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# COMPUTATION OF 3D ELECTROSTATIC FIELDS EXCITED BY THIN CONDUCTORS

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**Abstract:** Solution of 3D electric fields in a system of thin charged conductors of general shapes by classical differential methods (FDM, FEM etc.) is extremely difficult due to ignorance of boundary conditions and existence of geometrically incommensurable subdomains within the investigated area. Application of integral methods is also complicated because the kernel functions occurring in the first-kind Fredholm equations are integrable only in 2D and not in 1D. Conductors with 2D cross-section correspond, in fact, with the physical reality, but their introduction leads to very large fully populated matrices whose processing on common PCs is still practically impossible. The paper offers an alternative algorithm consisting in substitution of each thin conductor by a set of point charges placed on a helicoidal curve surrounding it. The theoretical analysis is supplemented with an illustrative example and discussion of the results (convergence, accuracy).

**Keywords:** Electrostatic field, thin conductor, point charge, numerical analysis.

## 1 Introduction

A lot of tasks in the domain of electrical power engineering (and not only there) is based on the knowledge of electric field in relatively large areas containing charged conductors or elements of general shapes (substations, various high-voltage apparatus, towers etc.). Computation of its distribution is, however, an uneasy business. Due to complexity of the arrangement, analytical methods are practically inapplicable, and also numerical methods do not often lead to acceptable results. Classical FDM or FEM based techniques may fail because of lack of the boundary conditions and problems with meshing (thin conductors versus large volume of ambient air). Situation can partially improve when using codes with implemented open boundary techniques; these are, on the other hand,

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extremely expensive. Even integral methods do not represent the best choice. As known, the first step is here calculation of the distribution of charge along the conductors by means of the first-kind Fredholm equations. Their kernel functions that are weakly singular are, however, integrable only in 2D. This is in accordance with the physical reality; no conductor is, in fact, infinitely thin. But surface discretisation of conductors of finite dimensions leads to very large system of equations characterised by a fully populated matrix whose processing on common PCs is often impossible.

The paper offers an alternative method. Real conductors in the system are first replaced by thin filaments and these are again replaced by sets of point charges located along a helicoidal curve closely surrounding the filament. Values of the point charges are calculated in such a manner that potential at the place of the filament is equal to potential of the original conductor. The field quantities at any point in the area may then easily be calculated from the Coulomb law.

## 2 Mathematical model of the problem

Let us consider a system of $n$ charged 1D filaments $C_i$ of general shape (Fig. 1) carrying potentials $\varphi_i$, $i = 1,...,n$. The filaments are placed in an isotropic medium with relative permittivity $\varepsilon_r$. The task is to find distribution of the electric field in such a system.

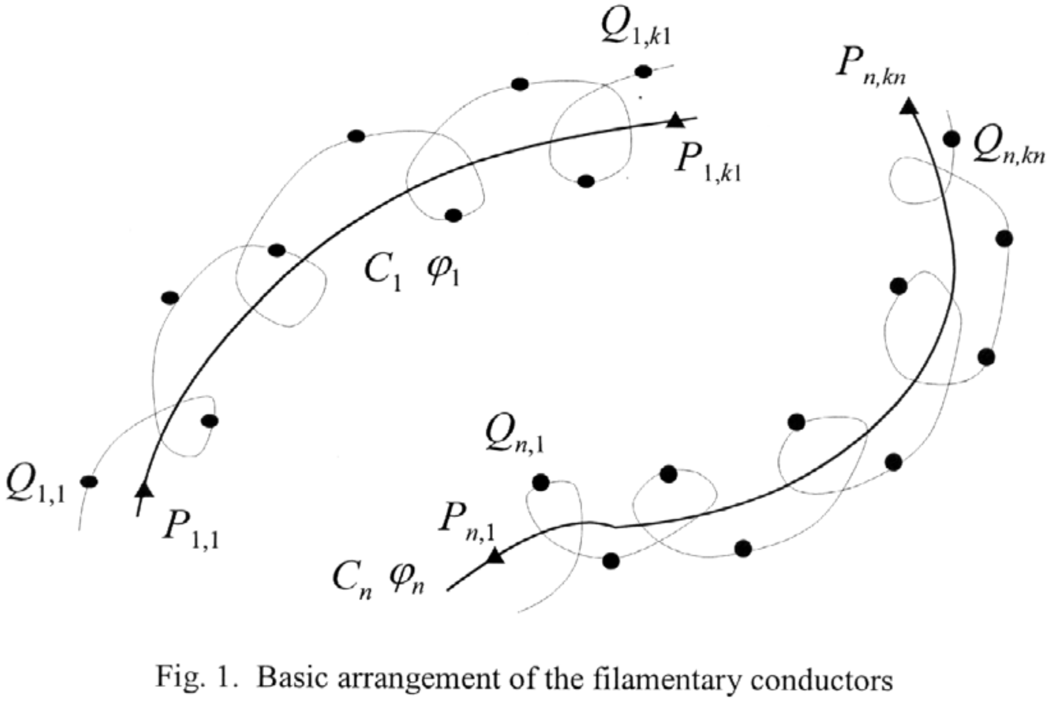


Fig. 1. Basic arrangement of the filamentary conductors

Thin conductors $C_1$, ... $C_n$ are replaced by sets of point charges (conductor $C_1$ by charges $Q_{1,1}$, ..., $Q_{1k1}$ etc.) located along helicoidal curves surrounding the filaments. While position of these point charges can be to some extent arbitrary, their values are determined from the condition that they produce the prescribed potential along the original filaments. Let us further choose (Fig. 1) points $P_{1,1}$, ..., $P_{1k1}$ located on conductor $C_1$ etc. For any point $P_{m,l}$ where $m \in \langle 1, n \rangle$ and $l \in \langle 1, k_m \rangle$ of this set we can write an equation

$$\varphi_m \left( P_{ml} \right) = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{n} \sum_{j=1}^{k_i} \frac{Q_{ij}}{\left| r_{Q_{ij}} - r_{P_{ml}} \right|} \tag{1}$$

where $\varphi_m$ denotes potential of the $m$-th conductor and expression in the absolute value in the denominator the distance between the charge $Q_{i,j}$ and point $P_{m,l}$. The system (1) provides the values of charges $Q_{i,j}$ that are immediately used for consequent computations of the field quantities within the investigated area. These are determined by means of relatively simple algebraic expressions.

## 3 Illustrative example

The methodology is illustrated on an example depicted in Fig. 2. The task is to calculate the distribution of electric field quantities (potential, electric field strength) near two equal circular loops with feeders made from thin conductors. Potential of loop $\mathbf{L}_1$ is 100 V while potential of loop $\mathbf{L}_2$ is 0 V. Both loops are placed in the air ($\varepsilon_r = 1$). The field has strongly expressed 3D character.

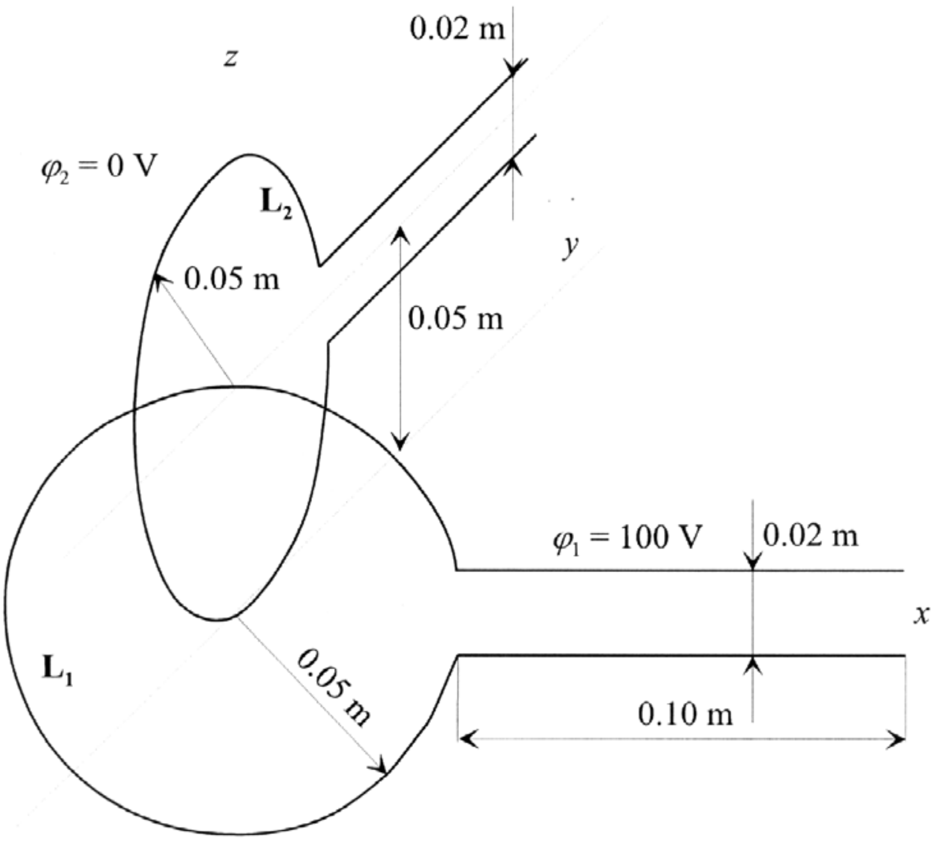


Fig. 2. The investigated arrangement

The radius of the helicoidal curve with the substitutive point charges was 0.1 mm with ten points per one turn. The task was solved in the Cartesian coordinates.

First, geometrical convergence of the method was tested, consisting in computation of the potential and electric field strength at selected points for increasing number of the point charges. The results are shown in Figs. 3 and 4.

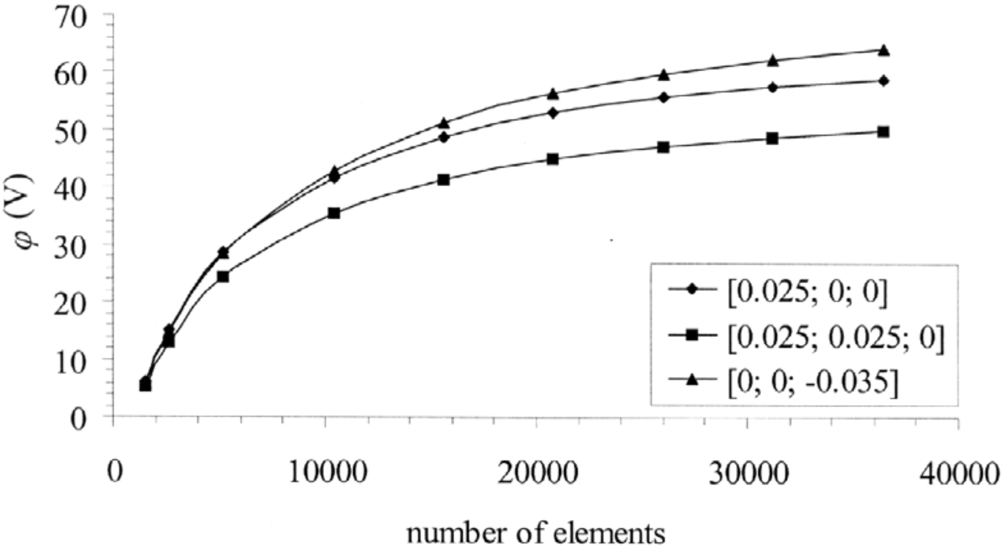


Fig. 3. Convergence of potential $\varphi$ at three selected points

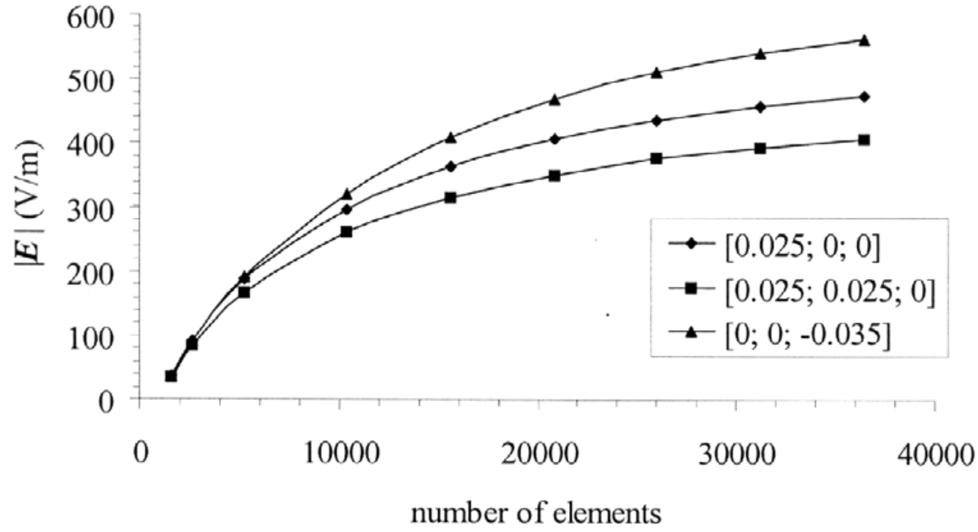


Fig. 4. Convergence of the module $|\boldsymbol{E}|$ of electric field strength at three selected points

Next figures show distribution of the individual components and module of electric field strength along several lines. Fig. 5 shows distribution of these components along axis $x$, Fig. 6 along axis $y$ and Fig. 7 along axis $z$.

The field quantities may be considered sufficiently precise at the distance about 3 mm and more from the conductors. Near the loops, however, they are strongly distorted due to used substitution. It can be seen, for example, in Fig. 6 (oscillations in the neighbourhood of point $y = 0$).

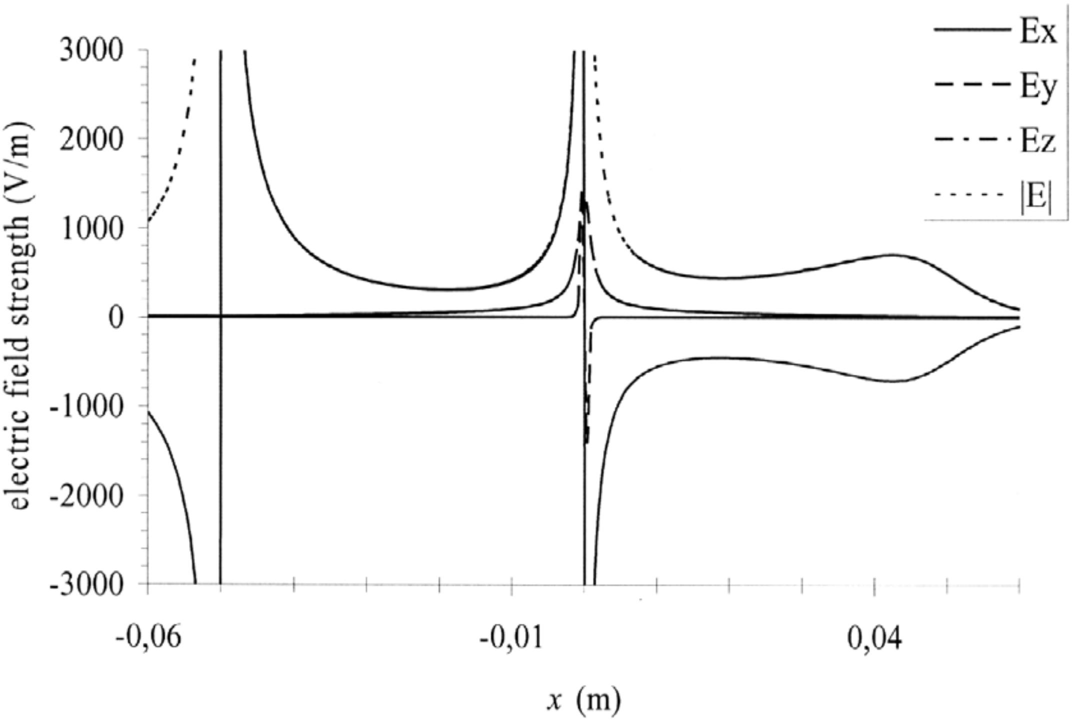


Fig. 5: Distribution of the module of electric field strength and its components along axis *x*

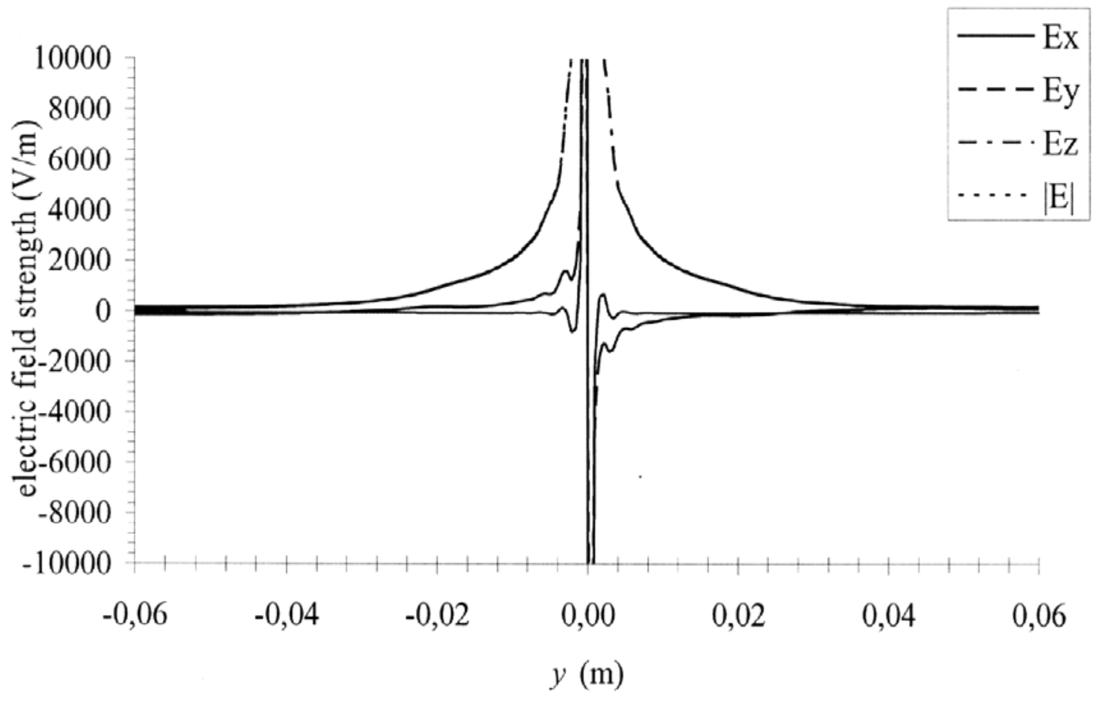


Fig. 6: Distribution of the module of electric field strength and its components along axis *y*

# 4 Conclusion

The method is well applicable in the cases where it is not necessary to know the field distribution in the neighbourhood of its sources. Next work in the domain will be aimed at testing the method in more complex arrangements and estimation of the errors by comparing the results for simple, analytically solvable dispositions.

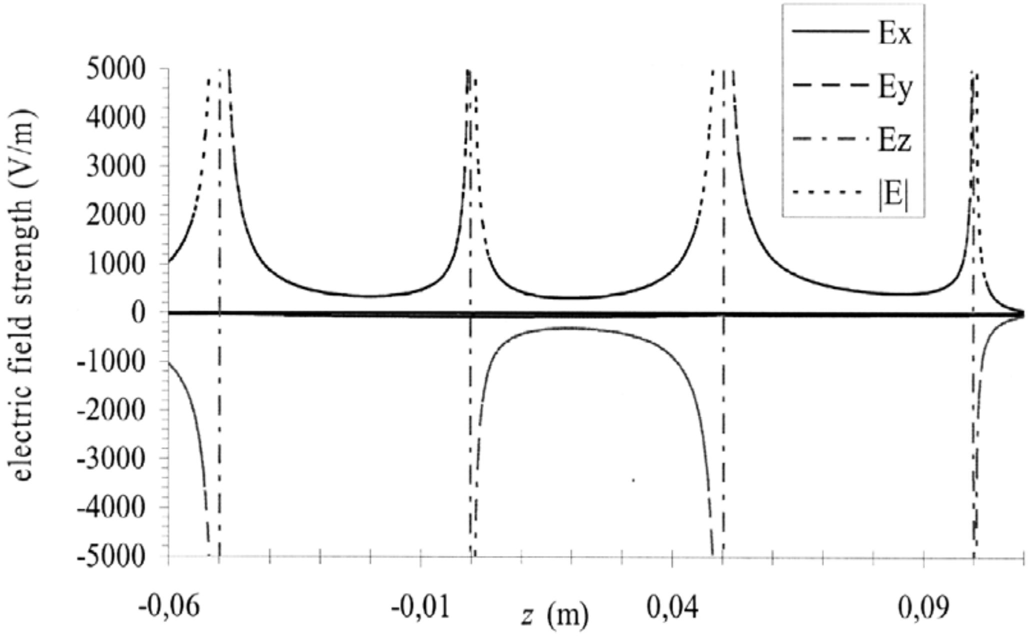


Fig. 7: Distribution of the module of electric field strength and its components along axis *z*

# 5 Acknowledgement

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