THEORY AND DESIGN OF SMALL MOTORS

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Abstract. Theory of small motors was generalized with the purpose to take into account of parametrical phenomenon in machines with magnetic asymmetry. Theory was used for electromagnetic computation of induction motors and direct current motors. The motors design drawings formed with the help of CAD program Pro/Engineer'2001.

Keywords. Small motor, magnetic asymmetry, space harmonic, parametrical phenomenon

1 Introduction

A small induction motors are non-symmetric machines. The most common case of asymmetric machines represents single-phase salient-pole motors. An apart from space and electrical asymmetry they have asymmetric construction of pole tips, stepped air-gap etc. Magnetic asymmetry and discretization of a stator winding are basic design features of salient-pole motors. Although construction of these motors is relatively simple, theoretical analysis is difficult such that much design work is accomplished empirically. The problem of theory generalization and design procedure creation is urgent.

The indicated design features are the reason of increased influence of high magnetomotive force (m.m.f.) harmonics on motor performances. Especially large influence on motor performances can render the third m.m.f. harmonic. Optimal using of asymmetric air-gap enables to improve the shape of a flux density distribution curve and motor performances.

A sinusoidal distributed m.m.f. produces an infinite series of space flux density harmonics in conditions of a non-uniform air-gap. The air-gap field is convenient for presenting as a product m.m.f. and air-gap permeance. The greatest amplitude has a second air-gap permeance harmonic [1,8].

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The availability of space air-gap permeance harmonics brings in a series of features to a character of formation of a magnetic field in air-gap and windings interlinkages. The features of energy transformation in induction motors with magnetic asymmetry are electromagnetic interactions between rotor currents with different space periods. The similar interactions are absent in machines with uniform air-gap.

The aim of this paper is to present generalized theory and design procedure of squirrel cage induction machines with non-uniform air-gap and non-sinusoidal distributed windings, in which took into account parametrical phenomenon. It relies upon one of the fundamental basis of electrical engineering - circuit theory. Special attention was given to use of computer-aided calculation subsystem for design optimization.

2 Theory development for asymmetric machines

Differential equations of rotating electrical machines in natural (phasic) coordinates always include variable coefficients [1,10]. Only for symmetric machines or machines with one-sided asymmetry and under condition of an invariance of magnetic properties of the core the differential equations manage to be reduced in the equations with constants coefficients with the help of known linear transformations [1,11]. It is possible to take into account only fundamental space m.m.f. harmonic in machines with non-uniform air-gap. The mathematical description in this case can be considered as the linear theory of electrical machines.

In case of the discrete arrangement and asymmetric distribution of windings, and also non-uniform air-gap equation it is impossible to re-arrange in the equations with constant coefficients by known linear transformations even in a neglect by nonlinear properties of magnetic circuit.

In these cases, a solution of the equations in general and analysis of machine properties become impossible and it is necessary to use the numerical methods. As a new stage of development of the analytical theory of electrical machines it is possible to consider development of the special transformation methods of the differential equations to the equations with constant coefficients when the usual linear transformations do not give such outcome. It is possible to divide objects circumscribed by the differential equations with variable coefficients, into two type [1]:

1) Parametrical (the equations coefficients depends on argument - time);

2) Nonlinear (the coefficients depends on unknown quantity of functions).

The modern level of the account of nonlinear phenomenon in electrical machines is achieved with the help of creations of mathematical models oriented on a numerical solution of a problem [2,6].

In machines with magnetic asymmetry there are parametrical phenomenon because variation of rotor inductances during its rotation. Essence of parametrical phenomenons is generation in the machine e.m.f. and currents of combinative frequencies depending on supply frequency and a speed of rotation. The parametrical phenomenon appear in rotating machines in the following cases:

1) Because of an operation of high space m.m.f. harmonics in machines with magnetic asymmetry (with non-uniform air-gap);

2) In machines with two-sided asymmetry (even in a case when act only basic space harmonics). This case meets in the following operating modes: asymmetric load and phase retardation, asymmetric short-circuits of synchronous generators, power supply of synchronous small motors from a single-phase circuit, asymmetric stator and rotor of induction motors.

These cases are of great importance however, known transformations did not allow to receive the equations with constant coefficients for mathematical exposition. Therefore were used only approximate or numerical methods. For modern practice such approach is insufficient.

The parametrical phenomenon are the reason influences on a rotor e.m.f. of the given frequency of currents all its frequencies. Other feature is generation in stator winding of an e.m.f. with frequencies which are not equal to supply frequency. One of the reasons of this phenomenon is discrete disposition of squirrel-cage bars.

The account of parametrical phenomenon is necessary condition of designing of an optimum motor construction [3,4].

The feature of squirrel-cage winding is that the number of poles of a fundamental m.m.f. harmonic of a rotor is equal to number of poles of a magnetic field of a stator.

The period of rotor current distribution round air-gap is equal to a space period of a stator field. Except for a fundamental m.m.f. wave of a rotor there are also harmonics with higher numbers because of a discontinuous distribution of squirrel-cage bars.

Harmonic number is noted from above on the right in brackets. The inferior indexes specify a winding, to which the given magnitude concerns.

Let's name as a rotor current of ν-th harmonics the current $I_R^{(\nu)}$, the space period of which distribution along a circle of air-gap is equal $2\tau/\nu$.

The higher magnetic field harmonics of a rotor reduce to spectrum of high currents harmonics, which frequency depends on rotor speed. The high air-gap permeance harmonics are the reason of electromagnetic interactions between separate circuits of the rotor winding on which the currents with various space periods of distribution and with various frequencies flow. The currents of a rotor of various harmonics generally are nonsymmetrical. The air-gap permeance harmonics are the reason of electromagnetic interactions between currents of a rotor of the same order, but various sequences.

For illustration, we select the second air-gap permeance harmonic

$$\lambda^{(2)}(x)=\lambda_m^{(2)}\cos\left(\frac{2\pi x}{\tau}\right), \tag{1}$$

where τ - pole pitch; x - coordinate round the air-gap; $\lambda_m^{(2)}$- amplitude value for second harmonic of air-gap permeance.

Let first rotor m.m.f. harmonic in positive direction of the coordinate x:

$$F_R^{(1)}(x,t)= F_R^{(1)}\cos\left(\frac{\pi x}{\tau}-\omega t\right), \tag{2}$$

where $F_R^{(1)}$ - amplitude value of the rotor m.m.f.

This m.m.f. harmonics acting on the second air-gap permeance harmonics (Fig.1) creates in an air-gap two flux density harmonics (first and third harmonics rotates in opposite directions with various speeds):

$$B_R(x,t)=F_R^{(1)}(x,t)\,\lambda^{(2)}(x)=\frac{F_R^{(1)}\lambda_m^{(2)}}{2}\left[\cos\left(\frac{\pi x}{\tau}+\omega t\right)+\cos\left(\frac{3\pi x}{\tau}-\omega t\right)\right]. \tag{3}$$

From this expression follows, that the first flux density harmonic rotates opposite to direction of m.m.f., from which it arises. The third flux density harmonic rotates according to with m.m.f.

Each harmonic induces in a rotor bars e.m.f. with an appropriate period of distribution round the air-gap. The first flux density harmonic induces an e.m.f. of an inverse sequence, and third harmonic induces an e.m.f. of direct sequence. In turn, the space waves of rotor currents of the first and third harmonics of various sequences, acting to the second harmonics of air-gap permeance $\lambda^{(2)}(x)$ induces the first flux density harmonic. The first flux density harmonic induces an e.m.f. of a direct sequence in the same circuits, where to flow the currents inducing m.m.f.

The scheme of mutual interaction between circuits of rotor currents $I_R^{(1)}$and $I_R^{(3)}$, due to the second air-gap permeance harmonic is shown on Fig. 1. The level of electromagnetic interactions depends on amplitude of appropriate air-gap permeance, which can achieve values close to a half by a constant component in salient-pole micromotors. Generally air-gap permeance with a serial number k causes interaction between a rotor currents, which create a m.m.f harmonics with a serial numbers μ and ν. Between magnitudes k, μ and ν the condition should be observed:

$$k=|\mu\pm\nu|. \tag{4}$$

Hereinafter if necessary instead of index k we shall use a double index μ,ν or ν,μ. If such indexes will have air-gap permeance, it will mean, that their serial number is determined by equality (4). If the double indexes will have reactances, they are mutual reactances between circuits with currents $I_R^{(\nu)}$ and $I_R^{(\mu)}$.

In salient-pole micromotors with magnetic asymmetry because of saturation of a stator yoke the magnetic resistances of circuits, on which the force lines of long and cross fields become closed, are various. In this connection, the amplitudes of air-gap permeances with the same serial number for long and cross-fields are various also [7, 10-13].

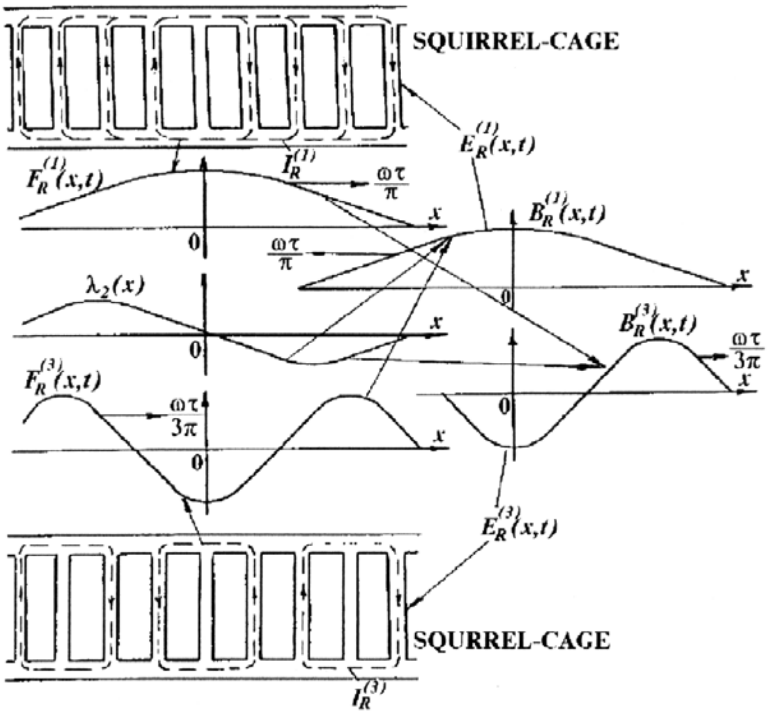

Fig.1. The scheme of electromagnetic interaction in induction motors with magnetic asymmetry

Salient-pole induction motors have skew of rotor slots usually. The m.m.f. of a rotor winding because of slots skew has unequal distribution in the various cross-sections of the machine. It is function of two variables: coordinate round the air-gap x and coordinate along axes of the machine.

All enumerated features of electromagnetic processes in micromotors with magnetic asymmetry are taken into account in mathematical models [7,12,13].

3 Computer-aided design

Because of theoretical researches and generalization of designing experience, the subsystem of computer-aided design of single-phase induction motors is created. It is based on the mathematical model described in [7,13]. A method of verifying calculations was used for synthesis of motors with the given level of performance. The essence of a method is that on each pitch of searching some variant of the designed machine is supposed. This variant is unambiguously determined by a set of input data, which describe the basic constructive elements of a motor. After calculation of operating performances of simulated variant the perfection is evaluated it. The modification even by one of initially assigned magnitudes reduces to development of new variant of a machine, for which operating performance also calculate. The evaluation will be carried out.

All constructive and technological singularities included in concrete variant are taken into account. The known restrictions (normative, constructive, and technological) are simple taken into account at a stage of generation of variants during computer-aided calculation. It is important to mark, that the verification of restrictions realization is rather simple formalized, so, and is automated.

It is possible to calculate parameters and operating performances for a selected construction, production process and factory assembly by calculating parameters of equivalent circuit. Through the same parameters those magnitudes are calculated also, on which judge realizability or working capacity of variant (for example, block coefficient of a slot by a wire or reheat temperature of windings, bearings etc.). It is important, that motor performances are determined unambiguously by method of verifying calculations for the given parameters of power supply and connection scheme. Just this factor is defining and ensures competitiveness of a method for computer-aided design of electrical machines.

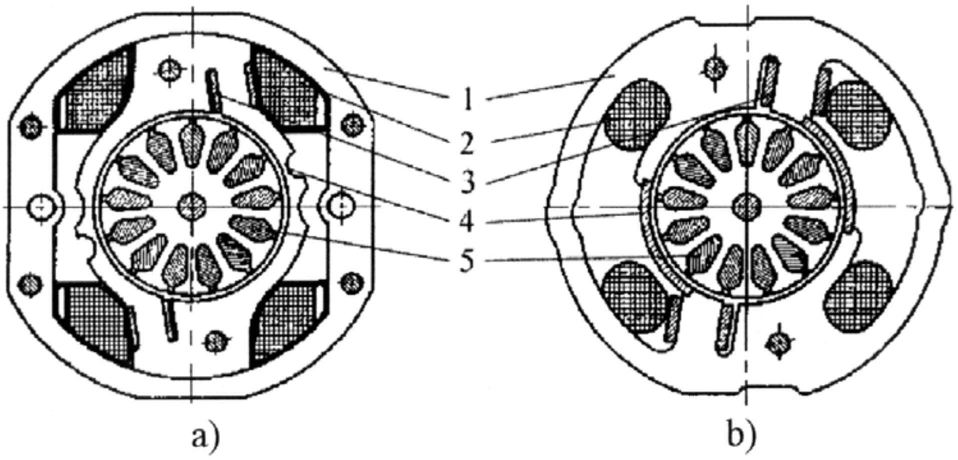

Fig. 2. Cross-sections of a shaded-pole motors AD-2,5-30/E (a) and DAO62-2,5 (b): 1 - yoke of stator core; 2 - main winding; 3 - auxiliary winding; 4 - magnetic shunt; 5 - squirrel-cage

The calculation subsystem of design synthesis and optimization was used for electromagnetic calculation of the unificated series of shaded-pole induction motors (Fig. 2). The computer-aided design program Pro/Engineer'2001 was used in engineering development subsystem.

Each constructive element of a motor represents the graphic module. The working drawings of motors print out by plotter Hewlett Packard Design Jet 430. The developed shaded-pole induction motor AD-2,5-2/30E by output power 2,5 W is intended for office heat blower TV-2 'ELARA', and also can be used in other electrical domestic appliances [5].

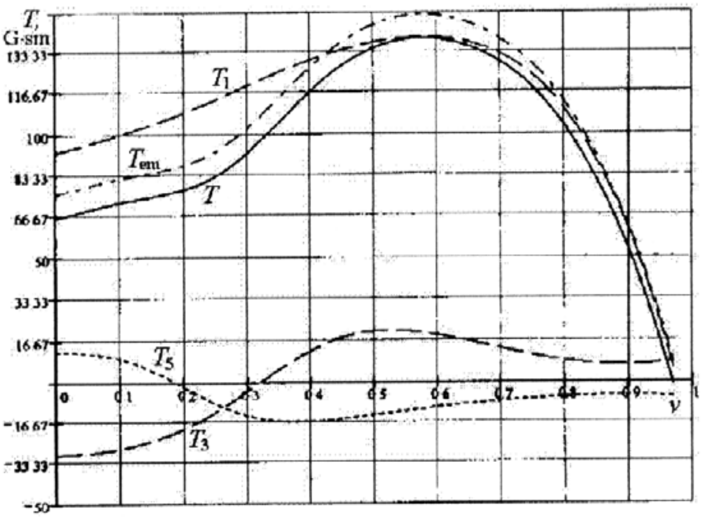

Fig. 3. Torque-speed characteristics of AD-2,5-30/E: T_1 - first harmonic torque; T_3 - third harmonic torque; T_5 - fifth harmonic torque; T - torque on the shaft; T_{em} - electromagnetic torque; v - relative speed

The performance attributes of the shaded-pole motor AD-2,5-2/30E are presented in Table 1 and Fig. 3. Space harmonic torques can influence to process of the electric motor start. First space harmonics produces greatest additional asynchronous torques of the number $n = kz/p \pm 1$, where k - integer; z - number of tooth; p - number of pair of poles. First space harmonic $n = 1$ is considered as basic, remaining space harmonics is considered as higher. The greatest additional asynchronous torques produces by high space harmonics with numbers $n = 3$ and $n = 5$.

The use of calculation subsystem has allowed at the stage of calculation to increase output power, to remove deeps in a curve of a torque, to reduce oscillations of a starting torque, it is essential to reduce breadboarding and to ensure a high level of quality of the electric motor.

Table 1. Data of shaded-pole induction motor AD-2,5-2/30E

Performance	Computed	Measured
Supply Voltage, *V*	220	220
Frequency, *Hz*	50	50
Output Power, *W*	2,5	2,5
Current, *A*	0,135	0,125
Efficiency, %	11,0	11,5
Speed, *rpm*	2400	2390
Breakdown Torque, *N·m*	0,0139	0,0140
Starting Torque, *N·m*	0,0066	0,0062

4 Conclusions

The theory of small induction motors was developed for took into account parametrical phenomenon in machines with magnetic asymmetry. The developed theory was used for computer-aided calculation and optimization of single-phase induction motors. The purpose of the designer is a minimization of motor cost. The advantage of the approach presented that it allows to reduce CPU time significantly.

5. References

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