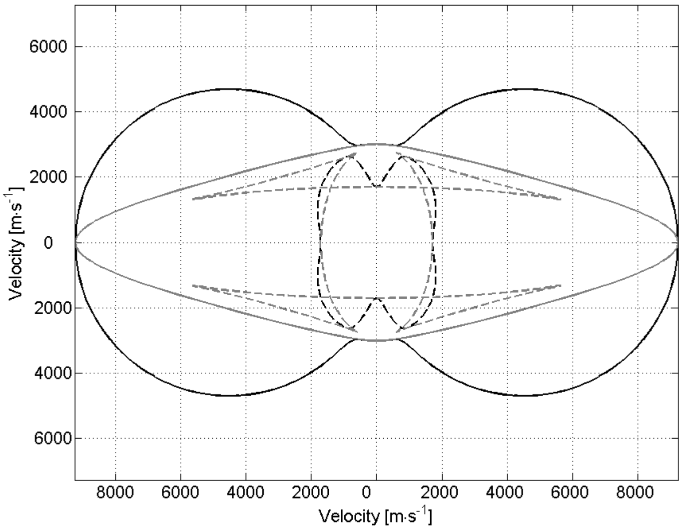

Fig. 2. Phase (black) and group (gray) velocities for manufacturer's material constants (quasi-longitudinal - solid line, quasi-transverse - dashed line).

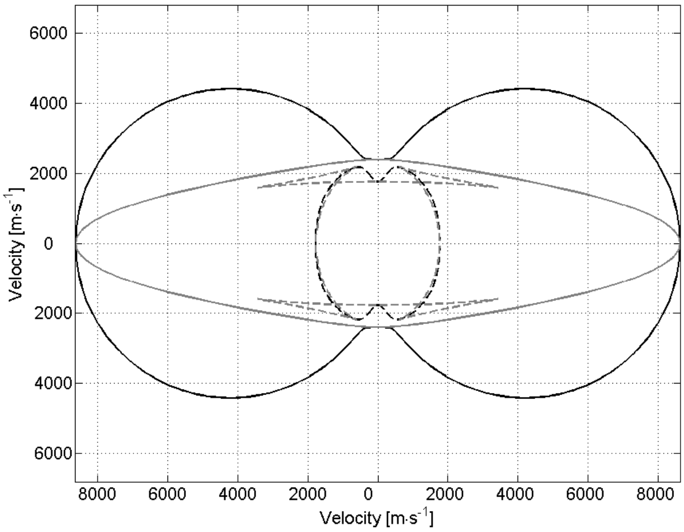

Fig. 3. Phase (black) and group (gray) velocities for modified material constants (quasi-longitudinal - solid line, quasi-transverse - dashed line).

Special type of waves, so called Rayleigh waves, propagates closely to the loaded edge of the material. This type of waves can be located closely to the free edge of the material and it vanishes with the increasing distance from the loaded edge. The velocities of the Rayleigh waves for the cases of loaded panels with fiber directions 0° and 90° are also shown in the tab. 1. The determination of c_R for chosen materials is closely discussed in [2] or [3]. It is important to know, that velocity c_R highly depends on the fiber direction in the case of composite material.

constant	manufacturer's	modified	velocities	manufacturer's	modified
E_L	$129.9\,$GPa	$113.8\,$GPa	c_L	$9223\,\mathrm{ms}^{-1}$	$8622\,\mathrm{ms}^{-1}$
E_T	$13.9\,$GPa	$8.8\,$GPa	c_T	$1709\,\mathrm{ms}^{-1}$	$1765\,\mathrm{ms}^{-1}$
G_{LT}	$4.5\,$GPa	$4.8\,$GPa	c_R - 0°	$1699\,\mathrm{ms}^{-1}$	$1745\,\mathrm{ms}^{-1}$
ν_{LT}	0.28	0.28	c_R - 90°	$1695\,\mathrm{ms}^{-1}$	$1726\,\mathrm{ms}^{-1}$

Tab. 1. Material constants and wave velocities.

3. Experiment

The dimensions of the whole measured panels, thickness, loading and measuring points a_{40}, a_{70}, a_{200}, a_{300} and fiber orientations of the composite panels are shown in the fig. 4.

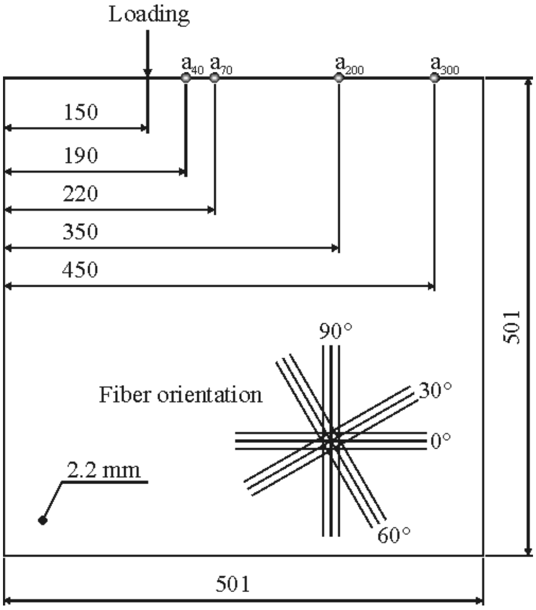

Fig. 4. Composite panel.

The Compact Laser Vibrometer 2000 (CLV) was used to measure velocities at selected points located on the loaded edges of the composite panels. The edges were loaded by in-plane impact of steel sphere with (diameter $4\,$mm) that was attached on the double sling and released from known height. The incident velocity of the sphere was calculated and used as initial condition in the FE contact analysis of the steel sphere impact on the edge of the composite sample with smaller dimensions and with fiber direction $0°$. Smaller dimension of the sample was used due to computational demands of three dimensional contact non-stationary analysis. The aim of the analysis was to obtain the time dependence of the loading force (as substitution of actual impact between two bodies) for plane stress analysis of the whole panels. The loading force and its frequency spectrum are shown in the fig. 5 .

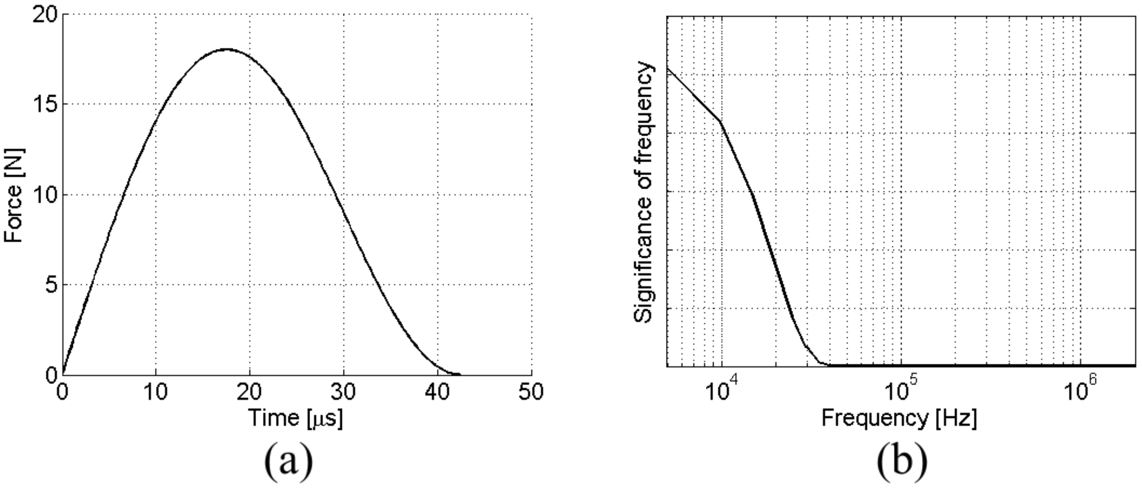

Fig. 5. Loading force (a) and its frequency spectrum (b).

4. Numerical analysis

Maximal significant frequency of the loading is $f_{max} = 44\,$kHz. It is necessary to use well sized elements to suppress the dispersion on the FE grid. Four node square elements with the edge length $a_{MKP} = 2.004\,$mm were employed in the presented work. It turned out that it was sufficient dimension of elements for analyses with both sets of mechanical constants. Considering the minimal wave length $\lambda_{min} = 10 \cdot a_{MKP}$ for which is FE grid able to transfer signals with low influence of the dispersion we can calculate maximum admissible frequency from the relation

$$\nu_{max} = \frac{c_T}{\lambda_{min}} \tag{7}$$

and maximal time increment as

$$\triangle t \leq \frac{a_{MKP}}{c_L}. \tag{8}$$

Tab.2 shows the parameters of FE net for both sets of material constants.

paramater	manufacturer’s	modified
ν_{max}	$85.3\,$kHZ	$88.1\,$kHz
$\triangle t$	$0.217\,\mu$s	$0.232\,\mu$s

Tab. 2. Parameters of the simulations.

The results obtained from numerical analysis and from experiments are shown in the figures from fig. 6 to fig. 9. Velocities in the direction perpendicular to the loaded edges in chosen measuring points for all types of panels are shown here. It may be seen that the greater angle fiber we have the smaller amplitude of measured signals we get. Also for increasing angle loading force becomes thinner and has greater amplitude. All numerical analyses were performed with use of loading force obtained from contact analysis of panel with fiber angle $\alpha = 0^\circ$.

5. Conclusion

The experimental and numerical solutions of the stress wave propagation problem in thin unidirectional long-fiber composite panel were obtained. Results of both approaches have been compared mutually and sufficient agreement has been found. On the contrary, certain interesting and important problems have appeared. The numerical results obtained with the use of the modified material constants still show minor differences compared to those gained by experiment and theory. Therefore the program for precise evaluation the velocity c_R from the numerical results was developed. It shows the difference about 1% against values of c_R obtained by analytical method. This discrepancy can be caused by grid dispersion or by choosing a big time increment etc. It will be subjected to further analysis. Also, it is very difficult to specify value of c_R from the experimental results with sufficient accuracy due to oscillations of signal. This can be caused by imperfection of the experimental devices or by the influence of the disturbances of the surroundings when performing the experiment. And so one of the main aims for the next research will be the development of program that will be able to specify values of c_R from experiments more accurately. The next aim of the further research will be the investigation of the influence of dispersion on the accuracy of the numerical solution of Rayleigh wave propagation in comparison with analytically obtained values of c_R and precise determination of velocity c_R for composite materials with various fiber orientations.

Acknowledgements

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References

[1] S. Abrate, Impact on composite structures, Cambridge university press, 1998.

[2] J. Červ, Rayleigh (Edge) Waves in a Thin Orthotropic Semi-Infinite Laminate, Proceedings of Eleventh International Congress on Sound and Vibration, 5-8 July 2004, St. Petersburg, Russia, pp. 3519-3526.

[3] J. Červ, R-Waves in Thin Specimens Made of Laminates or Some Anisotropic Metals, Proceedings of the International Scientific Conference, Session 8 - Applied Mechanics, September 7-9, 2005, Ostrava, Czech Republic, pp. 253 - 256.

[4] R. M. Jones, Mechanical of Composite Materials, Scripta Book Company, Washington, D.C., 1975.

[5] T. Kroupa, J. Červ, V. Laš, Vlny napětí v tenkém pásu s obecnou ortotropií, Proceedings of Computational mechanics 2004, Nečtiny, ZČU Plzeň, Czech Republic, 2004.

[6] T. Kroupa, R. Zemčík, V. Laš, Identification of composite material properties using non-linear stress-strain relation, Proceedings of Applied mechanics 2006, Srní, Czech Republic, 2006.

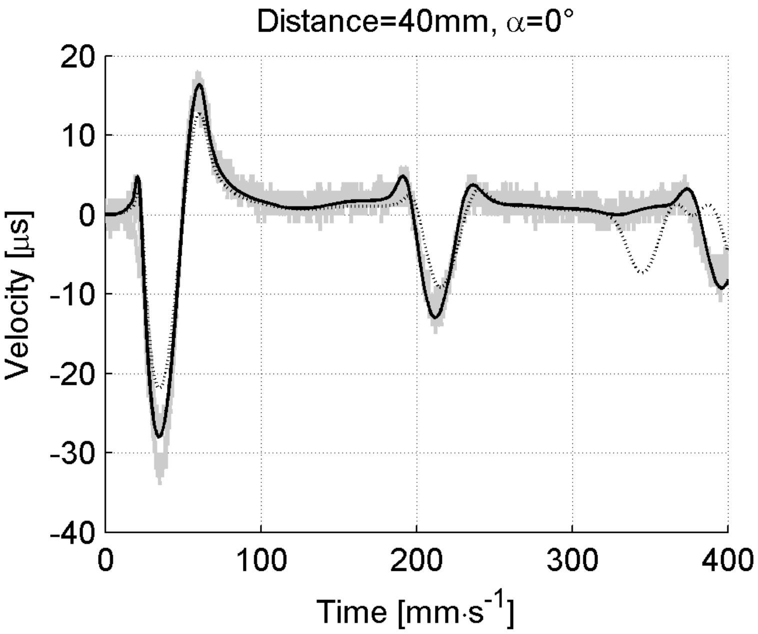

Fig. 6. FEM (black - modified, black dashed - manufacturer’s constants) and experimental (grey).

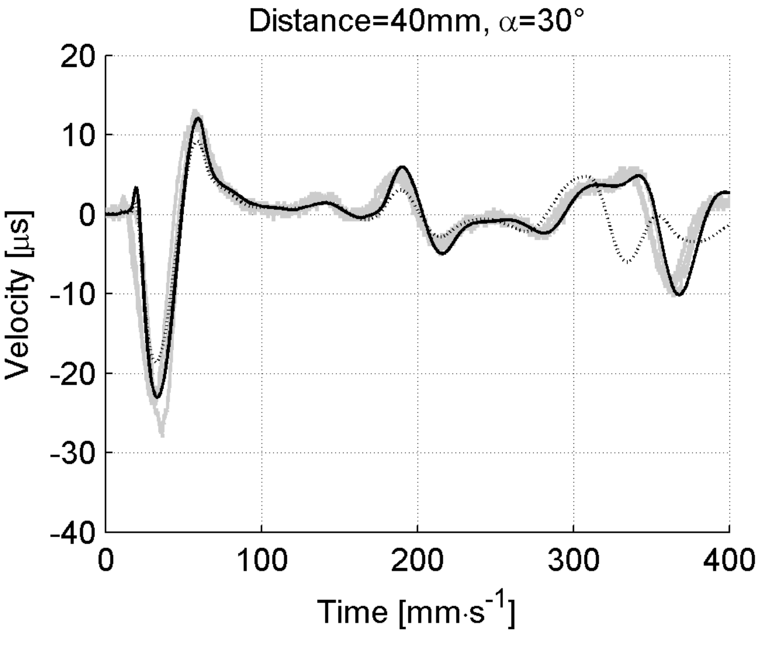

Fig. 7. FEM (black - modified, black dashed - manufacturer’s constants) and experimental (grey).

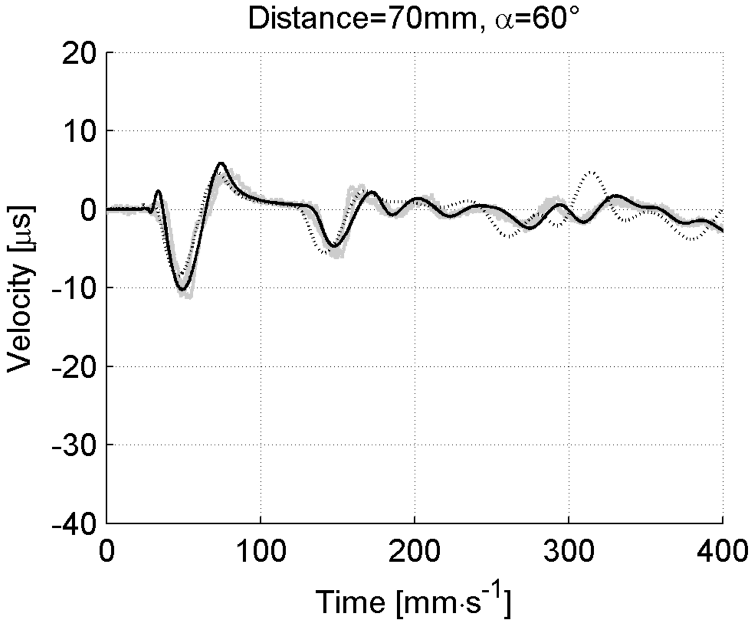

Fig. 8. FEM (black - modified, black dashed - manufacturer’s constants) and experimental (grey).

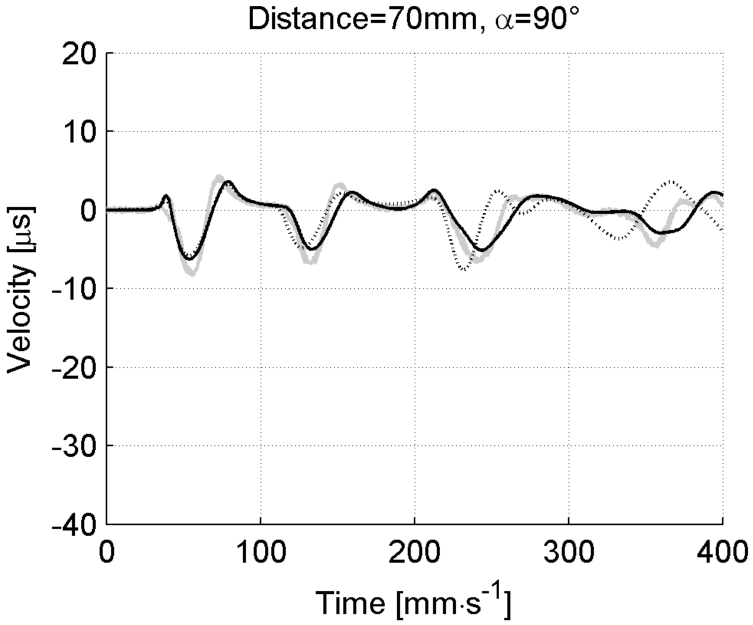

Fig. 9. FEM (black - modified, black dashed - manufacturer’s constants) and experimental (grey).

[7] T. Kroupa, R. Zemčík, V. Laš, Progressive failure analysis of orthotropic plate loaded by transverse low-velocity impact, Proceedings of 5th Inernational Congress of Croatian Society of Mechanics, Trogir/Split, Croatia, 2006.
[8] R. Zemčík, J. Červ, V. Laš, Numerical and experimental investigation of stress wave propagation in orthotropic strip, Proceeding of Computational Mechanics 2003, Nečtiny, ZČU Plzeň, 2003.
[9] R. Zemčík, Non-stationary progressive failure analysis of fiber-reinforced composites, Disertation thessis, ZČU FAV, Plzeň, 2004.