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NUMERICAL ANALYSIS OF INDUCTION HEATING-BASED ASSEMBLY AND DISASSEMBLY OF SHRINK FITS

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Abstract: The paper deals with mathematical modelling of assembly and disassembly of structural parts that are coupled with interference fit by means of induction heating. The process is formulated as a quasi-coupled electromagnetic-thermoelastic problem in 2D arrangement. Its solution is based on time stepping with built-in internal iterative processes for improvement of accuracy of parameters of linearised equations describing all three involved fields that are in each time step solved by the finite element techniques. The suggested methodology is illustrated on an example of fixing and disassembly of a drill in a chuck. Considered are real characteristics such as radial interference between the drill shank and chuck, power of the inductor, physical parameters of used materials, conditions of cooling and other quantities responsible for reliable operation of the system.

Keywords: Coupled problems, numerical analysis, thermoelasticity, electromagnetic field, temperature field, field of stresses and strains.

1 Formulation of the task

Hot pressing of metal parts is widely used in contemporary technologies (discs in steam turbines, shrunk-on rings, tires of railway wheels, armature bandages in electrical machines, fixing machine tools in chucks, etc.). The paper deals with investigation of a new technological process – assembly and disas-

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sembly of selected parts of machine tools with interference fit caused by induction heating.

Results and quality of the induction heating-based assembly and disassembly depends on the character of mutually coupled electromagnetic, temperature and thermoelastic (including thermocontact interactions) processes within the investigated structure. Time variable electromagnetic field generated by the inductor gives rise to eddy currents producing internal sources of heat in the metal parts, thus causing their temperature rise. The nonstationary and nonuniform distribution of temperature affects mechanical state of these parts and leads to their elastic or plastic deformations. The thermocontact interaction is characterised by contact domains between the parts that are not known in advance; this represents, in fact, a condition for coupling the temperature and thermomechanical processes. The electrophysical, temperature and mechanical properties in a general case depend on the temperature, which couples all three investigated processes.

During assembly the external part has to be heated to the temperature that causes necessary increase of its diameter. As for disassembly, both parts are heated in such a manner that provides necessary temperature difference between them. The process of heating has to be now sufficiently fast in order that the temperature rise of the internal part is only low and the gap increases at the expense of enlargement of the external part. As far as coefficients of the linear thermal expansion of both materials are approximately equal, disassembly by means of an inductor of smaller power may fail. Moreover, the process need not be accompanied by plastic deformations, as the joint can become not serviceable. Obviously, elastic deformations providing guaranteed interference are small, and that is why the surfaces of both parts have to be sufficiently smooth. And finally, even when the temperature rise during the process of assembly can be achieved by several different ways (gas burner etc.), induction heating for disassembly represents the only efficient alternative.

Computer modelling of electromagnetic and thermomechanical processes of the assembly and disassembly of thermoelastically pressed joints in high-revolution machines by induction heating requires simultaneous solution of nonstationary axisymmetric electromagnetic field, nonstationary temperature field with internal sources of heat and field of thermoelastic displacement including contact interactions of relevant parts. The paper deals with the mathematical model and methodology of its solution.

2 Mathematical model and its solution

Modelling of the task requires solution of three time-dependent physical fields. The corresponding equations will be linearly approximated (see equation (7)) in time using the time stepping method. Electromagnetic field will be solved independently of temperature and thermomechanical fields, regardless the dependence of the electrical conductivity and magnetic permeability on the temperature. In the first approximation, this assumption may be accepted for lower temperature rise – up to about 200 °C, which is just characteristic for the investigated process. As for the temperature and thermomechanical fields, they are tightly coupled and will be solved in turn during each step starting from the knowledge of distribution of internal sources of heat. While the mechanical problem of the task is linear, searching of the unknown contact domain between both parts in each step is realised by means of an iterative process. For bodies of general geometry the most effective tool for solution of similar problems is the finite element method (FEM) and application of a unique mesh substantially increases the effectiveness of the solution.

Distribution of the electromagnetic field is described by equation [1], [2]

$$\mathrm{rot}\left(\frac{1}{\mu}\mathrm{rot}\,\boldsymbol{A}\right)+\gamma\cdot\frac{\partial \boldsymbol{A}}{\partial t}=\boldsymbol{J}_{\mathrm{ext}} \tag{1}$$

where $\boldsymbol{A}$ denotes the vector magnetic potential, μ the magnetic permeability, γ the electric conductivity and $\boldsymbol{J}_{\mathrm{ext}}(\omega t)$ the vector of the external harmonic current density (within the inductor). Parameter γ is a function of the temperature T and μ a function of the temperature T and magnetic flux density $\boldsymbol{B}$. The definition area contains the inductor, heated bodies and dielectric subregions (usually air).

The eddy currents produced in electrically conductive bodies given by the second term on the left-hand side in (1) produce specific Joule losses w_{J}

$$w_{\mathrm{J}}=\gamma\cdot\left(\frac{\partial \boldsymbol{A}}{\partial t}\right)^2 \tag{2}$$

whose magnitude decreases practically exponentially with the distance from the surface of the heated body. In order to satisfy required accuracy of the solution, the finite element mesh for both the electromagnetic and temperature fields has to be made denser towards the surface.

Distribution of the temperature field is described by equation [1], [2]

$$\mathrm{div}(\lambda\cdot\mathrm{grad}T)=\rho c\cdot\frac{\partial T}{\partial t}-w_{\mathrm{J}} \tag{3}$$

where λ is the thermal conductivity, ρ the mass density and c the specific heat. All these parameters are temperature-dependent functions. When modelling the assembly, the internal sources exist only during operation of the inductor. Further we deal with redistribution of temperatures due to thermal conductivity in the process of forced convective cooling used for acceleration of the assembly. Thermal radiation is neglected because of relatively low temperatures.

Exchange of heat within the gap between both parts is realised through the contact zone that varies in time and that has to be determined in the course of process of solution of the thermoelastic problem. The contact thermal resistance is determined from empirical formula [3]

$$K_{\mathrm{N}}(T,\rho_{\mathrm{k}})=\frac{1.8\cdot\lambda_{\mathrm{m}}(T)}{h_1+h_2}+80\cdot\bar{\lambda}(T)\left(\frac{P_{\mathrm{k}}}{3\sigma_{\mathrm{t}}}\cdot K\right)^{0.86} \tag{4}$$

where h_1 and h_2 are the average roughnesses of the contact surfaces (μm), P_{k} the contact pressure, λ_{m} the thermal conductivity of medium within the gap, $\bar{\lambda}$ the equivalent value of the thermal conductivities λ_1, λ_2 of both materials

$$\bar{\lambda}=\frac{2\cdot\lambda_1\lambda_2}{\lambda_1+\lambda_2}, \tag{5}$$

σ_{t} is the ultimate strength of the less plastic material. Parameter K is given as

$$\begin{aligned} K&=1 && \text{for } h_1+h_2\geq 30\,\mu\mathrm{m},\\ K&=\sqrt[3]{\frac{30}{h_1+h_2}} && \text{for } 10<h_1+h_2<30\,\mu\mathrm{m},\\ K&=\frac{15}{h_1+h_2} && \text{for } h_1+h_2<10\,\mu\mathrm{m}.\end{aligned} \tag{6}$$

It is necessary to state that the main role in the process of heat exchange between two parts is played by existence or non-existence of the contact zone.

Solution of the nonstationary task of heat conductivity is performed by the steady Crank-Nicholson integration scheme

$$\frac{\partial T}{\partial t}(t+\Delta t)=\left[T(t+\Delta t)-T(t)\right]\cdot 2/\Delta t-\frac{\partial T}{\partial t}(t) \tag{7}$$

where Δt is the time step. At the moment the inductor is switched on and off this time step has to be strongly reduced.

Solution of the problems of thermoelasticity by the finite element method uses at each time step a linearized Lagrange variational equation for increments [4]

$$\iint_{S_0}\left(\Delta\sigma^{ij}\delta\Delta e_{ij}+\sigma^{ij}\delta\Delta\eta_{ij}-\Delta F^{i}\delta\Delta u_i\right)r\cdot\mathrm{d}S-\int_{L_0}\Delta P^{i}\delta\Delta u_i r\cdot\mathrm{d}L+$$

$$+\iint_{S_0}\left(\sigma^{ij}\delta\Delta e_{ij}-F^{i}\delta\Delta u_i\right)r\cdot\mathrm{d}S-\int_{L_0}P^{i}\delta\Delta u_i r\cdot\mathrm{d}L=0 \tag{8}$$

where S_0 and L_0 denote the surface and boundary of the meridian cross-section of the construction, σ^{ij}, $\Delta\sigma^{ij}$ components of the stress tensor and their increments, Δe_{ij}, $\Delta\eta_{ij}$ increments of the linear and nonlinear parts of the strain tensor, Δu_i increments of components of the displacement vector, F^i, ΔF^i components of the volumic load and their increments in one time step and, finally, P^i, ΔP^i components of the surface load and their increments.

Solution of the temperature and electrodynamic problems uses variational formulation corresponding to equations (1) and (3).

Modelling of the contact interaction between the parts is based on a concept of the contact layer having no thickness. The necessary conditions for the thermal and mechanical interactions are secured by means of a special thermocontact finite element [5]. This element controls conditions on the contact surfaces taking into account influence of the gap (interference) that can be a time variable function and allows evaluating the thermocontact resistance or boundary conditions of the heat exchange in the dependence of the contact pressure. It can take into account even heat generated due to mutual sliding of both parts. Functionals for the temperature problem and mechanical problem are supplemented with components corresponding to the thermal conductivity and energy of the deformed layer. At the points of the interpenetration of contact surfaces in the normal direction with condition

$$u_{\mathrm{n}}^1 - u_{\mathrm{n}}^2 - \delta_{\mathrm{n}} < 0 \tag{9}$$

where u_n^1, u_n^2 are the displacements of the contact surfaces and δ_{a} the width of the gap (interference) in the direction of the normal we introduce a sufficiently high contact stiffness C_{n} in the normal direction in order to prevent the surfaces from penetrating one to another. Should friction be taken into account, we further introduce an analogous tangential (axial) stiffness $C_{\tau.}$ Stresses in the contact layer are now given by formulas

$$\sigma_{\mathrm{n}} = C_{\mathrm{n}}\left(u_{\mathrm{n}}^1 - u_{\mathrm{n}}^2 - \delta_{\mathrm{n}}\right), \;\; \sigma_\tau = C_\tau\left(u_\tau^2 - u_\tau^1 - \delta_\tau\right). \tag{10}$$

As far as the condition $\sigma_{\mathrm{n}} \cdot f_{\mathrm{f}} < |\sigma_\tau|$ of slipping is satisfied, the tangential stresses are expressed as

$$\sigma_\tau = |\sigma_{\mathrm{n}}| \cdot f_{\mathrm{f}} \cdot \mathrm{sign}\left(\mathrm{u}_\tau^2 - u_\tau^1 - \delta_\tau\right) \tag{11}$$

where u_τ^2, u_τ^1 are the shifts of both surfaces and δ_τ is their difference at the beginning of the contact. In the zone of slipping the tangential stiffness is equal to zero and the functional is supplemented with the work of forces of friction on the corresponding shifts.

The thermocontact task is characterised by the contact zone that is not known in advance. This fact does not allow formulating conditions of interaction as is usual in thermal and mechanical tasks [6]. Solution of such nonlinear and nonstationary problems requires building a converging iterative process for obtaining more precise parameters of linearized equations.

3 Illustrative example

Investigated is the process of assembly and disassembly of an axisymmetric system tool-chuck, whose meridian cross-section (with a finite element mesh) is depicted in Fig. 1. The electromagnetic field was calculated by professional FEM-based code QuickField [1], [2]. Computations of the thermal and mechanical processes were realized by program KROK [4].

Material of the chuck: steel with the following parameters:

- thermal conductivity $\lambda = 47$ W/mK,

- heat capacity per unit volume $\rho c = 4{\cdot}10^6$ J/m^{3}K,
- coefficient of the linear thermal expansion $\alpha = 2{\cdot}10^{-5}$ 1/K,
- modulus of elasticity $E = 2{\cdot}10^5$ MPa,
- Poisson ratio $\nu = 0.3$,
- external radius of the chuck 1.1 cm,
- internal radius 0.7 cm.

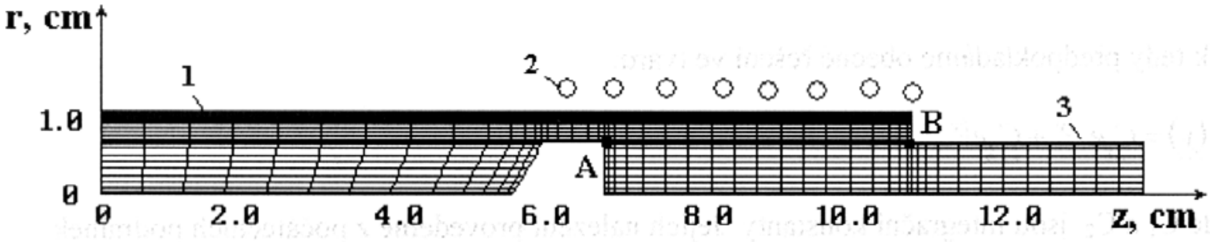

Fig. 1. Basic arrangement of the chuck (1) and tool (3), 2 denoting the inductor

Material of the tool (hard alloy):

- thermal conductivity $\lambda = 85$ W/mK,
- heat capacity per unit volume $\rho c = 2{\cdot}10^6$ J/m^{3}K,
- coefficient of the linear thermal expansion $\alpha = 0.5{\cdot}10^{-5}$ 1/K,
- modulus of elasticity $E = 5.3{\cdot}10^5$ MPa,
- Poisson ratio $\nu = 0.25$,
- external radius of the tool $r = 0.701$ cm,
- length 4 cm (6.5 cm $< z <$ 10.5 cm).

Temperature dependencies of material parameters were not taken into account. Coefficient of friction between both materials $f_{\mathrm{f}} = 0$.

Inductor (made from a hollow copper conductor cooled by water):

- current density $J_{\mathrm{ext}} = 60{\cdot}10^6$ A/m^2,
- frequency $f = 1$ kHz,
- number of turns $N = 10$,
- its geometry is depicted in Fig. 2.

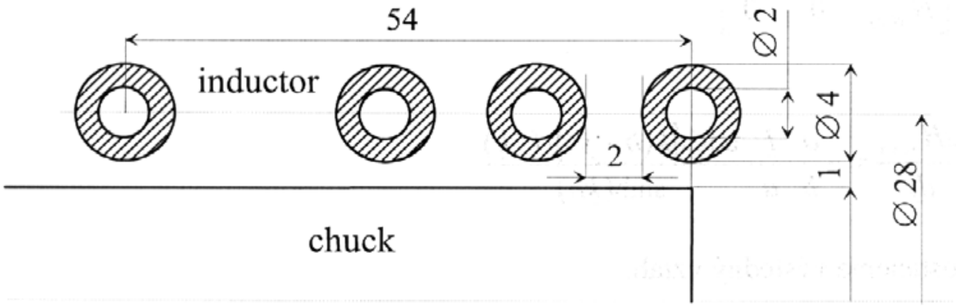

Fig. 2. Arrangement of the inductor (dimensions in mm)

Assembly – the maximum power of the internal heat sources at the external surface of the chuck was about 5 kW (parameters of the inductor were matched to this power by a series of computation of the electromagnetic field with total Joule losses in the chuck and regula falsi method) and the time of heating approximately 4 s. Depth of penetration of eddy currents was about 1 mm. At the end of the chuck ($z = 0$) the third kind boundary condition has been applied with $T_0 = 20$ °C and $\alpha = 10000$ W/m^{2}K, while otherwise $\alpha = 200$ W/m^{2}K.

At the end of the fourth second the maximum temperature in the chuck reached 190 °C. After switching off the inductor the temperature fast equalized all over the chuck and started to fall, particularly in the weakly heated part of the chuck $(z \to 0)$. After further 20 s ($t = 24$ s) the chuck got into contact with the instrument at point A (Fig. 1). At this moment there started fast heat transfer into the tool, through ever increasing contact surface. At the moment $t = 30$ s both surfaces got fully into contact. Distribution of the temperature for $t = 25$ s, $t = 28$ s and $t = 36$ s, respectively, is depicted in Fig. 3. The thick line denotes the contact zone.

The nonstationary process was investigated up to $t = 43$ s, when the temperature between the chuck and tool were practically balanced. Then we investigated the steady state corresponding to $t \to \infty$. The temperature of all parts in this time dropped to 20 °C and the assembly was completed with interference $\delta = 10^{-2}$ mm. The corresponding contact pressures are listed in Tab. 1.

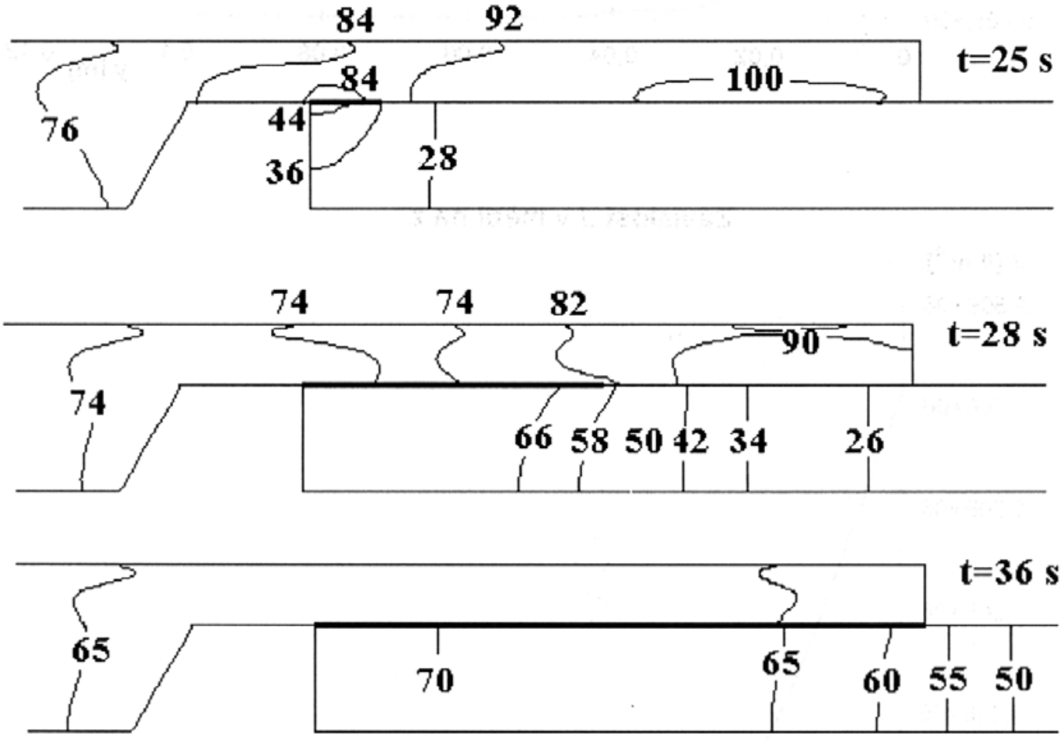

Fig. 3. Distribution of temperatures during the assembly

The maximum torque that can be transferred by the system is obtained from formula

$$M_\mathrm{k} = 2\pi f_\mathrm{f} r^2 \int_0^l \sigma_\mathrm{n} dl \approx 2\pi f_\mathrm{f} r^2 \sum_{i=1}^{N} \sigma_{\mathrm{n}i} \Delta z_i \tag{12}$$

where N denotes the number of elements of length Δz_i of the contact surface in direction z (points A and B, see Fig. 1). Considering the data in Tab. 1 we get $M_\mathrm{k} \approx 1650\, f_\mathrm{f}$ Nm. The maximum elastic tangential stress on a cylindrical surface is

$$\tau_\mathrm{max} = \frac{2M_\mathrm{k}}{\pi r^3} \approx 3060 \cdot f_\mathrm{f}\ \mathrm{MPa}. \tag{13}$$

For coefficient of friction $f_\mathrm{f} = 0.2$ the equivalent stress $\sigma_\mathrm{eq} = \sqrt{3}\, \tau_\mathrm{max} = 1060$ MPa, which exceeds the ultimate strength of many high strength steels and alloys. In this way the axial torque will be transferred with sufficient reliability, even if the contact stress would be two times lower, which is reached for time $t = 36$ s.

Disassembly – was investigated from the moment of switching on the inductor. The full disassembly was reached after 3.5 s. Distribution of the temperature for three times ($t = 1.5$, 2.5 and 3.5 s) is indicated in Fig. 4.

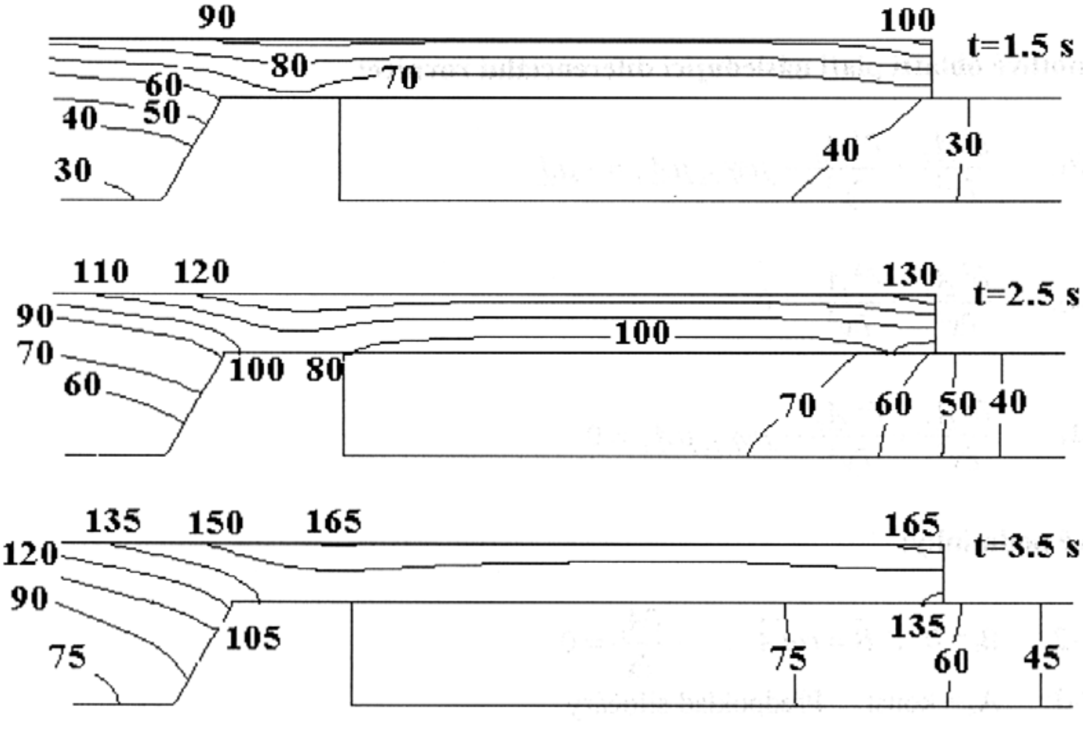

Fig. 4. Distribution of temperatures during the disassembly

Distribution of the contact pressure for these time levels is again in Tab. 1.

4 Conclusion

Computer modelling of electromagnetic, thermal and thermomechanical processes within the framework of a thermocontact problem allows selecting parameters of the technological process assembly-disassembly for particular metal parts. Respecting of friction during the assembly increases the contact pressures in the central part of the joint by (on average) 20 – 30 % and, consequently, working capacity of the system.

Further research in the area will be aimed at computer realisation of quasi-coupled algorithms for solving the task that would respect variations of the material parameters on the temperature.

Tab. 1: Distribution of the contact pressures (MPa) at the assembly and disassembly of the system chuck - tool

$z\cdot 10^2$ m	Assembly, t (s)					Disassembly, t (s)			
	28	30	36	43	∞	0,5	1,5	2,0	2,5
6,53	139	150	168	188	322	256	113	49	0
6,72	55	56	63	69	125	93	34	10	0
7,15	43	43	47	53	96	71	29	11	0
7,75	56	46	50	57	102	76	33	14	0
8,35	14	28	51	58	103	77	33	15	0
8,95	0	33	52	60	103	77	33	15	0
9,55	0	31	55	62	103	76	30	11	0
10,01	0	29	56	62	101	64	16	0	0
10,47	0	50	68	75	121	139	90	67	36

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