# Baseline equations for purposes of analysis of a three-phase three-winding transformer under asymmetric load 

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#### Abstract

The paper presents a set of equations enabling analysis of operation of a three-phase three-winding transformer under asymmetric load. For a selected asymmetric state of the transformer the characteristic output values of the transformer have been computed with the use of the Mathcad computer software.


Keywords Electrical machines, power transformers.

## I. Introduction

Asymmetric condition of a transformer is caused by asymmetric phase loads or by asymmetry of the supply voltage The three-winding transformers used in power systems are usually not loaded with single-phase receivers (with neutral wire). Nevertheless, they are often loaded with the receivers in two-phase arrangement. In the industry the transformers are used with various types of receiver connections. The paper presents a set of equations describing the effects of various connection types on the current flux in the transformer windings and, in consequence, on the voltage values in both secondary windings. For the purposes of the present paper the terms the upper (g), medium (s) and bottom(d) windings are used, in accordance with similar terms related to the power transformers, irrespective of actual values of the voltages.

## II. Characteristics of the problem

In order to enable investigation of the impact of three various values on output parameters of the transformer, the equations include three coefficients $k_{1}, k_{2}$ and $k_{3}$ as the independent values. The load impedances of particular phases of the medium voltage winding are defined by the equations (1):

$$
\begin{align*}
& Z_{z d u}\left(k_{1}\right)=k_{1}\left|Z_{o d n g}\right| \exp \left(j 0.107 \cdot 2 \frac{\pi}{3}\right) \\
& Z_{z d v}\left(k_{2}\right)=\left|Z_{o d n g} \cdot 0.85\right|\left(\exp j k_{2} 0.107 \cdot 2 \frac{\pi}{3}\right)  \tag{1}\\
& Z_{z d w}\left(k_{3}\right)=k_{3}\left|Z_{o d n g} \cdot 0.65\right| \exp \left(j 0.307 \cdot 2 \frac{\pi}{3}\right)
\end{align*}
$$

The impedances are transformed into symmetrical components $\quad \mathrm{Z}_{\mathrm{d} 1}\left(\mathrm{k}_{1}, \mathrm{k}_{2}, \mathrm{k}_{3}\right), \quad \mathrm{Z}_{\mathrm{d} 2}\left(\mathrm{k}_{1}, \mathrm{k}_{2}, \mathrm{k}_{3}\right), \quad \mathrm{Z}_{\mathrm{d} 0}\left(\mathrm{k}_{1}, \mathrm{k}_{2}, \mathrm{k}_{3}\right)$. Afterwards, the currents of particular symmetrical components were calculated for the medium and bottom voltage windings, and formulated in the matrix form (without the $s$ and $d$ indexes) $\mathrm{U}_{\text {ntg }}$ is the rated voltage of the upper (primary) winding (2)

$$
\left(\begin{array}{l}
I_{l}\left(k_{1}, k_{2}, k_{3}\right.  \tag{2}\\
I_{2}\left(k_{1}, k_{2}, k_{3}\right. \\
I_{3}\left(k_{1}, k_{2}, k_{3}\right.
\end{array}\right)=\left(\begin{array}{lll}
M_{11} & M_{12} & M_{10} \\
M_{21} & M_{22} & M_{20} \\
M_{01} & M_{02} & M_{00}
\end{array}\right)\left(\begin{array}{l}
1 \\
0 \\
0
\end{array}\right) \frac{U_{n t g}}{D\left(k_{1}, k_{2}, k_{3}\right)}
$$

The terms of the matrix (2) for the bottom voltage are specified as an example (3):

$$
\begin{aligned}
& M_{d 11}\left(k_{1}, k_{2}, k_{3}\right)=\left(Z_{u g d}+Z_{d 0}\left(k_{1}, k_{2}, k_{3}\right)\right) \times \\
& \left(Z_{d 0}\left(k_{1}, k_{2}, k_{3}\right)+Z_{\mu 0}\right)-Z_{d 1}\left(k_{1}, k_{2}, k_{3}\right) Z_{d 2}\left(k_{1}, k_{2}, k_{3}\right) \\
& \quad M_{d 2 I}\left(k_{1}, k_{2}, k_{3}\right)=\left(Z_{d 2}\left(k_{1}, k_{2}, k_{3}\right)\right)^{2}+ \\
& \quad-Z_{d 1}\left(k_{1}, k_{2}, k_{3}\right)\left(Z_{d 0}\left(k_{1}, k_{2}, k_{3}\right)+Z_{\mu 0}\right) \\
& M_{d 01}\left(k_{1}, k_{2}, k_{3}\right)=\left(Z_{d 1}\left(k_{1}, k_{2}, k_{3}\right)\right)^{2}+ \\
& -Z_{d 2}\left(k_{1}, k_{2}, k_{3}\right)\left(Z_{d 0}\left(k_{1}, k_{2}, k_{3}\right)+Z_{\mu d}\right)
\end{aligned}
$$

The output voltages are obtained with the use of the transformation matrix based on the components symmetrical to the phase values.

## III. Example calculation results

The formulae so derived were used for calculating some selected output values of a three-winding transformer $20 / 10 / 10$ MVA, $110,33,6.6 \mathrm{kV}$. It was assumed that the transformer is asymmetrically loaded at the side of the bottom voltage, with symmetric load at the side of medium voltage. Figure 1 show the plots of the currents vs. the selected coefficients.


Fig. 1. Variation of the current in the bottom voltage winding vs. the $\mathrm{k}_{3}$ parameter

## IV. Conclusions

The asymmetric load induces asymmetrical flux of the current in the transformer windings. In result, the output voltages and, in consequence, the voltage asymmetry coefficient, are asymmetrical too. It should be ensured that the load asymmetry cause no exceedance of rated current values at the side of the upper voltage.

## V. Literature

[1] L. Kasprzyk, Z. Stein, M. Zielińska, Analiza wpływu niesymetrycznych obciążeń transformatora na jego charakterystyczne wielkości wyjściowe. ZKwE 2010.

