MAGNETIC PRESSURE ACTING ON A CIRCULAR WORKPIECE IN THE PULSE MAGNETIC FIELD

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Abstract: In the paper we introduce a general formula describing the time- space distribution of the magnetic pressure in a conducting tubular workpiece placed in external magnetic field having the character of a damped sinusoid. For the case of a circular workpiece we determine the transient electromagnetic field using the solution of Bessel equation in cylindrical co-ordinates, and also applying the integral Lapalce transform. Next we determine the time-space distributions and instantaneous magnetic pressure inside the workpiece. The solution we obtained is illustrated by relevant graphs for different values of the parameter considering the external dimension and the depth of diffusion of the electromagnetic field.

Keywords: Electromagnetic forming; Magnetic pressure; Magnetic field

1. Introduction

Magnetic field of a character of a damped sinusoid is used in metal forming and consists of applying energy of impulse magnetic field to the process. The impulse of the field is obtained due to the flow of impulse current, generated by a high-current impulse generator, through an operating head (a solenoid or flat bobbin) [1-8].

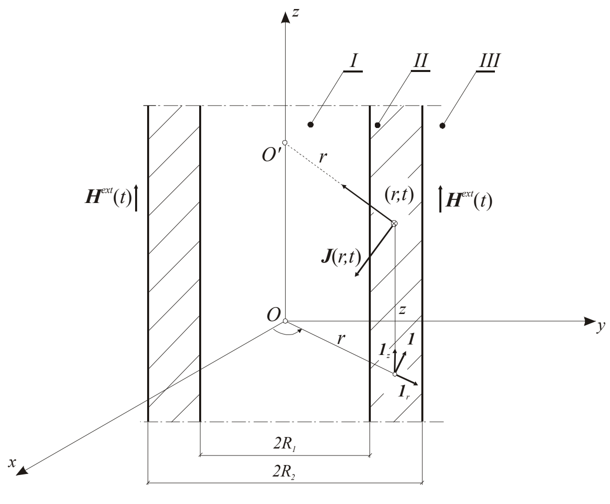

Fig.1. The tubular workpiece in the pulse magnetic field

In the conducting tubular workpiece (Fig. 1) the impulse magnetic field is external in relation to the workpiece and has one component along the Oz axis and it is determined by the following formula

$$\boldsymbol{H}^{ext}(t) = \mathbf{1}_z H_z^{ext}(t)\,, \tag{1}$$

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where the component of the magnetic field strength along the *Oz* axis

$$H_z^{ext}(t) = H_0 \, e^{-\eta t} \sin \omega t \, \mathbf{1}(t) \,, \tag{1a}$$

where: H_0 - magnetic field amplitude when, attenuation in A·m^{-1} is non-existent, ω - pulsation of proper oscillation of the system element being formed-operating head- capacitor bank in rad·s^{-1}, η - magnetic field attenuation coefficient in s^{-1}, $\mathbf{1}(t)$ - Headviside's unit step.

Typical average values of the above quantities appearing in the impulse magnetic field metal forming are: $\xi = 0$, $H_0 = 10^7 \,\mathrm{A \cdot m^{-1}}$, $\eta = 5 \cdot 10^3 \,\mathrm{s^{-1}}$, $\omega = \pi \cdot 10^4 \,\mathrm{rad \cdot s^{-1}}$ [5].

In the impulse magnetic field metal forming the basic problem consists of determining the space-time distribution of the pressure in the element being formed.

2. Magnetic pressure acting on the tubular workpiece

Magnetic pressure is determined after the successful calculation of the volume density of ponder-motor forces $\boldsymbol{f}(X,t)$ [1-3] and [9], the space-time distribution of current density $\boldsymbol{J}(X,t)$ and of the magnetic field strength $\boldsymbol{H}(X,t)$ [1-3] and [10], where t is the time while X is a point inside the workpiece in cylindrical co-ordinate system $X = X(r, \Theta, z)$. Magnetic pressure can be determined analytically [1-3] and [11-12] or numerically [3] and [13]. Often [1-3] and [11-12] the so-called 'universal formula' is given

$$p(t) = \frac{\mu_0}{2}\left[H_1^2 - H_2^2\right]. \tag{2}$$

where $H_1 = H_z(r = R_2, t)$ and $H_2 = H_z(r = R_1, t)$ are the magnetic field intensities on the workpiece surface on the field input side and on the opposite side.

The formula (2) expresses the instantaneous pressure in terms of instantaneous values of $H_z(r,t)$ and it corresponds to any law of magnetic field variation in time and space. However, it does not allow the determination of the magnetic pressure distribution and of the volume density of ponder-motor force distribution with respect to the workpiece thickness.

However, the space-time distribution of the magnetic pressure can also be expressed through the magnetic field intensity. In the case of a tubular workpiece (Fig. 1) and of the external magnetic field given by Eq. (1) the magnetic field in the area $R_1 \le r \le R_2$ has one component along the *Oz* axis, which depends only on the variable r of the cylindrical co-ordinate system, i.e.

$$\boldsymbol{H}(r,t) = H_z(r,t)\,\mathbf{1}_z \,. \tag{3}$$

Then the current density

$$\boldsymbol{J}(r,t) = \mathrm{curl}\,\boldsymbol{H}(r,t) = -\frac{\mathrm{d}\,H_z(r,t)}{\mathrm{d}\,r}\mathbf{1}_\Theta = J_\Theta(r,t)\,\mathbf{1}_\Theta \tag{4}$$

and the volume density of ponder-motor forces

$$\boldsymbol{f}(x,t) = \boldsymbol{J}(r,t) \times \mu_0 \,\boldsymbol{H}(r,t) = f_r(r,t)\;\mathbf{1}_r \,, \tag{5}$$

where

$$f_r(r,t) = -\mu_0 \, H_z(r,t)\frac{\mathrm{d}\,H_z(r,t)}{\mathrm{d}\,r} \,. \tag{5a}$$

The magnetic pressure (directed oppositely to the vector $\mathbf{1}_r$) is determined through the ratio of the force $F_r(r,t)$ and the surface S_r perpendicular to the *Or* axis, i.e.

$$p_r(r,t) = -\frac{F_r(r,t)}{S_r} = -\frac{\iiint\limits_V f_r(r,t)\,\mathrm{d}V}{\iint\limits_{S_r} \mathrm{d}S_r} = -\frac{\iiint\limits_V f_r(r,t)\,r\,\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}z}{\iint\limits_{S_x} r\,\mathrm{d}\Theta\,\mathrm{d}z} = -\frac{1}{r}\int\limits_r^{R_2} f_r(r,t)\,r\,\mathrm{d}r \,, \qquad (6)$$

where V – is the volume and R_2 - is the external radius of the tubular workpiece.
After the substitution of the formula (5a) into formula (6) we obtain

$$p_r(r,t) = \frac{\mu_0}{r}\int\limits_r^{R_2} H_z(r,t)\,\frac{\mathrm{d}\,H_z(r,t)}{\mathrm{d}\,r}\,r\,\mathrm{d}r = \frac{\mu_0}{2}\left[H_z^2(r=R_2,t) - H_z^2(r,t)\right]. \qquad (6a)$$

The formula (6a) defines the space-time distribution of the magnetic pressure through the space-time distribution of the magnetic field inside the tubular workpiece.

At time $t = t_0$ the highest value of the magnetic pressure appears on the inside surface of the tubular workpiece, i.e., for $r = R_1$ and it is

$$\begin{aligned} p_{r\max}(t_0) &= p_r(r=R_1,t_0) = -\frac{1}{R_1}\int\limits_{R_1}^{R_2} f_r(r,t)\,r\,\mathrm{d}r = \\ &= \frac{\mu_0}{2}\left[H_z^2(r=R_2,t_0) - H_z^2(r=R_1,t_0)\right] = \frac{\mu_0}{2}\left[H_1^2(t_0) - H_2^2(t_0)\right] \end{aligned} \,. \qquad (6b)$$

As the illustration of the formula (6a) we will discuss the space-time distribution of the magnetic pressure for a circular workpiece in the following chapters, as then the structure of the formulas describing the magnetic filed is relatively uncomplicated.

3. Magnetic field inside the circular workpiece

In the case of an infinitely long conducting circular workpiece in external longitudinal magnetic field (Fig.2) the values describing the electromagnetic field as for the symmetry of the system depend only on the r co-ordinate of the cylindrical co-ordinate system. Then, we deal here with a one-dimensional question with a constant magnetic permittivity of the cylinder $\mu = \mu_0$ and its constant conductivity γ.

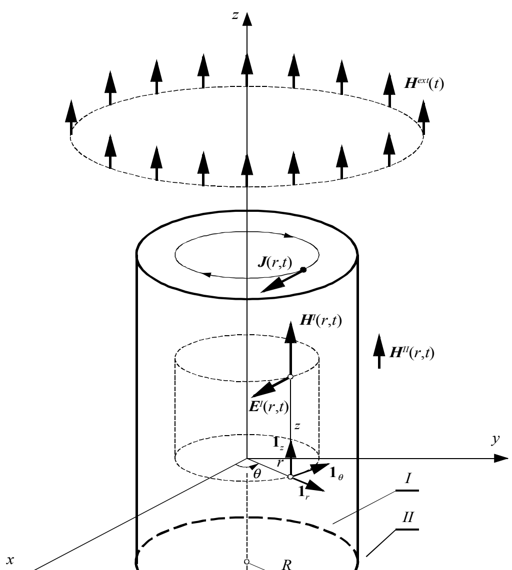

Fig. 2. Conducting circular workpiece in external longitudinal uniform magnetic field of a character of a damped sinusoid.

As the field $\boldsymbol{H}^{ext}(t)$ has got only one component along the *Oz* axis, form the second Maxwell's equation $\operatorname{curl}\boldsymbol{E}^{ext}(r,t) = -\mu\dfrac{\partial \boldsymbol{H}^{ext}(t)}{\partial t}$, the electric field strength has also got one component along the axis *Θ* , i.e. $\boldsymbol{E}^{ext}(r,t) = -\mathbf{1}_{\Theta}E_{\Theta}^{ext}(r,t)$. So we have to deal with a question of the cylindrical wave cast on the lateral surface of the conducting cylinder. In the general case of a conductor of a chosen kind placed in alternating electromagnetic field some currents are bound to appear, as the total electric field cannot equal zero everywhere in the whole conductor. Those currents are called Foucault currents and are determined by the current density vector $\boldsymbol{J}(r,t)$ - Fig.2. These currents generate the so-called return interaction magnetic field $\boldsymbol{H}^{ia}(r,t)$, which, in the system we are considering, has got one component along the *z* axis, thus $\boldsymbol{H}^{ia}(r,t) = \mathbf{1}_z H_z^{ia}(r,t)$. In papers [10] and [14] it has been shown that this field equals zero. The zero value of the return interaction magnetic field in $r > R$ area results form the fact that the lines of the density of current $\underline{\boldsymbol{J}}(r)$ induced in the tubular charge are concentric circles of *Oz* axis– Fig.2. Then they do not generate any magnetic field outside the tube, as it is also the case with the current in the infinitely long solenoid. Then the total magnetic field in the considered area

$$\boldsymbol{H}^{II}(t) = \mathbf{1}_z H_z^{II}(t) = \mathbf{1}_z H_z^{ext}(t) = \mathbf{1}_z \operatorname{Im}\{\underline{H}_z^{ext}(t)\}\,, \tag{7}$$

where

$$\underline{H}_z^{ext}(t) = \underline{H}_0\, \mathrm{e}^{-\eta t}\, \mathrm{e}^{\mathrm{j}\omega t}\, \mathbf{1}(t)\,, \tag{7a}$$

where the complex amplitude of the external magnetic field

$$\underline{H}_0 = H_0. \tag{7b}$$

The required magnetic field strength $H_z^I(r,t)$ in the area of *I* ($0 \le r \le R$) is written as $H_z^I(r,t) = \operatorname{Im}\{\underline{H}_z^I(r,t)\}$, where $\underline{H}_z^I(r,t)$ is the complex magnetic field function of real variables *r* and *t* . This function fulfils the scalar wave equation in cylindrical co-ordinates

$$\frac{\partial^2 \underline{H}_z^I(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial \underline{H}_z^I(r,t)}{\partial r} - \mu\gamma\frac{\partial \underline{H}_z^I(r,t)}{\partial t} = 0\,. \tag{8}$$

For *r = R* it has to be the case of the continuity of the magnetic field strength, i.e. we have the following boundary condition for the complex values:

$$\underline{H}_z^I(R,t) = \underline{H}_z^{\mathrm{zew}}(t)\,. \tag{8a}$$

Moreover we assume a zero initial condition, i.e. for $t = 0$

$$\underline{H}_z^I(r,0) = 0\,. \tag{8b}$$

We solve Eq. (8) with the boundary condition (3a) applying Laplace's integral transform. In order to do this let us denote by $\overline{H}_z^I(r,s)$ the Laplace's transform complex function $\underline{H}_z^I(R,t)$ in relation to variable *t* , and then we perform the Laplace's transformation of the following terms of the differential Eq. (8), taking into account the zero initial condition $\underline{H}_z^I(r,0) = 0$. Thus, we obtain the following equation [10] and [14]:

$$\frac{\partial^2 \overline{H}_z^I(r,s)}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{H}_z^I(r,s)}{\partial r} - s\mu\gamma\, \overline{H}_z^I(r,s) = 0 \tag{9}$$

with the boundary condition

$$\overline{H}^{I}_{z}(R,s)=\underline{H}_0\frac{1}{s-s_0}, \tag{9a}$$

where

$$s_0=-\eta+\mathrm{j}\,\omega. \tag{9b}$$

Eq. (9) is the Bessel equation of zero order of variable *r*, whose solution is the function

$$\overline{H}^{I}_{z}(r,s)=\underline{H}_0\frac{I_0(\sqrt{s}\sqrt{\mu\gamma}r)}{(s-s_0)\,I_0(\sqrt{s}\sqrt{\mu\gamma}R)}, \tag{10}$$

where $I_0(\sqrt{s}\sqrt{\mu\gamma}r)$ is the modified Bessel function of the complex variable $\sqrt{s}\sqrt{\mu\gamma}r$ of first kind and zero order. The function zeros of the denominator in Eq. (10) are $s=s_0$ and

$$s=s_k=-\sigma_k=-\frac{x_k^2}{\mu\,\gamma\,R^2}<0, \tag{10a}$$

where

$$x_k\cong\varphi_k+\frac{1}{8\varphi_k}-\frac{124}{3(8\varphi_k)^3}+\frac{120928}{15(8\varphi_k)^5}-\dots, \tag{10b}$$

where

$$\varphi_k=(k-\frac{1}{4})\pi,\quad (k=1,2,3,\dots). \tag{10c}$$

Then to calculate the original $\underline{H}^{I}_{z}(r,t)$ of the operational function $\overline{H}^{I}_{z}(r,s)$ we use the distribution theorem, obtaining [10] and [14]

$$\underline{H}^{I}_{z}(r,t)=\left[\underline{H}^{I}_{z,0}(r,t)+\sum_{k=1}^{\infty}\underline{H}^{I}_{z,k}(r,t)\right]\mathbf{1}(t), \tag{11}$$

where $\underline{H}^{I}_{z,0}(r,t)$ is the original of Eq. (5) in the pole $s=s_0$ (k = 0), while $\underline{H}^{I}_{z,k}(r,t)$ is the original of this function in the pole $s=s_k$ (k = 1, 2, 3,...). These originals are given by the following formulas:

$$\underline{H}^{I}_{z,0}(r,t)=\underline{H}_0\frac{I_0(\underline{\Gamma}\,r)}{I_0(\underline{\Gamma}\,R)}\,\mathrm{e}^{-\eta t}\,\mathrm{e}^{\mathrm{j}\omega t}, \tag{11a}$$

and

$$\underline{H}^{I}_{z,k}(r,t)=\underline{H}_0\frac{I_0(-\mathrm{j}x_k\frac{r}{R})}{\underline{A}_k(x_k)\,I_1(-\mathrm{j}x_k)}\exp[-\frac{x_k^2}{\mu\,\gamma\,R^2}t], \tag{11b}$$

where $I_1(-\mathrm{j}\,x_k)$ is the modified Bessel function of first kind and first order.

The complex propagation constant

$$\underline{\Gamma}=\sqrt{-\eta\,\mu\,\gamma+\mathrm{j}\,\omega\,\mu\,\gamma}=\Gamma\,\mathrm{e}^{\mathrm{j}\,\varphi}, \tag{12}$$

whose module

$$\Gamma=\sqrt{\omega\mu\,\gamma\sqrt{1+(\frac{\eta}{\omega})^2}}=\kappa\,k \tag{12a}$$

and the argument

$$\varphi = \frac{1}{2}\,\mathrm{arc\ tg}(-\frac{\omega}{\eta}) = \frac{\pi}{4} + \frac{1}{2}\,\mathrm{arc\ tg}\,\frac{\eta}{\omega}\,, \tag{12b}$$

where

$$\kappa = \sqrt{2\ \sqrt{1+(\frac{\eta}{\omega})^2}} \tag{12c}$$

and the coefficient

$$k = \sqrt{\frac{\omega\,\mu\,\gamma}{2}}\,, \tag{12d}$$

whose inverse is the depth of the diffusion of the wave inside the well conducting medium and it is

$$\delta = \frac{1}{k} = \sqrt{\frac{2}{\omega\,\mu\,\gamma}}\,. \tag{12e}$$

The complex propagation constant can also be written as

$$\underline{\varGamma} = k\,\sqrt{2\,(\mathrm{j} - \frac{\eta}{\omega})} = \underline{\kappa}\,k\,, \tag{12f}$$

where

$$\underline{\kappa} = \sqrt{2\,(\mathrm{j} - \frac{\eta}{\omega})} = \kappa\,\mathrm{e}^{\mathrm{j}\varphi}\,. \tag{12f}$$

Then the complex constant

$$\underline{A}_k\,(x_k) = \frac{1}{2x_k}[2k^2R^2 + \mathrm{j}\,(2k^2R^2\,\frac{\eta}{\omega}\,\text{-}x_k^2)] = A_k\,(x_k)\exp[\mathrm{j}\,\alpha_k\,(x_k)]\,, \tag{13}$$

where its module

$$A_k(x_k) = \frac{1}{2x_k}\sqrt{(2k^2R^2)^2 + (2k^2R^2\,\frac{\eta}{\omega} - x_k^2)^2} \tag{13a}$$

and its argument

$$\alpha_k\,(x_k) = \mathrm{arc\ tg}\,\frac{2k^2R^2\,\frac{\eta}{\omega} - x_k^2}{2k^2R^2}\,. \tag{13b}$$

The exponential form of the Bessel function appearing in formulas (6a) and (6b)

$$\left.\begin{aligned}
&I_0(\underline{\varGamma}\,r) = M_0(\underline{\varGamma}\,r)\exp[\mathrm{j}\,\beta_0(\underline{\varGamma}\,r)],\\
&I_0(\underline{\varGamma}\,R) = M_0(\underline{\varGamma}\,R)\exp[\mathrm{j}\,\beta_0(\underline{\varGamma}\,R)],\\
&I_0(\text{-j}\,x_k\,\frac{r}{R}) = M_{0,k}(\text{-j}\,x_k\,\frac{r}{R})\exp[\mathrm{j}\,\beta_{0,k}(\text{-j}\,x_k\,\frac{r}{R})],\\
&I_1(\text{-j}\,x_k) = M_{1,k}(\text{-j}\,x_k)\exp[\mathrm{j}\,\beta_{1,k}(\text{-j}\,x_k)].
\end{aligned}\right\} \tag{14}$$

lets us write the Eqs. (11a) and (11b) in real forms, i.e. as real functions of variable r of the cylindrical co-ordinate system and of time t. We obtain respectively

$$H^{I}_{z,0}(r,t) = H_0\,\frac{M_0(\underline{\varGamma}\,r)}{M_0(\underline{\varGamma}\,R)}\,\mathrm{e}^{-\eta t}\sin[\omega\,t + \beta_0(\underline{\varGamma}\,r) - \beta_0(\underline{\varGamma}\,R)]\,, \tag{15}$$

and

$$H^{I}_{z,k}(r,t) = H_0 \frac{M_0(\text{-j}\, x_k \frac{r}{R})}{A_k(x_k)\, M_1(\text{-j}\, x_k)} \exp[-\frac{x_k^2}{\mu\,\gamma\,R^2} t] \sin[\beta_{0,k}(\text{-j}\, x_k \frac{r}{R}) - \beta_{1,k}(\text{-j}\, x_k) - \alpha_k(x_k)] \tag{15a}$$

Finally the magnetic field strength in a conducting cylinder placed in external longitudinal magnetic field of a character of a damped sinusoid has the following form

$$H^{I}_{z}(r,t) = \left[H^{I}_{z,0}(r,t) + \sum_{k=1}^{\infty} H^{I}_{z,k}(r,t) \right] \mathbf{1}(t)\,. \tag{15b}$$

It is convenient to perform the analysis of the electromagnetic field in relative units. That is why we introduce the variable x corresponding to the variable r of the cylindrical co-ordinate system, as

$$x = \frac{r}{R}, \qquad 0 \le x \le 1\,. \tag{16}$$

The frequency of the sinusoidal external magnetic field and the conductivity of the charge with regard to its external radius are taken into account through the coefficient $\alpha = \frac{R}{\delta} = k\,R$. Thus, we have: $\underline{\Gamma}\, r = \underline{\kappa}\, k\, r = \underline{\kappa}\, \alpha\, x$, $\quad \underline{\Gamma}\, R = \underline{\kappa}\, k\, R = \underline{\kappa}\, \alpha$.

The magnetic field is then described by the following formulas:

$$H^{I}_{z}(x,t) = \left[H^{I}_{z,0}(x,t) + \sum_{k=1}^{\infty} H^{I}_{z,k}(x,t) \right] \mathbf{1}(t)\,. \tag{17}$$

where

$$H^{I}_{z,0}(x,t) = H_0 \frac{M_0(\underline{\kappa}\, \alpha\, x)}{M_0(\underline{\kappa}\, \alpha)} \mathrm{e}^{-\eta t} \sin[\omega\, t + \beta_0(\underline{\kappa}\, \alpha\, x) - \beta_0(\underline{\kappa}\, \alpha)]\,, \tag{17a}$$

and

$$H^{I}_{z,k}(x,t) = H_0 \frac{M_0(\text{-j}\, x_k x)}{A_k(x_k)\, M_1(\text{-j}\, x_k)} \exp[-\frac{\varpi\, x_k^2}{2\,\alpha^2} t] \sin[\beta_{0,k}(\text{-j}\, x_k x) - \beta_{1,k}(\text{-j}\, x_k) - \alpha_k(x_k)] \tag{17b}$$

4. Magnetic pressure inside the circular workpiece

Eq. (6) used for the case of the circular workpiece for the variable r has the following form:

$$p_r(r,t) = \frac{\mu_0}{r} \int_r^R H^{I}_{z}(r,t) \frac{\mathrm{d}\, H^{I}_{z}(r,t)}{\mathrm{d}\, r}\, r\, \mathrm{d}r = \frac{\mu_0}{2} \left\{ \left[H^{I}_{z}(r=R,t) \right]^2 - \left[H^{I}_{z}(r,t) \right]^2 \right\} \tag{18}$$

or for the variable x

$$p_x(x,t) = \frac{\mu_0}{x} \int_x^1 H^{I}_{z}(x,t) \frac{\mathrm{d}\, H^{I}_{z}(x,t)}{\mathrm{d}\, x}\, x\, \mathrm{d}x = \frac{\mu_0}{2} \left\{ \left[H^{I}_{z}(x=1,t) \right]^2 - \left[H^{I}_{z}(x,t) \right]^2 \right\}, \tag{18a}$$

where

$$H^{I}_{z}(1,t) = \left[H^{I}_{z,0}(1,t) + \sum_{k=1}^{\infty} H^{I}_{z,k}(1,t) \right] \mathbf{1}(t)\,, \tag{18b}$$

where

$$H^{I}_{z,0}(1,t) = H_0\, \mathrm{e}^{-\eta t} \sin \omega t\,, \tag{18c}$$

and

$$H^{I}_{z,k}(1,t) = H_0 \frac{M_0(\text{-j}\,x_k)}{A_k(x_k)\,M_1(\text{-j}\,x_k)} \exp[-\frac{\omega\,x_k^2}{2\,\alpha^2}\,t] \sin[\beta_{0,k}(\text{-j}\,x_k) - \beta_{1,k}(\text{-j}\,x_k) - \alpha_k(x_k)] \,. \qquad (18d)$$

The time-space distribution of the magnetic pressure is shown in Fig.3. This graph has been worked out for relative values, that is to say in relation to $\frac{\mu_0\,H_0^2}{2}$, i.e. as the coefficient $p_1 = \frac{2}{\mu_0\,H_0^2}\,p_x(x,t)$.

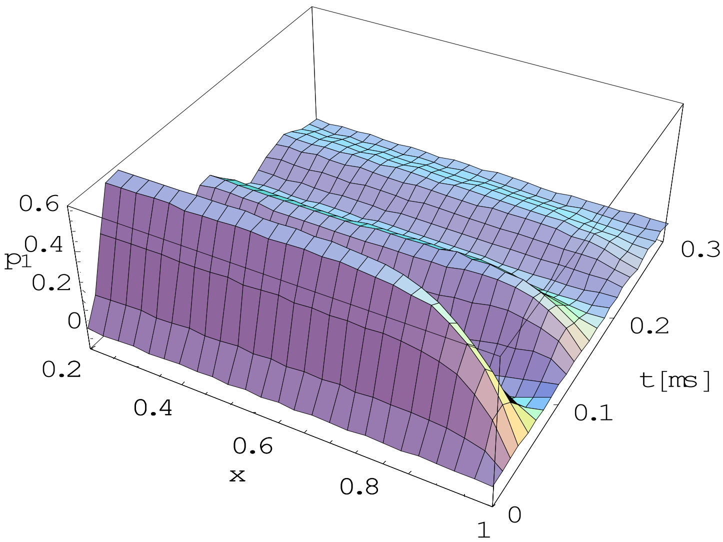

Fig. 3. Time-space distribution of the magnetic pressure; $\alpha = 5$, $\omega = \pi\cdot 10^4$ rad·s^{-1}, $\eta = 5\cdot 10^3$ s^{-1}, $\gamma = 58\cdot 10^6$ S·m^{-1}

The curve of the instantaneous magnetic pressure for different values of the variable x is shown in Fig. 4.

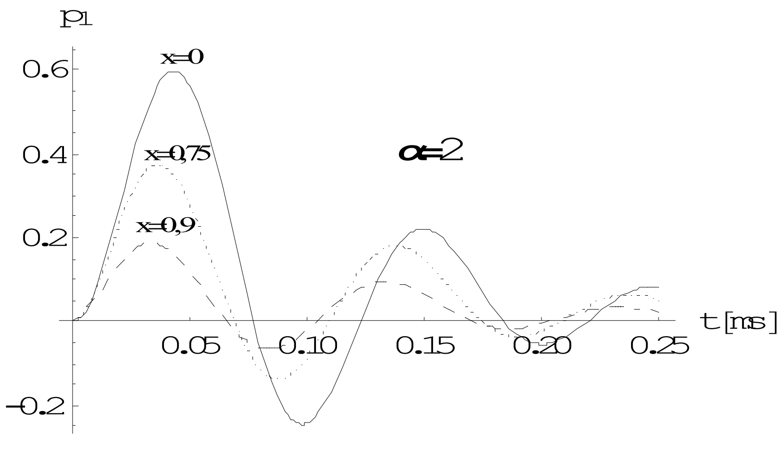

Fig.4. Instantaneous magnetic pressure; $\alpha = 5$, $\omega = \pi\cdot 10^4$ rad·s^{-1}, $\eta = 5\cdot 10^3$ s^{-1}, $\gamma = 58\cdot 10^6$ S·m^{-1}

The influence of the parameter α on the maximum (for x = 0) instantaneous magnetic pressure is shown in Fig. 5.

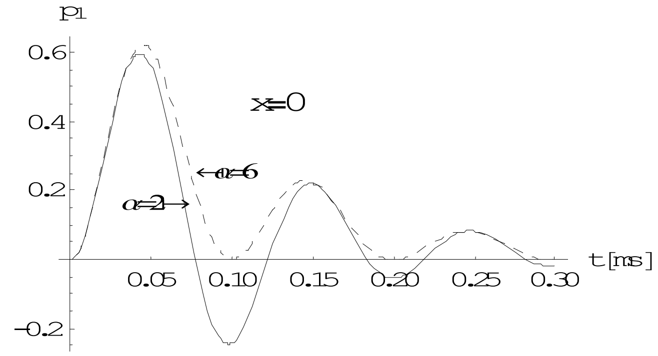

Fig.5. Maximal instantaneous magnetic pressure; $\omega = \pi \cdot 10^4$ rad·s^{-1}, $\eta = 5 \cdot 10^3$ s^{-1}, $\gamma = 58 \cdot 10^6$ S·m^{-1}

The influence of the parameter α on the distribution of the magnetic pressure a circular workpiece is shown in Fig. 6 at $t = T/4$, i.e. for the instantaneous value of the external magnetic field equal to its amplitude ($\xi = 0$). This graph has been worked out for relative values, , that is to say in relation to $\frac{\mu_0 H_0^2}{2}$, i.e. as the coefficient $p_2 = \frac{2}{\mu_0 H_0^2} p_x(x, t = T/4)$.

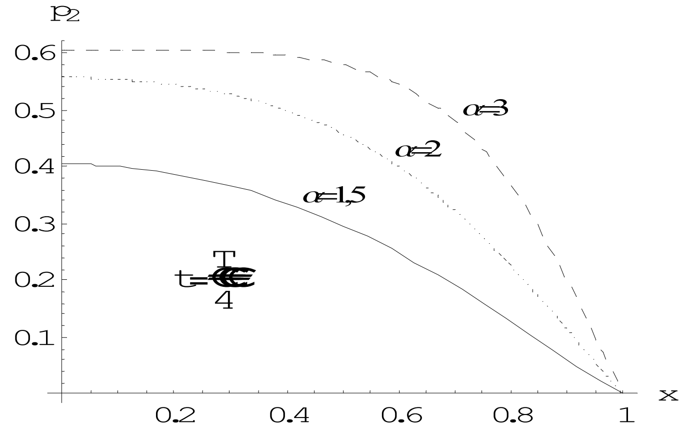

Fig.6. Distribution of the magnetic pressure a circular workpiece in external uniform magnetic field of a character of a damped sinusoid at $t = T/4$, $\omega = \pi \cdot 10^4$ rad·s^{-1}, $\eta = 5 \cdot 10^3$ s^{-1}, $\gamma = 58 \cdot 10^6$ S·m^{-1}

5. Conclusions

The so-called "universal" equation (2) defines in fact the instantaneous value of the magnetic pressure only on the inside surface of the tubular workpiece or the value of the pressure along its axis – curve *a* in Fig. 4 and Fig. 5. At chosen time $t = t_0$ this pressure is higher, as one could expect, from the pressures in other points of the workpiece – Fig. 3 and Fig. 4. The time-space distribution of the magnetic field should then be determined in the case of the tubular workpiece from the formula (6a), and from the formula (18) (or (18a)) for the case of a cylindrical workpiece. This distribution is expressed only in terms of the time-space distribution of the magnetic field inside the workpiece.

Important influence on the distribution of the pressure inside the workpiece is connected with the parameter α referring to the depth of the diffusion of the magnetic field into the workpiece – Fig. 5 and Fig. 6. The higher it is the larger is the area of the workpiece where the magnetic pressure is constant at time $t = t_0$ and equals the maximum pressure (for *x* = 0).

In some intervals of time Δt the pressure has negative values – Fig .4 and Fig. 5. It is due to the fact that eddy currents induced inside the workpiece are concentric circular loops and they form 'turns' with currents of the same sense. Then according to the Laplace's law electrodynamic interaction takes place between the 'turns', and it superimposes onto the basic interaction eddy currents-external magnetic field. The phenomenon of negative pressure appearing inside the workpiece diminishes with the increase of the parameter α (fig.5), so also with the increase of the ratio of the internal dimension of the workpiece and of the depth of diffusion of the electromagnetic field, then with the increase of the frequency and conductivity inside the workpiece as well.

From formulas (5) and (6a) and the pressure directed according to the sense of the vector $\mathbf{1}_r$ we obtain:

$$f_r(r,t) = -\mu_0 \, H_z(r,t) \frac{\mathrm{d}\, H_z(r,t)}{\mathrm{d}\, r} = \frac{\mathrm{d}\, p_r(r,t)}{\mathrm{d}\, r}, \tag{19}$$

That means that the time-space distribution of the volume density of ponder-motor forces can be determined by differentiation of the magnetic pressure.

If the current density is also determined according to the formula (3), then from the Umov-Poynting theorem

$$\boldsymbol{\Pi} = \boldsymbol{J}(r,t) \times \boldsymbol{H}(r,t) \tag{20}$$

we can determine the power diffused into the circular workpiece [15].

The determination of the magnetic pressure for the case of a tubular workpiece is similar to the case of a circular workpiece, but the pressure is then described by formulas having a more complicated structure.

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