September 10 - 12, 2003
Pilsen, Czech Republic

EXPRESSION OF THE UCERTAINTY OF MEASUREMENT USING EXCEL VBA

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Abstract: The technical community always underrates statistical processing of data. However the result of a measurement is complete only when accompanied by its uncertainty. This article originated from documents of BIPM, ISO, etc. for evaluating and expressing the results of measurement. Spreadsheet EXCEL is used in the computation.

Keywords: Uncertainty of measurement, spreadsheet EXCEL

1 Introduction

First material concerning uncertainty of measurement was published by *BIPM, Bureau International des Poids et Measures* in 1980 and was named "Recommendation INC-1 (1980)", see [8]. Uncertainties of "A" and "B" type are discussed here and an uniform approach for their evaluation is recommended.

From 1980 on many documents deal with this problem, see References. The documents [2], [3], [6] were published in Czech and Slovak Republic.

2 Example

Let us compute the *Body mass index (BMI)* of a group of 15 students. *Body mass index is defined as the ratio of mass in kg and the square of height in m,* see (1)

$$BMI = \frac{m}{l^2} \qquad (1)$$

Mass m and length l are the directly measured values. *BMI* is calculated for each student. The arithmetic mean, complete with uncertainty of

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measurement is calculated for the group. The personal weighting-machine LUXA and folding rule LOGAREX 38031 were used. *BMI* and health risk are related (see Table 1).

Table 1 BMI and health risk

BMI	18,5 – 24,9	25,0 – 29,9	30,0 – 34,9	35,0 – 39,9	40,0 and more
Status	Normal	Overweight	Obesity 1. level	Obesity 2. level	Obesity 3. level
Health risk	Minimal	Increased	Average	High	Very high

3 Basic terms

The uncertainty of measurement is a parameter, associated with the result of a measurement, which characterises the dispersion of the values. There are uncertainties of "A" and "B" type.

3.1 Uncertainties of type "A"

Uncertainties of "A" type originate from random errors. The evaluation of this uncertainty is based on a statistical analysis of series of observations.

The estimate of the quantity $\bar{x}_j$ is the arithmetic mean (2)

$$\bar{x}_j = \sum_{i=1}^{n} \frac{x_{ji}}{n}, j = 1,\ldots,m \tag{2}$$

The sample variance of $\bar{x}_j$ is (3)

$$u_{Ax_j}^2 = s_{\bar{x}_j}^2 = \frac{1}{n(n-1)} \sum_{i=1}^{n} \left(x_{ji} - \bar{x}_j\right)^2 \tag{3}$$

Experimental standard deviation $s_{\bar{x}_j}$ of the mean is used as a standard uncertainty u_{Ax_j} of "A" type, see Table 3, u_{Am}^2, u_{Al}^2

3.2 Uncertainties of type "B"

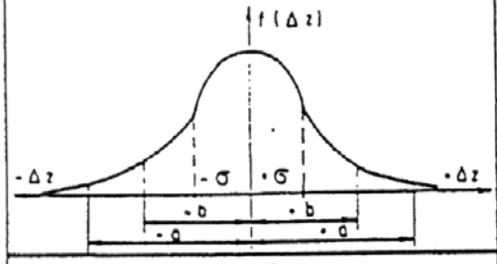
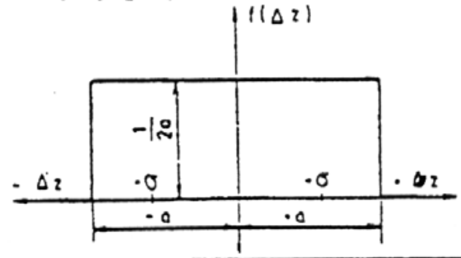
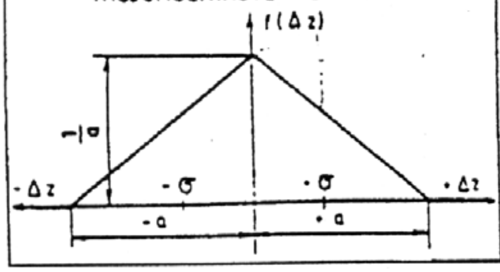
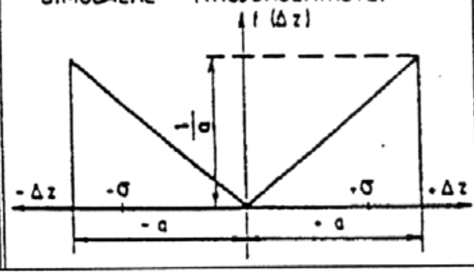
Uncertainties of "B" type originate from systematic errors. The evaluation of this uncertainty can not be based on statistical analysis of series of observations. Relevant information to estimate this type of uncertainty are previous measurement data, experience with relevant materials and instruments, technical documentation, etc. Maximal errors due to weighting-machine LUXA and folding rule LOGAREX 38031 are z_{m1} = 1 kg, z_{l1} = 1 mm respectively at full scale. Uniform (rectangular) distribution is assumed in both cases. To reduce the influence of clothes we suppose z_{m2} = 05 kg, and a normal probability distribution. The uncertainty of "B" type appropriate to source z u_{Bz} can be estimated as follows:

1. Estimate z_{max} - maximal deviation of value, appropriate to source z

2. Check probability curve and find in Table 2 the best approximation
3. Determine uncertainty of "B" type using (4). See Table 3, u_{Bm1}^2, u_{Bm2}^2, u_{Bl}^2

$$u_{Bz}^2 = \frac{z_{\max}^2}{\chi^2} \tag{4}$$

Table 2 Probability distribution and coefficients

ROZDELENIE	z_{max}	χ	ROZDELENIE	z_{max}	χ
NORMÁLNE – GAUSSOVO 	a b	3 2	ROVNOMERNÉ – PRAVOUHLÉ 	a	$\sqrt{3}$ ~1,73
TROJUHOLNÍKOVÉ – SIMPSONOVO 	a	$\sqrt{6}$ ~2,45	BIMODÁLNE (TROJUHOLNÍKOVÉ) 	a	$\sqrt{2}$ ~1,41

3.3 Combining uncertainty components

Let us assume that the output estimate is the result of a measurement calculated from the input estimates by the model function:

$$\bar{y} = f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m) \tag{5}$$

The simplest form of sample variance of $\bar{y}$ can be obtained from (6). This form is known as the *Gaussian law of propagation of uncertainties* and is based on a Taylor series approximation, where only part of the first order term is kept.

$$u_{Ay}^2 = \sum_{j=1}^{m} \left(\frac{\partial f}{\partial X_j} \right)^2_{X_1=\bar{x}_1, X_2=\bar{x}_1, \ldots, X_m=\bar{x}_m} u_{x_j}^2 \tag{6}$$

The partial derivatives $\partial f/\partial x_j$ (referred to as sensitivity coefficients) are equal to $\partial f/\partial x_j$ evaluated at $X_1 = \bar{x}_1, X_2 = \bar{x}_2, \ldots, X_n = \bar{x}_n$. Components of both types of uncertainty ("A" and "B") are combined using the same method, see (6). Finally both types of uncertainties are combined into a combined uncertainty u_c, see (7)

$$u_{Cy}^2 = u_{Ay}^2 + u_{By}^2 \tag{7}$$

4 Results

The result of measurement is given by (8) where k is the coverage factor. If $k = 2$, estimated values are approximately normally distributed and standard deviation is u_{Cy}, then the unknown value of y is believed to lie in the interval defined by u_{Cy} with a level of confidence of approximately 95%. Results of the example are in Table 3

$$Y = \bar{y} \pm k u_{Cy} \tag{8}$$

Table 3 Calculation of *BMI* and combined uncertainty

Body mass index BMI = m/l²

N= 15

Number of measurement	Complete		Calculated				
	m(kg)	l(m)	$m_i - \bar{m}$	$l_i - \bar{l}$	$(m_i - \bar{m})^2$	$(l - \bar{l})^2$	BMI = m/l²
1	73,5	1,83	6,5	0,06	41,8	0,0038	21,9
2	60,0	1,77	-7,0	0,00	49,5	0,0000	19,2
3	68,5	1,76	1,5	-0,01	2,2	0,0001	22,1
4	73,5	1,80	6,5	0,03	41,8	0,0010	22,7
5	62,5	1,81	-4,5	0,04	20,6	0,0017	19,1
6	71,0	1,79	4,0	0,02	15,7	0,0005	22,2
7	68,5	1,68	1,5	-0,09	2,2	0,0079	24,3
8	69,5	1,81	2,5	0,04	6,1	0,0017	21,2
9	72,5	1,75	5,5	-0,02	29,9	0,0003	23,7
10	59,5	1,73	-7,5	-0,04	56,8	0,0015	19,9
11	60,5	1,72	-6,5	-0,05	42,7	0,0024	20,5
12	62,5	1,79	-4,5	0,02	20,6	0,0005	19,5
13	76,5	1,78	9,5	0,01	89,6	0,0001	24,1
14	62,5	1,75	-4,5	-0,02	20,6	0,0003	20,4
15	64,5	1,76	-2,5	-0,01	6,4	0,0001	20,8
Σ	**1005,5**	**26,5**	**0,0**	**0,00**	**446,2**	**0,0218**	**321,5**
	$\bar{m}$	$\bar{l}$					$\overline{BMI}$
Arithmetical mean	67,0	1,77					**21,4**
	u_{Am}^2	u_{Al}^2			∂f/∂m	∂f/∂l	
Uncertainties "A"	2,12	0,0001		partial derivatives	0,32	-24,20	
	u_{Bm1}^2	u_{Bm2}^2	u_{Bl}^2		u_{cy}	**u**	
Uncertainties "B"	0,33	0,03	0,000008		0,57	**1,1**	
	u_{Ay}^2	u_{By}^2	**Result of measurement**				
Combined uncertainties	0,28	0,04	**BMI = (**		**21,4**	**±**	**1,1) kg/m²**

4.1 Commentaries to Table 3

- Measured values are shown in columns m(kg), l(m)
- The number of rows N has been computed by the application of function POČET
- Values in the column BMI are computed by (1)
- Values $\overline{m}, \overline{l}, \overline{BMI}$ are the arithmetical means of the corresponding columns
- Sample variances are computed by $s_{\bar{x}_j}^2 = \frac{1}{n(n-1)} \sum_{i=1}^{n} \left(x_{ji} - \bar{x}_j\right)^2$
- The partial derivatives $\partial f / \partial x_i$ are calculated from their analytical expression
- The combined uncertainty is computed from $u_C^2 = u_{A\bar{y}}^2 + u_{B\bar{y}}^2$
- The expanded uncertainty is $u = 2 \cdot u_c$.
- The true value lies with probability 95% in the interval defined by the value of expanded uncertainty
- The final result is given by $\mathrm{BMI} = \left(\overline{\mathrm{BMI}} \pm u\right)$

5 Conclusions

1 The computing of uncertainties of A and B type is described in this paper.
2 Computation has been mode according to documents WECC doc. 19-1990 and EAL-4/02.
3 A practical-oriented example has been solved.
4 Spreadsheet EXCEL has been used for computation.

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