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A COMPARISON OF DIFFERENT METHODS FOR THE GRADIENT EVALUATION IN THE INVERSE PROBLEMS OF THE IMPEDANCE TOMOGRAPHY

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Abstract: The standard approach for the evaluation of the gradient during the optimization process in the impedance tomography is based on the inverse of the global admittance matrix utilizing certain iterative procedure of nodal voltages. In this paper the benefit of the gradient evaluation by different position of the grounding node on the mesh are compared.

Keywords: Impedance tomography, inverse problem, finite element method, objective function, gradient evaluation.

1 Introduction

Electrical impedance tomography is used to locate non-homogeneities in otherwise homogenous media. The volume to be analyzed is accessible only at a limited number of external nodes. Some of these nodes can be supplied from an exciting current source I_e while the others carry potentials φ_b that reflects the conductivity of the volume. The main goal is finding the distribution conductivity γ within the volume using the electric potential measurements of the boundary of the volume. To determine this distribution we can use either deterministic or stochastic processes (for example Neural Networks it is in [1]). Another way is to minimize a suitable object function [2], [3]. The present paper

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describes one of the possibilities of evaluation gradient of the object function, which is used to speed up the optimization process.

2 FEM model and optimization

The model shown in Fig. 1 was used for the computer simulation. Region Ω under consideration is two-dimensional square area (side equal one unit) of specified electrical properties (conductivity $\gamma_0 = 1$ S/m), which is divided into set of triangle elements with different conductivities. The mesh of FEM model has NU = 100 nodes and NE = 162 triangle elements. There are two sub-domains Ω_1, Ω_2 inside with conductivity $\gamma_1 = 3$ S/m, $\gamma_2 = 3{,}5$ S/m.

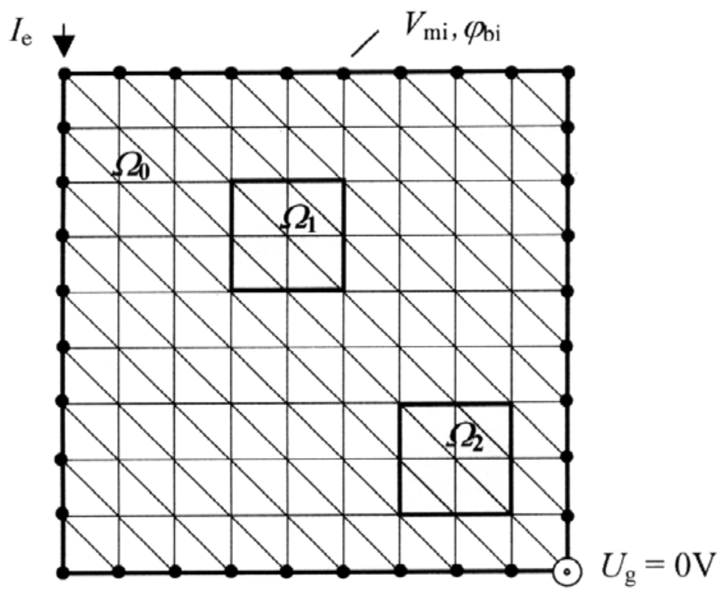

Figure 1. The FEM model

The exciting current $\boldsymbol{I}_e$ is connected only to one of the boundary nodes and one boundary node is grounded (node with $\boldsymbol{U}_g$ = 0V). Let us note Nsup the number of all different current source positions. In order to get as much information as possible, eighteen independent arrangements of current electrodes have been considered. The remaining nodes N_b along the boundary are used to measure electric potential $\boldsymbol{V}_{mi}$ and to calculate $\boldsymbol{\varphi}_{bi}$. The FEM notation of the system equations for nodal potential is

$$\boldsymbol{Y}_i \, \varphi_{bi} = \boldsymbol{f}_i \ , \quad i = 1, \text{Nsup} \ , \tag{1}$$

$\boldsymbol{Y}_i$ is matrix (NU × NU) of conductances, right-hand side vector $\boldsymbol{f}_i$ contains the exciting current $\boldsymbol{I}_e$. Because the potential of the grounded node is known, the system of equations (1) can be reduced to

$$\boldsymbol{Y}_{ri} \, \varphi_{bri} = \boldsymbol{f}_{ri} \ . \tag{2}$$

Now the object function for the minimization process is defined as

$$\Phi = 0,5\sum_{i=1}^{Nsup}\left(V_{mi}-\varphi_{bri}\right)^2 . \tag{3}$$

3 Solution for the different position of the grounded node

To speed up the optimization process using the quasi-Newton iteration method it is advantageous to prepare the gradient of the object function (3)

$$\frac{\partial \Phi}{\partial \gamma} = \sum_{i=1}^{Nsup} \frac{\partial \Phi_i}{\partial \varphi_{bri}} \frac{\partial \varphi_{bri}}{\partial \gamma} = -\sum_{i=1}^{Nsup}\left(V_{mi}-\varphi_{bri}\right)\frac{\partial \varphi_{bri}}{\partial \gamma} \tag{4}$$

in the space of γ. From equation (2) we can easily obtain the derivative

$$\frac{\partial \varphi_{bri}}{\partial \gamma} = -\boldsymbol{Y}_{ri}^{-1}\left[\frac{\partial}{\partial \gamma}\boldsymbol{Y}_{ri}\,\tilde{\varphi}_{bri}\right] . \tag{5}$$

The evaluation of the term (5) can be realized by different way depending on the grounded node location into the FEM mesh; two of them shows Fig. 2. If the grounded node is located according to Fig. 2a), matrix $\boldsymbol{Y}_{ri}$ is different for each current source position of Nsup. If it is located according to Fig. 2b), the matrix $\boldsymbol{Y}_{ri}$ is the same for each current source position of Nsup. How it will be shown, the second way is very efficient where evaluating the object function gradient during the iterative minimization process.

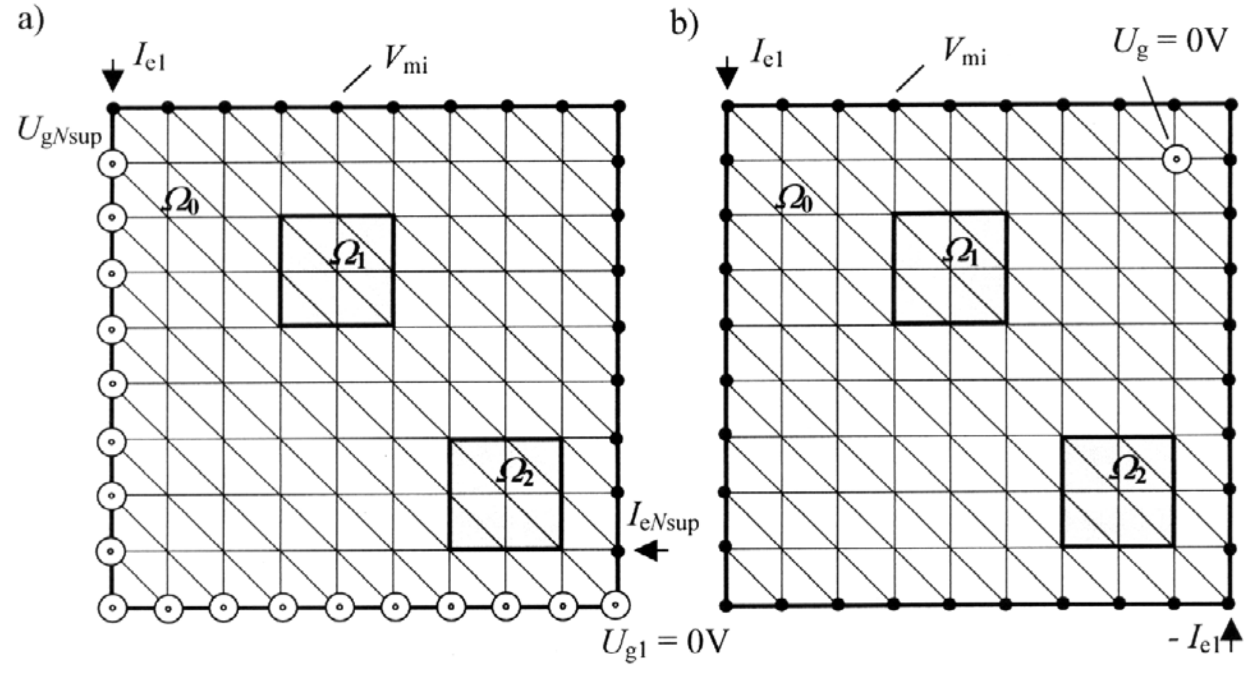

Figure 2. Position of the grounded node

4 Example of the optimization

The optimization process has been tested by own developed program on Notebook IBM with Intel Pentium 4, 2GHz processor. Different functions from FORTRAN IMSL library have been used to optimize (3). Fig. 3a) shows the conductivity distribution and Fig. 3b) shows errors on each element for the time accelerated variant of the optimization. The error for another variant of the optimization was greater.

a) b)

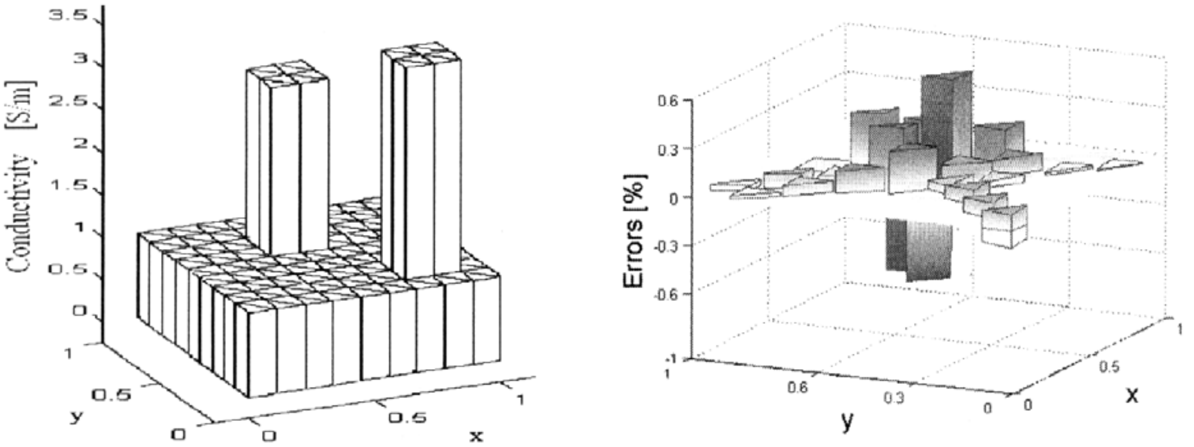

Figure 3. Conductivity and the errors of the conductivities evaluation

5 Conclusion

The present paper described some efficient possibility of gradient evaluation. The time consumption for gradient evaluation by the finite difference method for position according to Fig. 2a) was 360 min and by using (4) 80 min. For the position according to Fig. 2b) the time consumption was 90 min for gradient evaluation by the finite difference method and 21 min by using (4).

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