FACULTY OF ELECTRICAL ENGINEERING
KTE
September 10 - 12, 2003
Pilsen, Czech Republic

DISTRIBUTION OF THE ELECTRIC FIELD ON THE SURFACE OF BUNDLED CONDUCTORS

PROF. ING. DANIEL MAYER DRSC.[1]
ING. VÍT VESELÝ[2]

Abstract: The paper deals with computation of electric field distribution along the surfaces of a system of parallel conductors with various potentials. The method starts from integral equations and the elaborated algorithm is applied to *hv* and *uhv* overhead lines with bundle conductors. The results allow evaluating of danger of giving rise to corona (even with respecting the influence of rough surface of the cable) with all its interference effects. The algorithm was compared with other already published methods with remarkable agreement.

Keywords: *hv* and *uhv* overhead lines, bundle conductors, corona, method of integral equations.

1 INTRODUCTION

Hv and *uhv* overhead lines are mostly realised by bundle conductors. Electric field strength on their surfaces depends on potential of particular conductors, their radii, mutual distances and quality of their surface. Exceeding the critical value $E_{\text{crit}} \approx 21\,\text{kV/cm}$ gives rise to corona. One of the advantages of using bundle conductors is reduction of this discharge that, as is well known, increases losses along the line, unfavourably affects near telecommunication devices and produces acoustic noise.

Various methods have been used for determining the electric field near the bundle. Older papers, see, for instance, [2], [15] started from an analytical solution of electric field produced by one conductor and the resultant electric

[1] FEL ZČU, sady Pětatřicátníků 14, 30614 Plzeň, e-mail: mayer@kte.zcu.cz
[2] FEL ZČU, sady Pětatřicátníků 14, 30614 Plzeň, e-mail: vvesely@kte.zcu.cz

field of a bundle was calculated using superposition. The results obtained in this way are, however, only approximate. Other authors [3], [4] and [8] replace the particular conductors in the bundle by a system of suitably located current filaments and solve their electric field. Conformal mapping was used in [13]. Simulated charge method was successfully applied in [11] and [12]. Numerical calculation based on standard finite element techniques (and realised by means of professional codes) would be comfortable, nevertheless, problems with geometrical incommensurability (small cross-sections of the conductors versus their large mutual distances and domain containing air) would lead to using strongly nonuniform mesh and enormous number of equations. The task represents, moreover, an open boundary problem, which is often a source of further errors.

The principal advantage of this access consists in the fact that the numerical computation of the charge density is much simpler than computation of the distribution of potential near the conductors (which would, of course, also provide values of E_n). While distribution of potential in the solved arrangement represents a 2D problem, solution of the charge density is only a 1D problem.

2 MATHEMATICAL MODEL OF THE PROBLEM

Considered is homogeneous, linear and isotropic dielectrics (air) of permittivity ε_0 with a set of n direct parallel conductors with constant potentials φ_k, $k=1,\dots,n$. Electric charge on their surfaces is supposed to be distributed continuously, with charge density σ_k that does not change along their length. The electric field near the conductors is obviously two-dimensional. Potential at a general point B of this field is given by expression [7]

$$\varphi(\boldsymbol{r}_\mathrm{B}) = \frac{1}{2\pi\varepsilon_0}\sum_{k=1}^{n}\oint_{l^{(k)}}\sigma_k(\boldsymbol{r}^{(k)})\ln\frac{1}{\left|\boldsymbol{r}^{(k)}-\boldsymbol{r}_\mathrm{B}\right|}\mathrm{d}l^{(k)}, \quad k=1,\dots,n$$

where $l^{(k)}$ is a simply connected curve in which the cross-section $S^{(k)}$ intersects the plane of the field perpendicular to the conductors, $\boldsymbol{r}^{(k)}$ is the radiusvector of the element $\mathrm{d}l^{(k)}$, $\boldsymbol{r}_\mathrm{B}$ is the radiusvector of point B and $\left|\boldsymbol{r}^{(k)}-\boldsymbol{r}_\mathrm{B}\right|$ the distance of a variable point A of the planar curve $l^{(k)}$ from point B outside the conductors (see Fig. 1). We put, moreover, that

$$\lim_{|\boldsymbol{r}_B|\to\infty}\varphi(\boldsymbol{r}_B)=0\,.$$

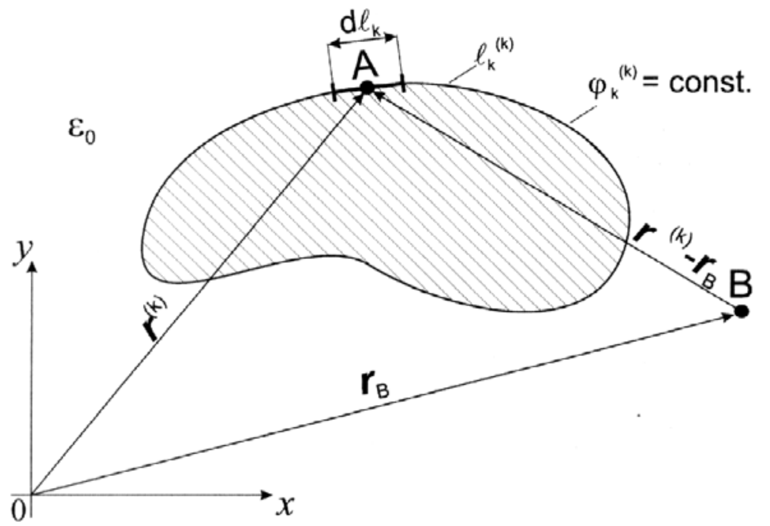

Fig. 1. The k-th conductor in the set of n parallel conductors

Let point $\mathrm{B} \in l^{(k)}$. Now (1) transforms into the first-kind Fredholm integral equation with unknown distribution of charge density σ_k.

$$2\pi\,\varepsilon_0\varphi_k = \sum_{k=1}^{n} \oint_{l^{(k)}} \sigma_k(\boldsymbol{r}^{(k)}) \ln \frac{1}{\left|\boldsymbol{r}^{(k)} - \boldsymbol{r}_{\mathrm{B}}\right|} \mathrm{d}l^{(k)}, \quad k = 1, \ldots, n.$$

Solution of the system (2) provides σ_k at an arbitrary point $\mathrm{B} \in l^{(k)}$. This knowledge enables consequent computing the field quantities at any point outside the conductors. The potential φ would be determined from (1) while the electric field strength from relation

$$\boldsymbol{E}(\boldsymbol{r}_{\mathrm{B}}) = -\operatorname{grad} \varphi(\boldsymbol{r}_{\mathrm{B}}).$$

This is, however, beyond our interest.

Let us return to our case, when $\mathrm{B} \in l^{(k)}$ (Fig. 2). The electric field strength at such a point has only normal component and may be determined from formula

$$E_{\mathrm{n}}^{(k)}(\boldsymbol{r}^{(k)}) = \frac{\sigma_k(\boldsymbol{r}^{(k)})}{\varepsilon_0}.$$

In this way we solved our problem because the value of electric field strength decides about giving rise to corona on the surface of any conductor. Now it remains to carry out numerical solution of (2). First we divide the contour lines $l^{(k)}$ of cross-sections of the particular conductors into $N^{(k)}$ parts of length $\Delta l_i^{(k)}$, $i = 1, \ldots, N_k$. When this division is sufficiently fine, each part may be supposed to carry a constant value of the charge density

$$\sigma_i^{(k)} \cong \text{const} \ \left(i = 1, \ldots, N_k,\ k = 1, \ldots, n\right).$$

The distance between the midpoints of parts $\Delta l_i^{(k)}$ and $\Delta l_j^{(l)}$ is given as

$$\left|\boldsymbol{r}_i^{(k)} - \boldsymbol{r}_j^{(l)}\right| = \sqrt{(x_i^{(k)} - x_j^{(l)})^2 + (y_i^{(k)} - y_j^{(l)})^2}\ .$$

Equation (2) for the discretised model now reads (in order to avoid dividing by zero we leave the term for $i = j$ and $k = l$ in the form of a definite integral)

$$2\pi\,\varepsilon_0\varphi_k = \sum_{l=1}^{n}\sum_{\substack{i=1\\ i\neq j\vee k\neq l}}^{N^{(l)}} \sigma_i^{(l)} \ln\frac{1}{\left|\boldsymbol{r}_i^{(k)}-\boldsymbol{r}_j^{(l)}\right|}\Delta l_i^{(l)} + 2\sigma_i^{(k)}\int_0^{\Delta l_i^{(k)}/2} \ln\frac{1}{\left|\boldsymbol{r}_{ii}\right|}\mathrm{d}l_i^{(k)},\quad k=1,\dots,n\,.$$

As far as $\Delta l_i^{(k)}$ is a line segment, calculation of the integral in (6) is easy and its result is

$$\int_0^{\Delta l_i^{(k)}/2} \ln\frac{1}{\left|\boldsymbol{r}_{ii}^{(k)}\right|}\mathrm{d}l_i^{(k)} = \frac{\Delta l_i^{(k)}}{2}\left(1-\ln\frac{\Delta l_i^{(k)}}{2}\right).$$

In this manner we obtain a system of algebraic equations whose number is

$$m = \sum_{k=1}^{n} N^{(k)}\,.$$

The system can be rewritten into a matrix form

$$\boldsymbol{A}\,\boldsymbol{\sigma} = 2\pi\,\varepsilon_0\,\boldsymbol{\varphi}$$

where

$$\boldsymbol{A}(m,m) = \left[a_{pq}\right],\qquad p,q=1,\dots,m\,,$$

$$\text{for } p\neq q:\quad a_{pq} = \Delta l_p\cdot\ln\frac{1}{\left|\boldsymbol{r}_p-\boldsymbol{r}_q\right|},$$

$$\text{for } p=q:\quad a_{pq} = \Delta l_p\cdot\left(1-\ln\frac{\Delta l_p}{2}\right),$$

Here $\sigma(m,1)$ is the column vector of charge densities in the particular segments and $\varphi(m,1)$ their potentials.

3 ILLUSTRATIVE EXAMPLES

For the conductors placed in vertices of a regular square (see. Fig. 2) we calculated distribution of the normal component of E_n on along the surface of one conductor. All conductors in the bundle have the same potential $\varphi = 100$ kV. Perimeters of all conductors were discretised into $N^{(1)} = \dots = N^{(4)} = 50$ line segments. The results are depicted in the polar co-ordinates in Fig. 3. The maximum and minimum values of the electric field strength are $E_{\text{n max}}$ = 14.30 kV/cm, $E_{\text{n min}}$ = 10.58 kV/cm. The total electric charge per unit length of one conductor of the bundle is

$$Q = \int_S \sigma\,\mathrm{d}S \cong \sum_{i=1}^{N}\sigma_i\,\Delta l_i = 6.8783.10^{-7}\ \text{C/m}\,.$$

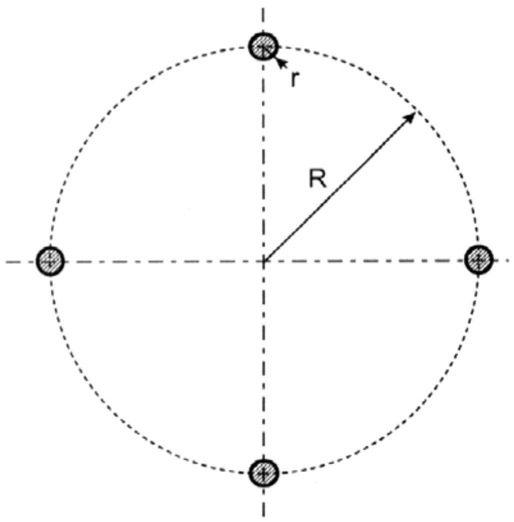

Fig. 2. A bundle conductor for $n = 4$

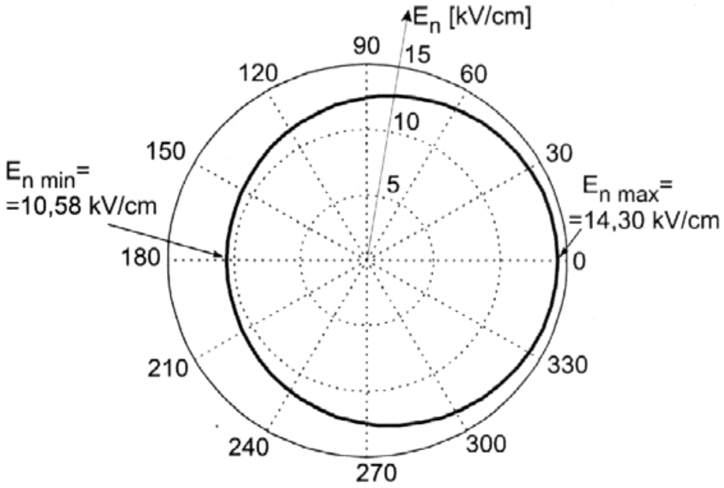

Fig. 3. Distribution of the normal component of electric field strength along the surface of one conductor of the bundle ($n = 4$)

If we replace the bundle conductor by a single conductor of radius $r = 2r_0$ (its cross-section being the same as the total cross-section of the bundle conductor), the electric field strength E_n would be distributed uniformly and its value would be

$$E_n = \frac{nQ}{2\pi\varepsilon_0}\frac{1}{r} = \frac{4\cdot 6.8783\cdot 10^{-7}}{2\pi\varepsilon_0}\frac{1}{0.02} = 24.73 \ \text{kV/m}\,.$$

If we use a bundle conductor where the distance between individual conductors is very high (their mutual electrostatic interaction is negligible), the electric field strength E_n will again be uniform and its value would be

$$E_n = \frac{Q}{2\pi\varepsilon_0}\frac{1}{r_1} = \frac{6.8783\cdot 10^{-7}}{2\pi\varepsilon_0}\frac{1}{0.01} = 12.36 \ \text{kV/m}\,.$$

4 CONCLUSION

The presented algorithm for computation of dielectric stress on the surface of a conductor in a bundle was compared with results obtained by other methods. Tab. 1 contains some results obtained by different algorithms for a bundle containing $n = 4$, 8 and 12 conductors characterised by $d/r = 26.099$ (d being the distance of axes of two neighbouring conductors in the bundle and r their radius). The perimeter of each conductor was then divided into $N = 32$ line segments. The agreement between particular methods is outstanding.

Tab. 1. Comparison of results

E_{min}/E_{max}	$n = 4$	$n = 8$	$n = 12$
the described method	0.7217	0.6624	0.6454
by Cahen [3]	0.7203	0.6593	0.6418
by Timascheff [13]	0.7222	0.6632	0.6463

5 ACKNOWLEDGEMENT

This work has been financially supported from the grant No. 102/01/1401 of the Grant Agency of the Czech Republic.

References

[1] Allan, R. N., Salman, S. K.: Electrostatic fields underneath power lines operated at very high voltages. Proc. IEE 121, 1974, pp. 1404–1408.

[2] Brűderlink, R.: Induktivität und Kapazität der Statrkstrom-Freileitungen. G. Braun, Karlsruhe 1954.

[3] Cahen, F., Pelisseur, R.: L´emploi de conducteurs en faisceaus pour l´armement des linges a tres haute tension. Bull. Soc. Fr. Electr., March 1948, pp. 111–160.

[4] King, S. Y.: The electric field near bundle conductors. Proc. IEE, Vol. 106C (1959), pp. 200 – 206.

[5] Foo, P. Y., King, S. Y.: Bundle-conductor electric field by integral-equation method. Proc. IEE 123, 1976, pp. 702–706.

[6] Mayer, D., Ulrych, B.: Solution of 2D and 3D electrostatic and magnetostatic fields. Elektrotechn. obzor, 69 (1980), No. 8, pp. 458–463 (in Czech).

[7] Mayer, D., Polák, J.: Methods of solution of electric and magnetic fields. SNTL/ALFA, Praha 1983 (in Czech).

[8] Miroljubov, N. N., Kostenko, M. V., Levinštejn, M. L., Tichodějev, N. N.: Methods of calculation of elctrostatic fields. Edited by „Vysšaja škola", Moskva 1963 (in Russian).

[9] Sandell, D. H., Shealy, A. N., White, H. B.: Bibliography on bundled conductors. IEEE Trans. on Power Apparatus and Systems. December 1963, No. 69, pp. 1115–1128.

[10] Sarma, M. P., Janischewskyj, W.: Electrostatic field of a system of parallel cylindrical conductors. Trans IEEE, Vol. PAS-88, 1969, pp. 1069–1079.

[11] Singer, H., Steinbigler, H., Weiss, P.: A charge simulation method for the calculation of high voltage feldes. IEEE Trans, paper T74 085–7, 1973.

[12] Steinbigler, H., Singer, H., Berger, S.: Berechnung der Randfeldstärken von Bundelleitern in Drehstromsystemen. ETZ-A, 92, 1971, pp. 612–617.

[13] Timascheff, A. S.: Fast calculation of gradients of a three-phase bundle conductor line with any number of subconductors. AIEE Trans., PAS-90, 1971, pp. 157–164; PAS-94, 1975, pp. 104–107.

[14] Tozoni, O. V.: Method of secondary sources in electrical engineering. „Energija, Moskva, 1975 (in Russian).

[15] Veverka, A.: High-voltage engineering. SNTL/ALFA, Praha 1976 (in Czech).