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A DYNAMICS ANALYSIS OF DC-DC BUCK CONVERTERS

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Abstract: An analysis of nonlinear phenomena in power electronic circuits is carried on by different methods. Results of this analysis are obtained using analytical, simulating and experimental methods. The continuous models of keys have been proposed for a dynamics analysis of the buck converter in this article. The proposed method can be used to qualitative analysis of different power converters without decomposition their structures.

Keywords: Nonlinear systems dynamics, DC-DC converters, equilibrium points, qualitative analysis.

1 Introduction

Power electronics is the branch of electronics concerned with the processing of electrical energy. Power converters are used to convert electrical energy from one form to the other form. There are four basic types of converters: AC-DC, DC-DC, AC-AC, DC-DC. Many research concern analysis of DC-DC buck converters. This is generally realised by chopping and filtering the input voltage through a suitable switching circuits [7].

An analysis of nonlinear phenomena in power electronic circuits is carried on by different methods. Results of this analysis are obtained using analytical, simulating and experimental methods. These results show a variety of phenomena [4] occurring in nonlinear systems. An occurrence of: one-periodic and multi-periodic orbits, subharmonics waveforms, bifurcations and chaotic attractors in these systems was an object of a lot of research works [1], [2], [3], [5] and [6]. Usually DC-DC buck converters can be modelled by piecewise linear models, whose topologies change according to the given switching law,

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Fig. 1. The switching law usually implemented as Pulse Width Modulation (PWM).

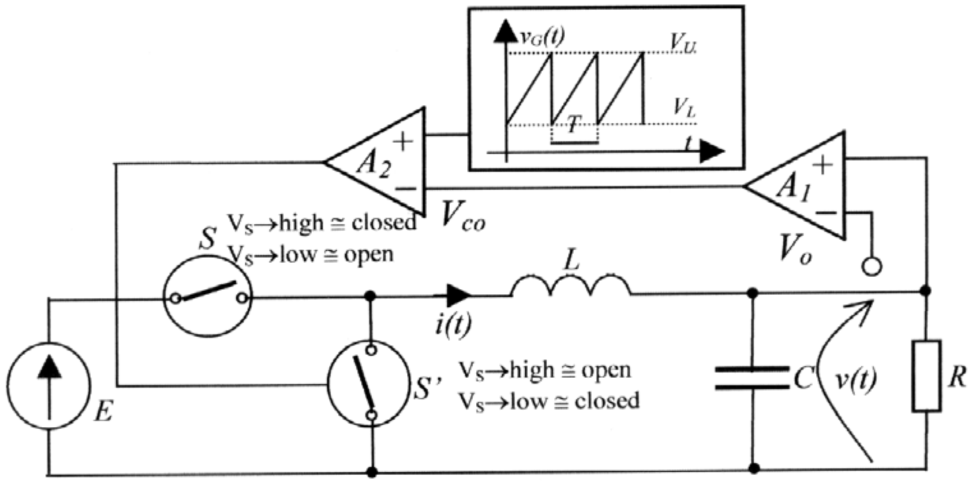


Fig. 1. The switching model of buck converter

The mathematical model of buck converter using a nonlinear controlled source has been proposed in this article as opposed described results. Selected applications of this model in a dynamics analysis have been given too.

2 Formalisation of the model

A generalised model of converters from Fig. 1 can be presented in the form shown on Fig. 2.

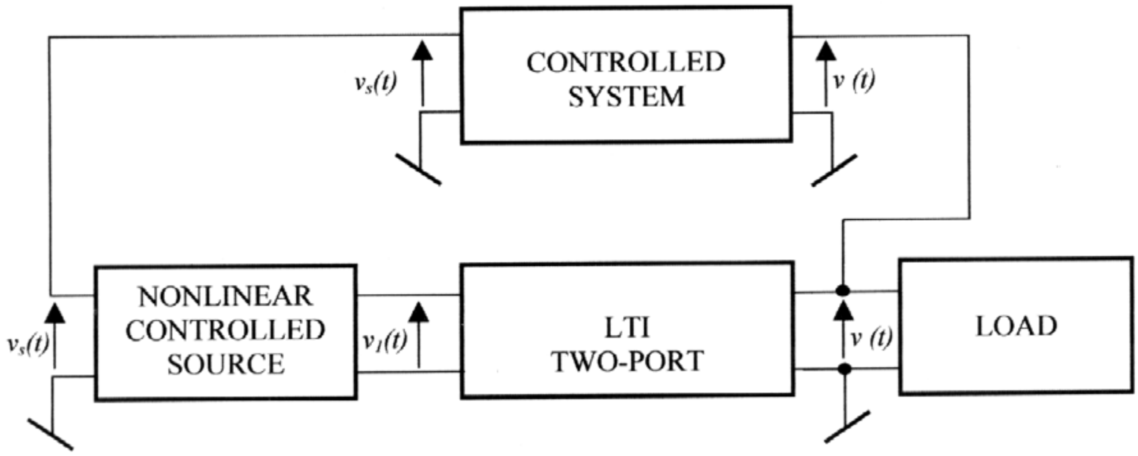


Fig. 2. The block diagram of a continuous models of buck converters

A mathematical model from Fig. 2 depends on specific properties of two-ports included in that model. Limiting to the system shown in Fig. 1, the system of equations describing the switching model of buck converters (Fig. 1) can be written in the following form:

$$\frac{dv}{dt}(t) = -\frac{1}{RC}v(t) + \frac{1}{C}i(t), \tag{1}$$

$$\frac{di}{dt}(t) - \frac{1}{L}v(t) + \frac{1}{L}EF(v_s(t)), \tag{2}$$

where:

E,R,L,C - the concentrated parameters of the system (Fig. 1),
$F(v_s(t))$ – switching function and it's approximations (Fig. 3).

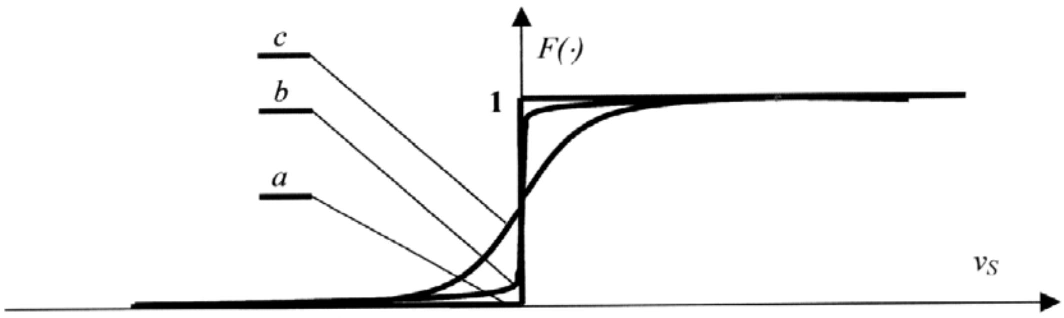


Fig. 3. The approximations of switching function: (a) non-continuous, (b) sigmoidal (c) polynomial

The function $F(\cdot)$ is described by the following formula:

$$F(v_S(t)) = F_1(v(t),t) = F(A_2(v_G(\omega t)) - A_1(v(t) - V_o)), \tag{3}$$

where:

$v(t)$- the state variable occurring in equations (1), (2),
$v_G(t)$-the periodic voltage of generator (i.e. saw tooth oscillator),
A_1, A_2 – the amplifiers gain from Fig. 1,
V_0 – the reference voltage.

Introducing additional variable $\theta=\omega t$, the nonautonomous system of equations (1), (2) have been reduced to an autonomous form:

$$\frac{dv}{dt}(t) = -\frac{1}{RC}v(t) + \frac{1}{C}i(t), \tag{4}$$

$$\frac{di}{dt}(t) = -\frac{1}{L}v(t) + \frac{E}{L}F(A_2(v_g(\theta) - A_1(v(t) - V_o)), \tag{5}$$

$$\frac{d\theta}{dt} = \omega \,. \tag{6}$$

A qualitative analysis of the system of equations (4), (5), (6) can be carried out using known methods [4]. The example of this analysis has been presented below.

3 Example

In order to simply presentation of obtained results, the considered example concerns a cause with linear loads. The simulating experiment has been carried out for the following details of the system (Fig. 1): $A_1=10000$, $A_2=10000$, $L=20mH$, $C=47\mu F$, $R=22\Omega$, $V_o=11.3V$, $V_L=3.8V$, $V_U=8.2V$, $T=400\mu s$, $E=12V$, $E_P=4.4V$. The waveform of the saw tooth oscillator presented in Fig. 4a is given in the Fourier series form:

$$v_G(t)=\frac{E_P}{2}-\frac{E_P}{\pi}\sum_{h=1}^{100}\frac{1}{h}\sin(h\omega t)+V_L,\quad where\quad \omega=\frac{T}{2\pi}. \tag{7}$$

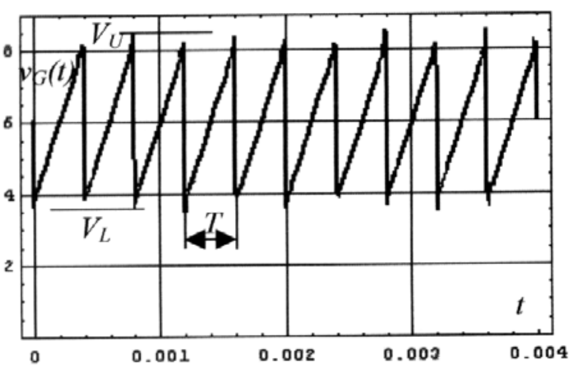
Equilibrium points of considered system have been determined basis of the system of equations:

$$\begin{cases} -\frac{1}{RC}v(t)+\frac{1}{C}i(t)=0 \\ -\frac{1}{L}v(t)+\frac{E}{L}F(v_s(\theta,t))=0, \end{cases} \tag{8}$$

where the switching function $F(\cdot)$ is given as sigmoidal model:

$$F(v_s(\theta,t))=\frac{1}{1+\exp(-\beta v_s(\theta,t))},\quad \beta>0. \tag{9}$$

The equilibrium point (11.3006, 0.5136) has been obtained for $\theta=0.0002$ *[rad]* as a result of solution of the system of equations (8). A graphic illustration of that solution is presented in Fig. 4b. The waveform and trajectory in the phase space of the state variable $v(t)$ and $i(t)$occurring in the equations (1) and (2) have been shown in Fig. 5a,b.



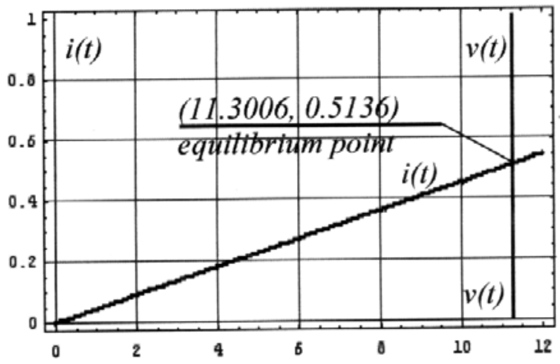


Fig. 4. The wave-form of the voltage of the ramp $v_G(t)$(a) and the graphic interpretation of the solution of the system of equations (b) (8)

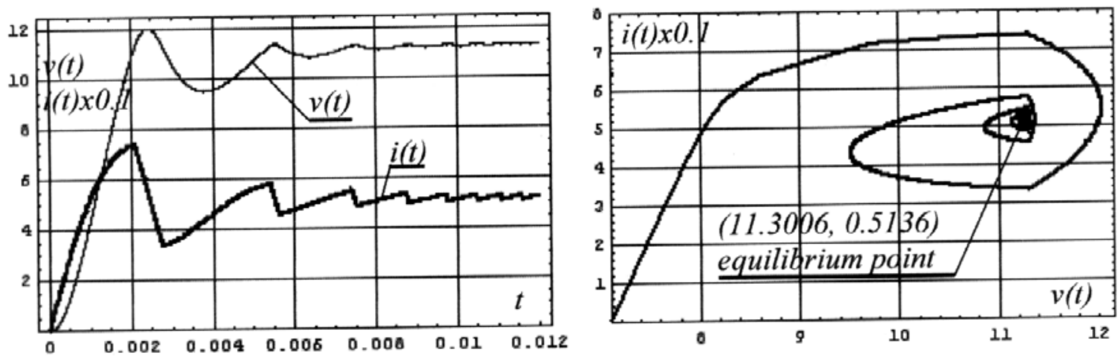


Fig. 5. The waveform (a) and trajectory in the phase space (b) of the state variable *v(t)* and *i(t)*

A continuation of dynamics analysis of considered converters concerning of bifurcation of equilibrium points and stability of oscillations will be presented on the conference.

4 Conclusion

The methods of analysis of DC-DC converters presented in literature suppose a discreet models of the switch. The method described in this article proposes a continuous models of the switch. This approach enables an analysis of the considered system described by the same system of equations in all state of work.

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