CONTRIBUTION TO THE THREE-DIMENSIONAL TOMOGRAPHY MODEL

PROF. ING. LIBOR DEDEK, CSC.[1]

Abstract: Electrical impedance tomography (EIT) is an imaging tool for clinical and process applications, in which maps of the electric conductivity (or, more generally admittivity) distribution inside a body are formed from the current-to-voltage map of the body's surface. EIT is usually a three-dimensional problem, as the current lines in practical problems do not remain localized in a plane. The transposition of two-dimensional (2-D) EIT to three-dimensional (3-D) EIT is a challenging task and imposes significant increase in computational power demands and storage requirements. This work demonstrates most advance techniques, which are used in EIT. Described is 3-D image reconstruction algorithm based on the Newton method in which optimal results are obtained for the reconstruction process. More importantly, the algorithm incorporates 3-D forward modeling, based on the finite element method (FEM), in which is proposed accurate modeling of current fed and voltage collecting electrodes. Two programs as well as experimental vessel with objects of different conductivity are being developed.

Keywords: Electrical impedance tomography, Three-dimensional problem, Inverse problems, Finite Element Method, Floating potential electrodes, Current driven electrodes.

1 Introduction

EIT is a new imaging technique based on the evaluation of internal distribution of the conductivity inside conductive objects. An array of electrodes is attached on the boundary of the object and small alternating currents are injected through the selected pairs of electrodes. The resulting voltages are measured between the other adjacent pairs of electrodes according to Fig. 1 and sometimes even between the current electrodes. The volume conductivity γ distribution or it inverse ρ is evaluated from the potentials measured on the boundary, forming vector $\mathbf{U}_\mathrm{m}$. There are many methods how to solve the problem from fast and less accurate to more sophisticated and time consumable.

---

[1] Libor Dědek, FEKT VUT, Purkyňova 118,112 00 Brno, e-mail: dedek@feec.vutbr.cz

This work was supported by the Grant Agency of the Czech Republic (GAČR) under Grant No. 102/03/1108.

In this work we use for the conductivity modelling the FEM and for the inverse method the generalized Tichonov regularization.

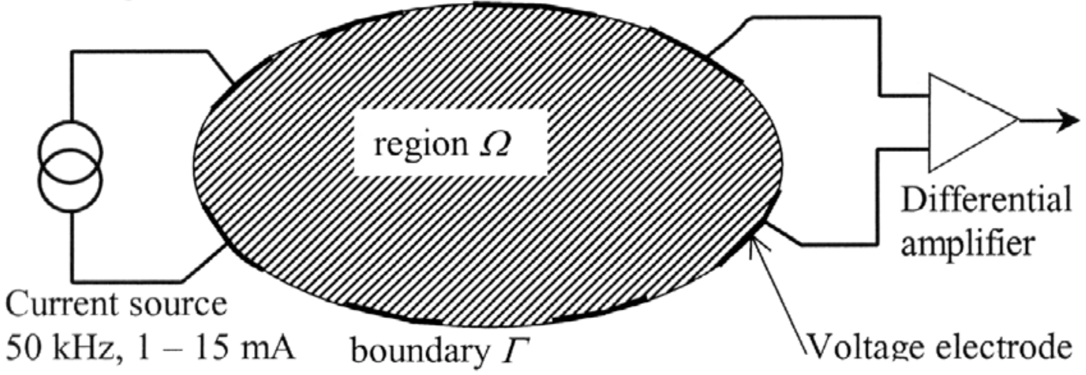

Fig. 1. Principle of EIT apparature

Compared with techniques like positron emission or x-ray tomography, EIT is at least thousand times cheaper and smaller. The method is very fast and so it can also be applied to partially moving objects.

2-D EIT problems are discussed in some books, e.g. [1], [2] and in a lot of papers. Thousands of references can be found on different web sites. Only few 3-D EIT problems are discussed, for example, in [3] to [6].

In this paper we discuss the estimation of difference and static 3-D conductivity distribution. We use, according to [6], simplified complete electrode model to describe the EIT measurements. FEM is used to obtain a discretization of the governing equations. Moreover, we use accurate models of both current and voltage electrodes using the Galerkin method.

2 The forward problem

The field in the forward model is supposed to satisfy the Laplace equation for potential u in a region Ω of unknown conductivity γ (see Fig. 1)

$$\nabla \cdot (\gamma \nabla u) = 0. \tag{1}$$

The boundary Γ consists of three parts, namely of a pair of current source electrodes $\Gamma_{I1,2}$, a set of voltage electrodes Γ_{Vj}, j = 1, … N_v, and the free surface Γ_n among electrodes. The following boundary conditions hold

$$\int_{\Gamma_{Ij}} \gamma \frac{\partial u}{\partial n} d\Gamma = I_j \text{ on } \Gamma_{Ij}, \quad \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma_n. \tag{2a, b}$$

Further u_j is the measured floating potential of an unloaded j-th voltage electrode, forming vector $\mathbf{U_E}$. Usually only two electrodes are fed from a current source, so that holds the law of conservation of the charge. Voltage set is measured between adjacent pairs of other electrodes. It is unusual to make voltage measurement from electrodes that are carrying current, since the voltage drop across the electrode-skin resistance is unknown. The EIT system then switches the current to another pair of electrodes, and measures a second set of voltages and so on. At least one point or an electrode is set to zero to preserve the existence and the uniqueness of the solution.

2.1 FEM implementation

The FEM is used to turn the continuous problem into a discrete formulation. The solution domain Ω is discretized into 3-D finite elements. An example of meshing of a cylindrical tank under development is in Fig. 2. The mesh consists of 336 linear prismatic elements with 7 vertical element layers. The number of nodes is 216. From totally 40 electrodes are darkened two in the bottom layer.

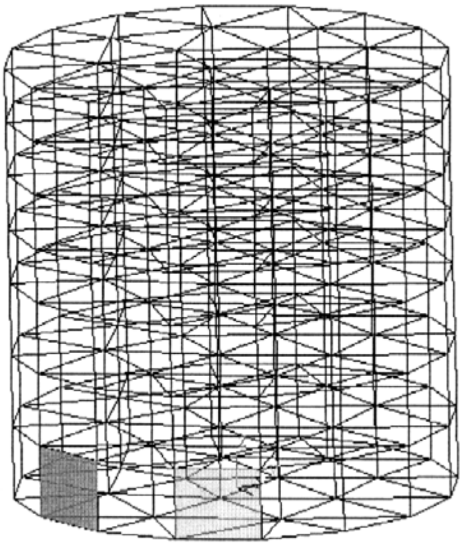

Fig. 2. 3-D mesh of a cylindrical tank using upright linear prismatic elements

The Galerkin method is used to transform (1) using approximation

$$u = \sum_{j=1}^{n} u_j W_j(x, y, z) \tag{3}$$

Here u_j and W_j are nodal potential and weighting functions. We obtain a system of equations, which can be written in the matrix form as

$$\mathbf{G}(\mathbf{U}_{\text{int}} + \mathbf{U}_{\text{E}})^T = \mathbf{I} \tag{4}$$

where $\mathbf{U}_{\text{int}}, \mathbf{U}_{\text{E}}$ is vector of internal and electrode nodal potentials, respectively. From the last equation vector $\mathbf{U}_{\mathbf{E}}$ is calculated. Procedure described in [8], [9] is used to model current supplied and voltage electrodes with minimal error.

3 The inverse problem

EIT image reconstruction is a non-linear ill-posed inverse problem for which reason we have to use regularization techniques. We use the Tikhonov regularization, such as in [5], [6] which can be written in the form

$$\min_{\rho}\left\{\left\|\mathbf{U}_{\mathbf{m}}-\mathbf{U}_{\mathrm{E}}\left(\boldsymbol{\rho}_{\mathbf{0}}\right)+\mathbf{J}\boldsymbol{\rho}-\mathbf{J}\boldsymbol{\rho}_{0}\right\|^{2}+\left\|\mathbf{L}\left(\boldsymbol{\rho}-\boldsymbol{\rho}_{0}\right)\right\|^{2}\right\} \tag{5}$$

where **J** is the Jacobian of $\mathbf{U}_{\mathrm{E}}(\boldsymbol{\rho})$ with respect to $\boldsymbol{\rho}$, ($\boldsymbol{\rho}$ is the vector of the element resistivities, **L** is a regularization matrix). The regularization matrix is usually chosen so that it approximates the directional derivatives in three dimensions. The solution of (5) is

$$\delta\boldsymbol{\rho}=\left(\mathbf{J}^{\mathrm{T}}\mathbf{J}+\mathbf{L}^{\mathrm{T}}\mathbf{L}\right)^{-1}\left(\mathbf{J}^{T}\delta\mathbf{U}\right) \tag{6}$$

where $\delta\boldsymbol{\rho}=\boldsymbol{\rho}-\boldsymbol{\rho}_{0}$ is estimated small resistivity change and $\delta\mathbf{U}=\mathbf{U}_{\mathrm{m}}-\mathbf{U}_{\mathbf{0}}$ is the measured voltage change. This is so called difference model. Also a static model for the large resisitivity changes can be obtained from (6) ([5]).

4 Conclusion

Two independent programs are written in FORTRAN and MATLAB - FEMLAB environment to solve the above formulated problem. The simple system of a saline filled tank according to Fig. 2 is chosen for its easy FEM model generation when a triangular grid on the circle is translated in the z – axis direction. This form of the tested body is classical and the results can be compared with the results published previously.

References

[1] Williams, R. A., Beck, M. S., editors: Process Tomography, Principles, Techniques and Applications. Butterworth – Heinemann Ltd, Oxford, 1995.

[2] Sikora J.: Numerical Algorithms in Impedance and Eddy Current Tomography. WPW, Warszawa, 2000 (in Polish)

[3] Metherall, P., et al.: Three-dimensional electrical impedance tomography. Nature, 380, pp. 509 – 512, 1996

[4] Pinheiro, P. A.T., Loh, W. W., Waterfall,R. C.: Developments of 3-D Reconstruction Algorithms for ERT. 1st World Congress on Industrial Process Tomography. Buxton, 1999.

[5] Vaukhonen, P. et al.: Three-dimensional electrical impedance tomography based on the complete electrode model. IEEE Trans Biomed Eng, vol. 46, pp. 1150 – 1160, 1999.

[6] Vaukhonen, M. et al.: Tikhonov regularization and prior information in electrical impedance tomography. Report No. 3/1996 University of Kuopio, Department of applied physics, report series ISSN 0788-4672

[7] Dědek, L.: Implementation of total prescribed electrode fluxes into a FEM potential program. 18th SPETO, Gliwice, 1995, Vol. 1, pp. 111 - 116.

[8] Dědek, L.: Implementation of electrodes with a floating potential into FEM (in Czech). In: Proc. of 7th international conference, section High voltage. TU FEI Košice, Slovakia, 1994, pp. 217-221.