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DIFFERENCE BETWEEN SUPPLE AND STIFF NON-LINEAR ELEMENTS

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Abstract: – In the previous paper [1] a need for a distinction of non-inert (supple) and inert (stiff) non-linear elements has been introduced. There it has been shown, that an influence of an external phenomenon that evokes changes of parameters of the non-linear element can be explained using two kinds of ideal elements (e.g., two resistors) with the same static characteristic function: E = f_s(C), where E is an effect and C is a cause. These two ideal elements are:

- an ideal non-inert element (an ideal supple element), in which a change of a parameter occurs immediately with the change of the cause;
- an ideal inert element (an ideal stiff element), in which the change of the parameter does not occur after the change of the cause until time $t \to \infty$.

In this paper an example of a simple circuit with a non-linear resistor, supplied with a sinusoidal voltage, is considered. A so-called "capacitance effect" can be stated on the basis of analysis of the equation describing this circuit. For voltage frequencies greater than 10 Hz, the circuit is a quasi-linear system, as for a sinusoidal voltage the current waveform is also sinusoidal. However, for frequencies lower than 5 Hz, the current changes in time accordingly to a non-linear characteristic function of this resistor, thus the current is no more sinusoidal. The first current harmonic is then shifted ahead of the voltage as if the circuit contained a capacitor.

Keywords: Non-linear elements. Non-linear resistor. Capacitance effect.

1. Introduction

According to the classical approach, a circuit is considered non-linear if at least one of its resistive (R), inductive (L), or capacitive (C) elements is dependent on the respective current, voltage, magnetic flux and/or electric

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charge. However, the dependence of these parameters on frequency is typically not taken into account for the classification of the system to be linear or non-linear.

A truly linear system, in which the **effect** is in all circumstances proportional to the **cause**, is rather a rarity in the nature. Yet in the practical engineering analyses of various problems, a strong attempt to the approximate linear approach is very typical. This is because the assumption of linearity leads to a much simpler analysis, allows for the use of the superposition law, which is intuitively clear, and in many cases yields results, which are enough close to the reality. If, however, the non-linearity is a strong effect (such as a ferromagnetic saturation) or is introduced deliberately for the required system properties (as in the case of self-excitation of oscillations or in the rectification), a non-linear analysis is essential. Deviation from linearity results first of all due to "external" physical effects such as thermal, chemical, mechanical, structural and other phenomena, which accompany the electromagnetic effects.

In this contribution, attention is paid to the chosen properties of non-linear elements, i.e., to those, which, as a rule, are omitted in the most typical analyses of non-linear circuits.

2. Inertia of a nonlinear element

The following functions and parameters are of primary importance for the description of properties of a non-linear element:

- a static characteristic $E_s = f_s(C_s)$ describing the relation between the electromagnetic effect E_s and the electromagnetic cause C_s under assumption that the changes of the cause are very slow (ideally – infinitely slow);
- a family of dynamic characteristics describing the relation $E_d = f_d\,[C_d\,;\,f_t(t)]$ for the cause C_d varying in time according to the function $f_t(t)$. If the function $f_t(t)$ is sinusoidal varying in time then the family of dynamic characteristics is given by $E_d = f_d\,[C_d\,;\,\omega]$ or alternatively by $E_d = f_d\,[C_d\,;\,f\,]$, where $\omega = 2\pi f$ and f is frequency;
- static and differential parameters of the non-linear element;
- time variation of the effect to cause ratio; this variation is caused by the external phenomena in the circuit (i.e., those different from the electromagnetic phenomena).

If in a chosen point P_0 of the static characteristic of an element (e.g., of a resistor), the voltage and the current in instant $t = 0$ are equal to u_0 and i_0, respectively, then after a step increase of the voltage to the value $u_0 + \Delta u$, the current will change (increase or decrease) to the value $i_0 \pm \Delta i$, but this change occurs with some delay with respect to a complicated function of time. This function depends on the influence of other (external) physical phenomena on the considered element. If the succeeding voltage increase occurs in a short time, not sufficient for the stabilisation of current, than the process will not run along the static characteristic. If the cause is changing in time according to the function C(*t*), we cannot, as a rule, express the effect change using the function:

$$E_s(t) = f_s[C_s(t)] \ . \tag{1}$$

However, for particular elements and situations, e.g., if changes of the cause are not too quick, (i.e., if the frequency is not too high) and no external factors do act on the element, we can use equation (1) without omission of a significant error. For example, the dynamic characteristics for a non-linear inductor with barium-ferrit core in a frequency range of 0 ÷ 250 Hz coincide with the static characteristics. On the other hand, in the inductor with ferromagnetic core in which eddy-currents occur, the dynamic characteristics change with frequency very significantly.

We can explain the influence of the cause changes on the parameters of a non-linear element using the following two kinds of ideal elements with the same static characteristics (e.g., two resistors):

- an ideally non-inert element (an ideally supple element), in which the change of the parameter occurs immediately with the change of the cause;
- an ideally inert element (an ideally stiff element), in which after the change of the cause, the change of the parameter does not occur until time $t \rightarrow \infty$.

Equation (1) is valid only for non-inert circuits, independently from the speed of the change of the cause.

Up to the present, a general measure for the inertia of non-linear elements, which would describe the direct dependency of their parameters on time, does not exist.

Among external physical phenomena, acting together with a specific electromagnetic phenomenon and on parameters of a non-linear element, one of them is, as a rule, dominant upon the others. It can be assumed that only this dominant phenomenon acts on the element. If several equally important phenomena occur (e.g., in a non-linear inductor with ferromagnetic core), it can be assumed that only one equivalent phenomenon exists and encloses all effects. For example, the dominant effect in a non-linear resistor is the thermal phenomenon, which depends on the Joule losses in the conductor.

In this work it is proposed to express the effect of external influences on the change of element parameters by an exponential (increasing or decreasing) function with time constant T equal to T_R , T_L , or T_C for an R, L, or C element, respectively; i.e., by function of the general form:

$$g = a_1 g_1 + a_2 g_2, \tag{2a}$$

$$g_1 = c_1(k_1, k_2, \ldots, k_n)\left\{1 - \exp\left[-\frac{t}{T(k_1, k_2, \ldots, k_n)}\right]\right\}, \tag{2b}$$

$$g_2 = c_2(k_1, k_2, \ldots, k_n)\exp\left[-\frac{t}{T(k_1, k_2, \ldots, k_n)}\right], \tag{2c}$$

where: a_1, $a_2 \in \{0;1\}$ are parameters for the choice of the function g_1 and g_2; k_1, k_2,..., k_n are parameters describing shape, dimensions, material properties of the element and also the relation between the cause and the external physical phenomenon; t is time and T is time constant of the external phenomenon.

This time constant is assumed as a measure for inertia of the element. The greater the time constant the more inert is the element and *vice versa.* For an ideal supple element, time constant is equal to $T = T_R = T_L = T_C = 0$, and for an ideal inert (stiff) element the time constant approaches infinity: $T \to \infty$.

According to the period $T_e = 1/f$ of the waveform of the cause, which imposes the electromagnetic phenomena in the circuit, and according to the time constant T (describing changes of the element parameters in time), the electromagnetic phenomena in the circuit have different character.

1. If $T_e \gg T$, we can recognize the non-linear elements as non-inert. The steady-state in the circuit can be calculated on the basis of differential parameters of the elements. The response on a sinusoidal excitation is in this case deformed (i.e., non sinusoidal).

2. If $T_e \ll T$, we can recognize the non-linear elements as inert. The steady-state in e.g. a resistor can be calculated on the basis of the static resistance.

For the **cause** C(t) varying sinusoidal in time the **effect** in a non-linear circuit is usually not monoharmonic (sinusoidal) and depends on the amplitude and frequency of the cause. Due to this dependency, a family of dynamic characteristics is commonly used for the analysis of non-linear circuits. For example, the family of characteristics can by given in a form: $I = f(U; \omega)$, where I is the effective or maximum value of current, U is the effective or maximum value of voltage, $\omega = 2\pi f$ and f is frequency.

In the case $T_e \ll T$, the elements are quasi-linear (constant for a given point on the characteristic) and the circuit with such elements is named quasi-linear. The response of the circuit has the same shape as the excitation, but may be phase-shifted.

A hypothesis may be stated that for a properly high frequency every non-linear circuit will become a quasi-linear system. Indeed, the characteristic of the non-linear inductor with ferromagnetic core is more and more linear with the growing frequency [1; 3; 5]. Similarly, we can see this effect in a semiconductor diode [4]. But the range of frequency is different for various elements. For a bulb the frequency higher than 0.01 Hz is sufficient, for an inductor with a core made of 0.5 mm thick steel sheets – the frequency higher than 1000 Hz is necessary, but for a semiconductor element, depending on the technology, it can be higher than $10^4 \div 10^5$ Hz.

3. Capacitance effect of a non-linear resistor

As an example in this paper the so-called "capacitance effect" occurring in a circuit with a non-linear resistor, supplied with a sinusoidal voltage is considered. The dominant effect in a non-linear resistor is the thermal phenomenon, which depends on the Joule losses in a conductor. The resistance R is a temperature dependent quantity described as follows:

$$R(\vartheta) = R_0\left[1 + \alpha(\vartheta - \vartheta_0) + \beta(\vartheta - \vartheta_0)^2\right], \tag{3}$$

where R_0 is the resistance in temperature ϑ_0, α and β are the temperature coefficients of resistivity, and ϑ is the temperature.

The basis for computation of a current in the considered circuit is the energy balance equation, which can be transformed to the following differential form:

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}t} = \frac{u^2(t)}{cmR(\vartheta)} - \frac{aS}{cm}\vartheta, \tag{4}$$

where c is the specific heat, m is the mass, S is the cooling surface, a is the specific heat-dissipation (emissivity) of the resistor, ϑ is the temperature rise, and t is the time.

Solving equation (4) we obtain the temperature function $\vartheta(t)$ for a given voltage $u(t)$. Then using equation (3) we can compute the resistance $R(t)$ as a function of time. Finally we can find the time function of the current:

$$i(t) = \frac{u(t)}{R(t)}; \tag{5}$$

and of the power losses

$$p(t) = \frac{u^2(t)}{R(t)}. \tag{6}$$

As it is well known, an example of such a non-linear resistor is an electric bulb with a non-linear static characteristic. For a sinusoidal supply voltage with frequency of, e.g., 50 Hz, the current is also sinusoidal. However, the effective value of current depends non-linearly on the effective value of voltage.

In order to solve the initial value problem for the non-linear differential equation (4) the classical fourth order Runge-Kutta method with an automatic step-size control mechanism has been used. The elaborated program for the analysis and visualization of the heating processes in the described circuit has been prepared with Delphi 5 programming environment and has been ran on a Pentium IV computer. Before starting this application, the parameters of the considered scheme should be set up.

The program allows to control the following quantities with respect to time: temperature: $\vartheta(t)$, resistance $R(t)$, power losses $p(t)$ in the resistance, supply voltage $u(t)$, and current $i(t)$. All these quantities can be calculated both for the transient and for the steady state. At first, the transient state is calculated, starting with instant t_0, in which the voltage is switched on. The computations are continued until the steady state is reached in instant t_k. Time interval $t_k - t_0$ depends on the frequency and the other system parameters. This interval can be determined on the basis of numerical experiments [2].

Computations for a fixed set of parameters consist of 100 000 steps. This simulation lasts about 25 s on a 1.9 GHz Pentium IV computer.

If the steady state is reached, i.e., if $t \geq t_k$, the computations are continued in the time interval equal to the period T_e of the supply voltage.

The program allows also for a spectral analysis of the obtained current the $i(t)$. It computes the phase shift between the voltage $u = U \sin(2\pi/T_e)$ and the first current harmonic $i_1(t)$. This phase shift is frequency dependent. In analyzed illustrative example this shift was equal to 0 for frequencies over 3 Hz and was equal to 5.2° for the frequency of 0.005 Hz.

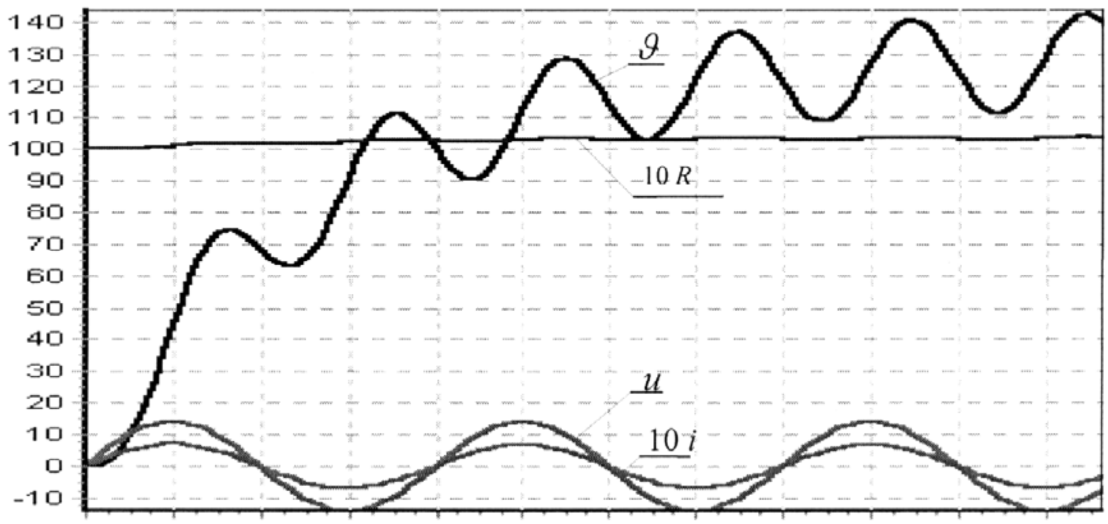

Fig. 1. The transient waveforms (explanation in the text)

Illustrative waveforms of the temperature, current, voltage, and resistance for the sinusoidal voltage excitation with frequency of 0.5 Hz are plotted in

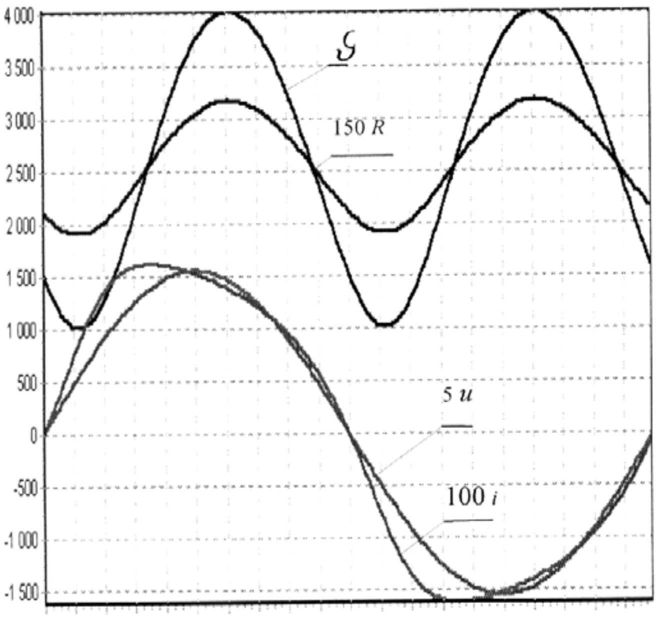

Fig. 2. The steady state waveforms (explanation in the text)

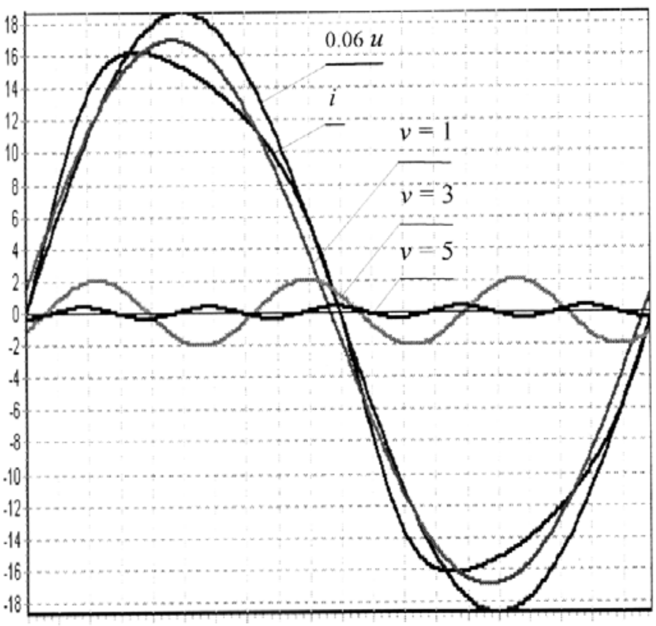

Fig. 3. Harmonics of the steady state current (explanation in the text)

Fig. 1. Particularly interesting is a pulse-rising temperature waveform. It is worth to note that the temperature minima are shifted against the zero-crossing instants of the voltage waveform. Similarly, the temperature maxima are shifted against the positive and negative voltage extremes.

The steady state waveforms of the temperature, current, voltage, and resistance are shown in Fig. 2. The respective computations were performed with the voltage of 220 V (rms), frequency of 0.005 Hz, resistance $R_0 = 10$ Ohm, temperature resistivity coefficients: $\alpha = 2.7 \times 10^{-4}$ and $\beta = 1.86 \times 10^{-9}$, specific heat $c = 250$ J/(kg·K), mass $m = 0.1$ kg, $S = 0.1$ m^2, and the specific heat- dissipation $a = 10$ W/(K·m^2).

In Fig. 3 waveforms of the voltage, the total current, as well as the current 1st, 3rd, and 5th harmonics are illustrated.. From Fig. 3 we conclude that the 1st current harmonic is shifted ahead of the voltage. The computed angle equals 5.2°.

4. Conclusions

Non-linear properties of the elements of electric circuits result due to "external" phenomena, which accompany the electromagnetic processes, such as thermal, chemical, mechanical, structural and other physical effects. The response in such circuits is delayed against the excitation (the imposed cause). In this contribution the calculation method for changes of the circuit parameters is proposed. For this purpose an exponential function with time constant T equal to T_R, T_L, or T_C for an R, L, or C element, respectively, has been formulated. If the electromagnetic excitation varies sinusoidal in time with period T_e, phenomena in the circuit depend on the relation of the period T_e to the time constant T. The following situations have been considered: (1) $T_e \gg T$, (2) $T_e \ll T$. In the first case, the effect changes in time accordingly to the non-linear characteristic of the element and process may be calculated using the function (1). In the second case, the respective circuit is referred to as the "quasi-linear", because for the sinusoidal cause the effect is also sinusoidal. A hypothesis has been formulated, that for a sufficiently high frequency every non-linear circuit will become a quasi-linear system.

As an example a non-linear resistance supplied with a sinusoidal voltage, is considered. From the presented computations it follows that the behavior of a non-linear resistor depends on the excitation frequency. The resistor operates as a quasi-linear element for a sufficiently high frequency, i.e., the waveform of the resistor current has the same shape as this of its voltage.

On the contrary, for very low frequencies the current is strongly deformed and its first harmonic is shifted ahead of the voltage. In this case, the non-linear resistor operates as an element of resistive-capacitive character. This phenomenon is referred in this paper as a "capacitance effect" of the resistor.

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