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ALGORITHM FOR COMPUTATION OF INDUCTANCES OF THREE-PHASE OVERHEAD LINES

PROF. ING. ZDEŇKA BENEŠOVÁ, CSC.[1]
PROF. ING. DANIEL MAYER, DRSC.[1]

Abstract: Inductances of every overhead line of given geometrical configuration may be characterised by an inductance matrix. Knowledge of this matrix represents the starting point for easy derivation of voltages on inductances of particular conductors. The paper is aimed at building of the inductance matrix for a three-phase overhead line with one earth wire.

Keywords: Overhead line, inductance matrix.

1 Introduction

Our previous papers [1] and [2] show a possibility how to correctly introduce (by means of the static definition) the self-inductance of a single conductor and mutual inductance of one pair of conductors. Another result is that inductances of any overhead line may be characterised by an inductance matrix.

This topic has been dealt with by a number of authors but the knowledge they acquired does not result in a simple and from the theoretical viewpoint correct method applicable to any type of three-phase overhead line. The algorithm we suggested in the above paper makes it possible to derive the inductance matrix for any type of three- or multiphase overhead line in a simple and mathematically exact manner. This algorithm is here illustrated on a three-phase line with earth return.

The earth current passes through both earth wire and earth via towers. In practice, estimation is sometimes accepted that each of these two ways carries one half of it. Impedance of the earth return and distribution of the earth current

[1]FEL ZČU, Sady Pětatřicátníků 14, 30614 Plzeň, e-mail: {bene, mayer}@kte.zcu.cz

to the earth wire and earth was investigated in details in [3]. Further we shall consider arrangement of the conductors according to Fig. 1 and assume that the earth current passes exclusively through the earth conductor.

2 Non-transposed four-conductor line with earth wire

Voltage on the inductance of phase a in the arrangement in Fig. 1 is given as

$$u_{\mathrm{a}} = L_{\mathrm{aa}}\frac{\mathrm{d}i_{\mathrm{a}}}{\mathrm{d}t} + L_{\mathrm{ab}}\frac{\mathrm{d}i_{\mathrm{b}}}{\mathrm{d}t} + L_{\mathrm{ac}}\frac{\mathrm{d}i_{\mathrm{c}}}{\mathrm{d}t} + L_{\mathrm{a0}}\frac{\mathrm{d}i_{0}}{\mathrm{d}t}. \tag{1}$$

The self-inductance and mutual inductances may be expressed as

$$L_{\mathrm{aa}} = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{r_0}{R}\right)$$

$$L_{\mathrm{ab}} = \frac{\mu_0}{2\pi}\ln\frac{r_0}{d_{\mathrm{ab}}}, \quad L_{\mathrm{ac}} = \frac{\mu_0}{2\pi}\ln\frac{r_0}{d_{\mathrm{ac}}}, \quad L_{\mathrm{a0}} = \frac{\mu_0}{2\pi}\ln\frac{r_0}{d_{\mathrm{a0}}} \tag{2}$$

where d_{ij} is the distance between the phase conductors ($i, j = a, b, c$) or between the phase conductor and earth wire ($i = a, b, c; j = 0$).

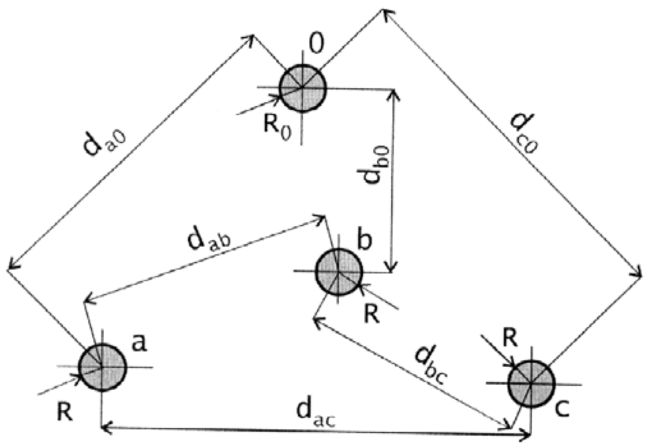

Fig. 1. Three-phase line with earth wire

The currents generally satisfy the condition

$$i_{\mathrm{a}} + i_{\mathrm{b}} + i_{\mathrm{c}} + i_0 = 0. \tag{3}$$

After elimination of i_0 from (1), (2) and (3) we get

$$u_{\mathrm{a}} = \frac{\mu_0}{2\pi}\left[\frac{\mathrm{d}i_{\mathrm{a}}}{\mathrm{d}t}\left(\frac{1}{4} + \ln\frac{d_{\mathrm{a0}}}{R}\right) + \frac{\mathrm{d}i_{\mathrm{b}}}{\mathrm{d}t}\ln\frac{d_{\mathrm{a0}}}{d_{\mathrm{ab}}} + \frac{\mathrm{d}i_{\mathrm{c}}}{\mathrm{d}t}\ln\frac{d_{\mathrm{a0}}}{d_{\mathrm{ac}}}\right]. \tag{4}$$

Analogously, for voltages in phases b, c and earth wire we have

$$u_{\mathrm{b}} = \frac{\mu_0}{2\pi}\left[\frac{\mathrm{d}i_{\mathrm{a}}}{\mathrm{d}t}\ln\frac{d_{\mathrm{b0}}}{d_{\mathrm{ab}}} + \frac{\mathrm{d}i_{\mathrm{b}}}{\mathrm{d}t}\left(\frac{1}{4} + \ln\frac{d_{\mathrm{b0}}}{R}\right) + \frac{\mathrm{d}i_{\mathrm{c}}}{\mathrm{d}t}\ln\frac{d_{\mathrm{b0}}}{d_{\mathrm{bc}}}\right], \tag{5}$$

$$u_c = \frac{\mu_0}{2\pi}\left[\frac{\mathrm{d}i_a}{\mathrm{d}t}\ln\frac{d_{c0}}{d_{ac}} + \frac{\mathrm{d}i_b}{\mathrm{d}t}\ln\frac{d_{c0}}{d_{bc}} + \frac{\mathrm{d}i_c}{\mathrm{d}t}\left(\frac{1}{4}+\ln\frac{d_{c0}}{R}\right)\right], \tag{6}$$

$$u_0 = \frac{\mu_0}{2\pi}\left[\frac{\mathrm{d}i_b}{\mathrm{d}t}\ln\frac{d_{a0}}{d_{b0}} + \frac{\mathrm{d}i_c}{\mathrm{d}t}\ln\frac{d_{a0}}{d_{c0}} + \frac{\mathrm{d}i_0}{\mathrm{d}t}\left(\frac{1}{4}+\ln\frac{d_{a0}}{R_0}\right)\right], \tag{7}$$

R_0 denoting the radius of the earth wire. Now we express the voltages in the matrix form

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_0 \end{bmatrix} = \mathbf{L}_n \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_0 \end{bmatrix} \tag{8}$$

where the matrix of inductances is

$$\mathbf{L}_n = \frac{\mu_0}{2\pi}\begin{bmatrix} \frac{1}{4}+\ln\frac{d_{a0}}{R} & \ln\frac{d_{a0}}{d_{ab}} & \ln\frac{d_{a0}}{d_{ac}} & 0 \\ \ln\frac{d_{b0}}{d_{ab}} & \frac{1}{4}+\ln\frac{d_{b0}}{R} & \ln\frac{d_{b0}}{d_{bc}} & 0 \\ \ln\frac{d_{c0}}{d_{ac}} & \ln\frac{d_{c0}}{d_{bc}} & \frac{1}{4}+\ln\frac{d_{c0}}{R} & 0 \\ 0 & \ln\frac{d_{a0}}{d_{b0}} & \ln\frac{d_{a0}}{d_{c0}} & \frac{1}{4}+\ln\frac{d_{a0}}{R_0} \end{bmatrix}. \tag{9}$$

Due to geometrical non-symmetry of non-transposed lines, induced voltage in the earth wire may appear even when symmetric current load. This voltage induces current in the closed loop consisting of the earth wire and other conductive paths. When considering symmetric harmonic current load ($\boldsymbol{I}_a = I_a, \boldsymbol{I}_b = I_a\boldsymbol{a}^2$, $\boldsymbol{I}_c = I_a\boldsymbol{a}$, where $\boldsymbol{a} = e^{j120°}$) the phasor of voltage induced in 1 m of the earth wire is (using (8) and (9))

$$\boldsymbol{U}_0 = j\omega I_a\left(L_{a0} + \boldsymbol{a}^2 L_{b0} + \boldsymbol{a}L_{c0}\right) = j\omega I_a \frac{\mu_0}{2\pi}\left(\ln\frac{\sqrt{d_{b0}d_{c0}}}{d_{a0}} - j\frac{\sqrt{3}}{2}\ln\frac{d_{c0}}{d_{b0}}\right) \tag{10}$$

Analysis of non-symmetric systems is often based on the symmetric component method. An important value is here the inductive reactance of the zero-sequence component. For its determination we put $i_a = i_b = i_c = i$, so that we get using (3) $i_0 = -3i$. The zero-sequence component of the voltage in the earth wire may be obtained from (8) and (9) in the form

$$u_0 = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \frac{1}{3}\ln\frac{(d_{a0})^3}{R_0^{\,3}}\frac{d_{b0}d_{c0}}{d_{a0}^{\,2}}\right]\frac{\mathrm{d}i_0}{\mathrm{d}t} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{\sqrt[3]{d_{a0}d_{b0}d_{c0}}}{R_0}\right]\frac{\mathrm{d}i_0}{\mathrm{d}t}. \tag{11}$$

Denoting

$$D_{g0} = \sqrt[3]{d_{a0} d_{b0} d_{c0.}} \, , \tag{12}$$

the total inductance of this component can be expressed as

$$L_0 = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{D_{g0}}{R_0}\right). \tag{13}$$

3 Transposed four-conductor line with earth wire

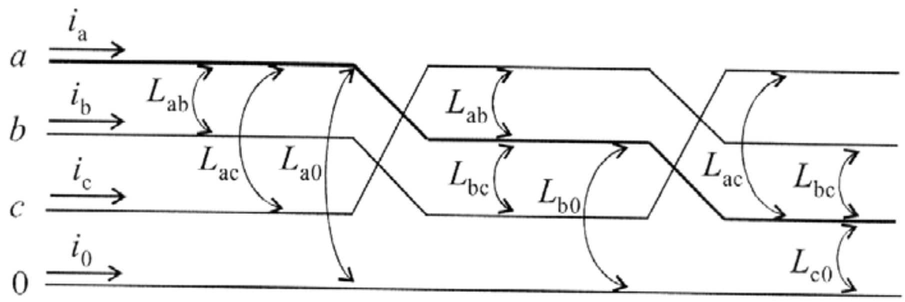

Fig. 2. Transposed three-phase overhead line with earth wire

Its basic arrangement is in Fig. 2. As far as during the one-line-to-ground fault the current i_0 passes only through the earth line, the voltage in phase a is

$$\begin{aligned} u_a = & \frac{1}{3}\Big(3L_{aa}\frac{di_a}{dt} + L_{ab}\frac{di_b}{dt} + L_{ac}\frac{di_c}{dt} + L_{a0}\frac{di_0}{dt} + L_{ab}\frac{di_c}{dt} + \\ & + L_{b0}\frac{di_b}{dt} + L_{b0}\frac{di_0}{dt} + L_{bc}\frac{di_c}{dt} + L_{ac}\frac{di_b}{dt} + L_{c0}\frac{di_0}{dt}\Big) = \\ = & \frac{1}{3}\left\{\frac{di_a}{dt}\left[3L_{aa} - \left(L_{ab} + L_{ac} + L_{bc}\right)\right] + \frac{di_0}{dt}\left(L_{a0} + L_{b0} + L_{c0} - L_{ab} - L_{ac} - L_{bc}\right)\right\} \end{aligned} \tag{14}$$

and after substitution for the self- and mutual inductances we have

$$u_a = \frac{\mu_0}{2\pi}\left[\frac{di_a}{dt}\left(\frac{1}{4} + \ln\frac{\sqrt[3]{d_{a0}d_{b0}d_{c0}}}{R}\right) + \frac{di_0}{dt}\ln\frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{\sqrt[3]{d_{a0}d_{b0}d_{c0}}}\right]. \tag{15}$$

Analogously

$$\begin{aligned} u_b = & \frac{1}{3}\Big(3L_{aa}\frac{di_b}{dt} + L_{ab}\frac{di_a}{dt} + L_{cb}\frac{di_c}{dt} + L_{b0}\frac{di_0}{dt} + L_{c0}\frac{di_0}{dt} + \\ & + L_{ca}\frac{di_c}{dt} + L_{bc}\frac{di_a}{dt} + L_{a0}\frac{di_0}{dt} + L_{ab}\frac{di_c}{dt} + L_{ca}\frac{di_a}{dt}\Big) \end{aligned} \tag{16}$$

and hence, after substituting the inductance from (2)

$$u_b = \frac{\mu_0}{2\pi}\left[\frac{di_b}{dt}\left(\frac{1}{4} + \ln\frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{R}\right) + \frac{di_0}{dt}\ln\frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{\sqrt[3]{d_{a0}d_{b0}d_{c0}}}\right]. \tag{17}$$

Let us denote the average geometrical distance between the phase conductors and earth wire $D_{g0} = \sqrt[3]{d_{a0}d_{b0}d_{c0}}$ and the average geometrical distance between the phase conductors $D_{\mathrm{gf}} = \sqrt[3]{d_{\mathrm{ab}} d_{\mathrm{ac}} d_{\mathrm{bc}}}$. Relation between voltages and currents is now the same as in (8), but the inductance matrix reads

$$\mathbf{L}_{\mathrm{t}} = \frac{\mu_0}{2\pi}\begin{bmatrix} \frac{1}{4}+\ln\frac{D_{\mathrm{gf}}}{R}, & 0, & 0, & \ln\frac{D_{\mathrm{gf}}}{D_{\mathrm{g0}}} \\ 0, & \frac{1}{4}+\ln\frac{D_{\mathrm{gf}}}{R}, & 0, & \ln\frac{D_{\mathrm{gf}}}{D_{\mathrm{g0}}} \\ 0, & 0, & \frac{1}{4}+\ln\frac{D_{\mathrm{gf}}}{R}, & \ln\frac{D_{\mathrm{gf}}}{D_{\mathrm{g0}}} \\ 0, & 0, & 0, & \frac{1}{4}+\ln\frac{D_{\mathrm{g0}}}{R_0} \end{bmatrix}. \quad (18)$$

Equations (8) and (18) provide the voltage in the earth wire

$$u_0 = L_0 \frac{\mathrm{d}i_0}{\mathrm{d}t} \quad (19)$$

where L_0 is the total inductance of the earth wire

$$L_0 = \frac{\mu_0}{2\pi}\left(\frac{1}{4}+\ln\frac{D_{\mathrm{g0}}}{R_0}\right). \quad (20)$$

Inductance of the earth wire for transposed line and non-symmetric current load is identical with the inductance of the zero-sequence component of the non-transposed line according to (13). Using (20), we obtain the voltage on a phase conductor of the transposed line as

$$u_{\mathrm{a}} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\ln\frac{D_{\mathrm{gf}}}{R}\right)\frac{\mathrm{d}i_{\mathrm{a}}}{\mathrm{d}t} + \ln\frac{D_{\mathrm{gf}}}{D_{\mathrm{g0}}}\frac{\mathrm{d}i_0}{\mathrm{d}t}\right]. \quad (21)$$

Voltages on the phase conductors may generally be expressed by formula

$$u_i = L_{\mathrm{v}}\frac{\mathrm{d}i_i}{\mathrm{d}t} + L_{\mathrm{v0}}\frac{\mathrm{d}i_0}{\mathrm{d}t} \qquad i = a,b,c \quad (22)$$

where L_{v} is the total inductance of a conductor of the transposed line

$$L_{\mathrm{v}} = \frac{\mu_0}{2\pi}\left(\frac{1}{4}+\ln\frac{D_{\mathrm{gf}}}{R}\right) \quad (23)$$

and the mutual inductance between the phase conductor and earth wire is

$$L_{\mathrm{v0}} = \frac{\mu_0}{2\pi}\ln\frac{D_{\mathrm{gf}}}{D_{\mathrm{g0}}}. \quad (24)$$

At symmetric current load of a three-phase transposed line with earth wire $i_0 = 0$. Voltages on the phase conductors may be now determined by means of a diagonal submatrix $\mathbf{L}_{\mathrm{t}}$ (3,3) that is obtained from matrix in (18) omitting the last row and column:

$$\mathbf{L}_{\mathrm{t}} = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{D_{\mathrm{gf}}}{R}\right)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{25}$$

4 Conclusion

Further to works [1] and [2] we showed how to derive the inductance matrix for a three-phase overhead line with earth wire (non-transposed as well as transposed) and how these quantities may be used for consequent analysis of the line.

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References

[1] Benešová, Z., Mayer, D.: Determination of inductance of multiphase lines. Journal EE, 8, 2002, No. 4, pp. 32 – 34, No. 5, pp. 8 – 9.

[2] Benešová, Z., Mayer, D.: Calculation of inductances of three-phase lines with bundle conductors. Proc. IC SPETO 26, Ustroň, Poland, 2003), Vol. 1, pp. 47 – 50.

[3] Benešová, Z., Mayer, D.: One-line-to-ground fault on overhead lines. Acta Technica CSAV, 46, 2001, No. 2, pp. 181 – 195.