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THE NEW METHOD FOR ANALYSING OF DYNAMIC CHARACTERISTICS OF A NON-LINEAR ACTUTOR OF A MECHATRONIC SYSTEM

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Abstract: *The paper contains the crucial features of the research programme for building complex mathematical tool that enables theoretical investigation of analysing of dynamic characteristics of a non-linear actuator of a mechatronic system. We suppose that the actuator with the non-hysteresis and non-linear magnetic circuit is consisted of a magnetic, mechanic and electric part – feeding circuit. This electromechanical system can be described by a canonic system of non-linear differential equations of order 1 by means of state variables method. Next, permeability of the core lamination is influenced by the non-linearity of the magnetic circuit. Therefore we have to consider the magnetizing curve of a magnetic circuit to count the permeability and variable inductance and magnetic force. This given system of equations can be solved numerically and for several basic cases (harmonic or DC feeding circuit) it is possible to use available professional numerical program packages for solution of the differential equations (Matlab, Mathematica, etc.). According to our experience it is necessary to develop a special user program. The output data are time dependences of current i (electric state variable), of velocity of deflection v and of size of deflection x (mechanic state variables). In this article there is created a mathematical model and algorithm for numerical solution of the above described problems.*

Key words: *mechatronic system, actuator with the non-hysteresis and non-linear magnetic circuit, dynamic characteristics, state variables method.*

INTRODUCTION

Robotics belongs to one of the most developing present industrial branches and it occupies itself with the research in the area of building of intelligent movable robots. The majority of produced robots uses an actuator to realize its motoric activities. In technical practice there are a lot of different types of actuators. Some robots employ electric motors, another robots prefer solenoids (electromagnetic circuits) or hydraulic or pneumatic system (system driven by compressed gases). In some cases robots can use all these types of actuators together. An actuator (without consideration of its type) secures robot's mobility – for example – by switching its electromagnetic circuits or by the motions of its robotic arms including control of its end effectors.

1 ELECTROMECHANICAL CIRCUIT

Fig. 1 shows a simplified electromechanical circuit of an actuator, which is consisted of three main parts - an electric part (feeding circuit), a magnetic part (fixed magnetic circuit with a movable anchor), and a mechanic part (two springs – rigidity K_1, K_2 and damping B_1, B_2).

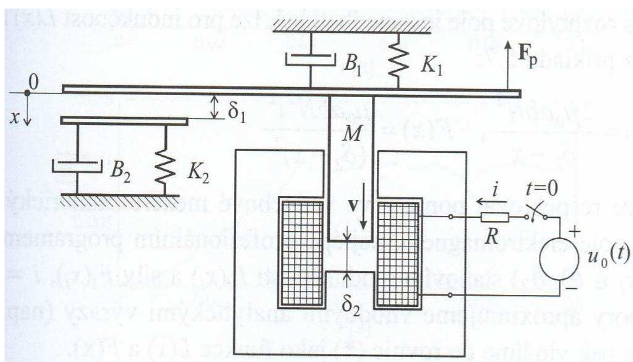

Fig. 1: Electromechanical circuit

2 MATHEMATIC MODEL

Mathematic model of this mechatronic system is given by a canonic system of non-linear differential equations of order 1 for the calculation of dynamic characteristics (time dependences of current *i* (electric state variable), of velocity of deflection *v* and of size of deflection *x* (mechanic state variables)) – see (1). Note: This system of equations is non-linear, because first equation of these three given equations is non-linear.

$$\begin{aligned}\frac{di}{dt} &= -\frac{R}{L(x)}i - \frac{1}{L(x)}\frac{dL(x)}{dx}vi + \frac{1}{L(x)}u_0 \\ \frac{dv}{dt} &= -\frac{B_1}{M}v - \frac{K_1}{M}x - c(x)\frac{B_2}{M}v - \\ &\quad - c(x)\frac{K_2}{M}(x-\delta_1) - p(x)\frac{F_P}{M} + \frac{F(x,i)}{M} \\ \frac{dx}{dt} &= v\end{aligned} \tag{1}$$

For constants *c(x)* and *p(x)* applies:

$$\begin{aligned}&c = 0 \text{ for } x \in (0;\delta_1) \\ &c = 1 \text{ for } x \in (\delta_1;\delta_2) \\ &p = 0 \text{ for } x \le x_0 \ (\text{"dead travel"}) \\ &p = 1 \text{ for } x \succ x_0\end{aligned} \tag{2}$$

3 FIELD CHARACTERISTIC OF A MAGNETIC CIRCUIT OF AN ACTUATOR

This new method for analyzing dynamic characteristics of an actuator considers the influence of non-linearity of a magnetic circuit, respectively influence of iron saturation, and so it can lead to the improvement of the gained results. Therefore we use a field characteristic of a magnetic circuit – see Fig. 2. Blue marked points are gained from the field characteristic of the dynamo core lamination. We can approximate these blue points by way of interleaving by the least square method in „Curve Fitting Library“ in Matlab 7.0 and find the red marked curve with the general formula: *B(H)* = *c*.arctg*(d.H)*. Note: This approximation is relatively exact (R^2 = 0,9982), unknown constants *c*,*d* – see Fig. 2.

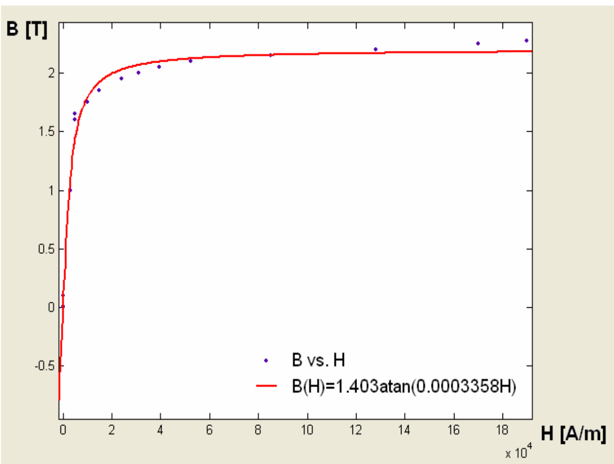

Fig. 2: Field characteristic of the magnetic circuit of the actuator

4 THE CALCULATION OF INDUCTANCE *L(X)* AND MAGNETIC FORCE *F(X,I)*

The main part of the computation of dynamic characteristics is the calculation of inductance *L(x)* and magnetic force *F(x,i)* in individual time instants of the calculating of dynamic characteristics.

By deliberation of constant magnetic permeability of the magnetic circuit (μ_Z = *m* = *const*) we can describe magnetic circuit with equation system (3) in two unknown variables H_V and H_Z:

$$\begin{aligned}&Ni = H_V(\delta_2 - x) + H_Z(\ell + x) \\ &\mu_0 H_V = mH_Z\end{aligned} \tag{3}$$

Note: H_V – magnetic field intensity in the air-gap
H_Z – magnetic field intensity in the core iron
ℓ – mean length of the magnetic circuit
N – number of turns of the coil

Following solution of the unknown variables H_V and H_Z is:

$$\begin{aligned}H_V &= \frac{Ni}{\delta_2 - x + (\ell + x)\frac{\mu_0}{m}} \\ H_Z &= \frac{\mu_0}{m}\frac{Ni}{\delta_2 - x + (\ell + x)\frac{\mu_0}{m}}\end{aligned} \tag{4}$$

The cardinal part of this new method is embodiment of the non-linearity of the magnetic circuit (i.e. influence of saturation), when applies: $B = \mu_Z H_Z$, where $\mu_Z \neq const$. Progress for the calculation of the unknown variables H_V and H_Z will be analogical.

We describe the magnetic circuit with the assistance of equation system (3) and with the formula $B = f(H_Z)$ for the field characteristic (Fig. 2):

$$\begin{aligned}&Ni = H_V(\delta_2 - x) + H_Z(\ell + x) \\ &\mu_0 H_V = B(H_Z) = 1{,}403 arctg(0{,}0003358 H_Z)\end{aligned} \tag{5}$$

After solving of this equation system we obtain a non-linear equation (6) for the unknown variable H_Z in the form:

$$\begin{aligned}Ni = \frac{1{,}403}{\mu_0} arctg(0{,}0003358 H_Z)(\delta_2 - x) + \\ + H_Z(\ell + x)\end{aligned} \tag{6}$$

We define a function $f = f(H_Z)$, which describes our given non-linear equation:

$$\begin{aligned}f(H_Z) = \frac{1{,}403}{\mu_0} arctg(0{,}0003358 H_Z)(\delta_2 - x) + \\ + H_Z(\ell + x) - Ni\end{aligned} \tag{7}$$

Fig. 3 shows the graph of the function (7) in the time instant t = 0,015 s. Another required parameters: N = 400, δ_2 = 0,012 m, ℓ = 0,2 m. We completed the calculation of the dynamic characteristics and in the time instant t =

0,015 s we found the following values of the current i and deflection x: i = -5,518 A, x = 0,001967 m.

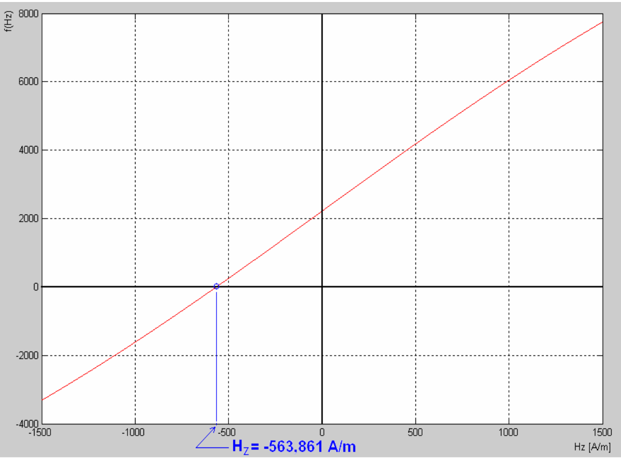

Fig. 3: The graph of the function $f(H_Z)$ for entered values of all used parameters

Function $f(H_Z)$ - see (7) - describing the non-linear equation (6) is made up of the sum of two uniquely invertible functions, i.e. function (7) is uniquely invertible in the whole domain of definition. So, desired solution of the equation (6) will be one and only. To solve this non-linear equation (6) we can use numeric method like Newton's method or simple iterative method. Function (7) is gradually monotonous because of the function 'arctg', that's why the use of Newton's method is unsuitable, because we can achieve the exact result after doing approximately 10^4 iterations. This progress can lengthen the whole calculation. When we use simple iterative method, we can get the exact result after 10 – 20 iterations and the time of the calculation is incomparably shorter.

Next, we use the value of the variable H_Z to calculate the unknown variable H_V according to the formula (8), which is found by solving the equation system (5).

$$H_V = \frac{1,403}{\mu_0} arctg\left(0,0003358 H_Z\right) \tag{8}$$

We need the found value of H_V only for the calculation of the magnetic permeability μ - see (9).

$$\mu = \frac{\mu_0 H_V}{H_Z} \tag{9}$$

Then, we count the total magnetic stream flowing through the magnetic circuit:

$$\Phi_C = N\mu_0 H_V S \text{, where } S = 2ab \tag{10}$$

Note: Parameters $2a$,b – magnetic circuit size

For next calculations contributing to determine the inductance $L(x)$ and magnetic force $F(x)$ we use magnetic field intensity H_V according to the formula (4), which is used for the calculation of H_V provided that μ_{Fe} = *const*.

We make a certain error in calculation, because we presume, that the magnetic permeability μ_{Fe} calculated according to the formula (9) is approximately constant around the certain time instant. But by drawing of the time dependence of H_V according to the formula (4) and (8) we can see, that the graphs are intermeshing – see Fig. 4, i.e. the error caused by this exchange is practically insignificant.

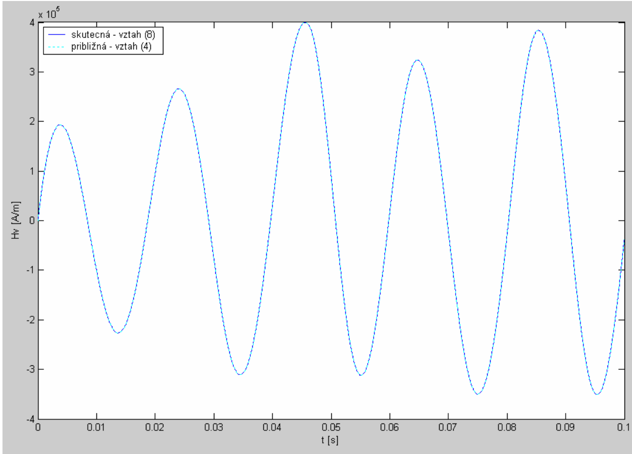

Fig. 4: Comparison of the behaviour of time dependences of H_V according to the formula (4) a (8)

The advantage of this exchange is noticeable, because after this exchange for the calculation of the inductance – see formula (11) – there is no current dependence, but only deflection dependence. This can considerably simplify our used mathematical model for solution of dynamic characteristics.

$$L(x) = \frac{\Phi_C}{i} = \frac{N\mu_0 S}{i} \frac{Ni}{\delta_2 - x + (\ell + x)\dfrac{\mu_0}{\mu}} = \\ = \frac{2\mu_0 N^2 ab\mu}{(\delta_2 - x)\mu + (\ell + x)\mu_0} \tag{11}$$

We can determine the magnetic force $F(x,i)$ by using the energy of the magnetic field:

$$F(x,i) = \frac{1}{2}\frac{dL}{dx} i^2 = \\ = \frac{1}{2} i^2 \frac{2\mu_0 N^2 ab\mu(\mu - \mu_0)}{((\delta_2 - x)\mu + (\ell + x)\mu_0)^2} \tag{12}$$

5 Drawing of Dynamic Characteristics

Final part of the whole calculation includes the drawing of dynamic characteristics for entered input data. Electromechanical circuit is described by the following values of parameters:

- values of electromagnetic circuit: ℓ = 0,2 m, a = 0,04 m, b = 0,04 m, R = 26 Ω, N = 400, $\mu_0 = 4\pi \times 10^{-7}$ H/m.
- values of mechanic circuit:

M = 2 kg, B_1 = 50 Ns/m, B_2 = 1000 Ns/m, K_1 = 1000 N/m, $K_2 = 8.10^6$ N/m, F_P = 60 N, x_0 = 0,002 m, δ_1 = 0,01 m, δ_2 = 0,012 m.

- source voltage
  - a) $u_0 = 200\sin(2\pi ft+\pi/3)$ V, f = 50 Hz – see Fig. 5
  - b) u_0 = 100 V – see Fig. 6

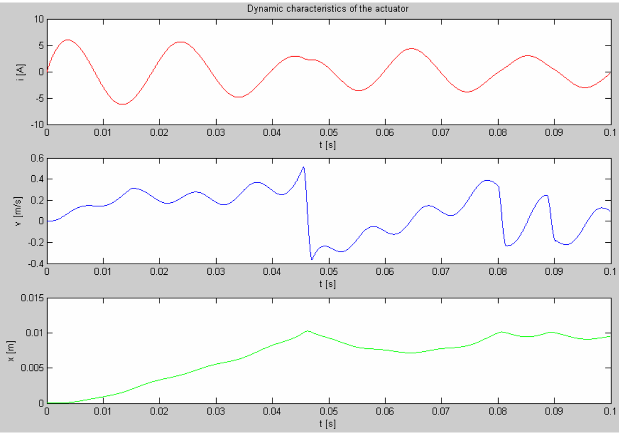

Fig. 5: Dynamic characteristics for harmonic feeding

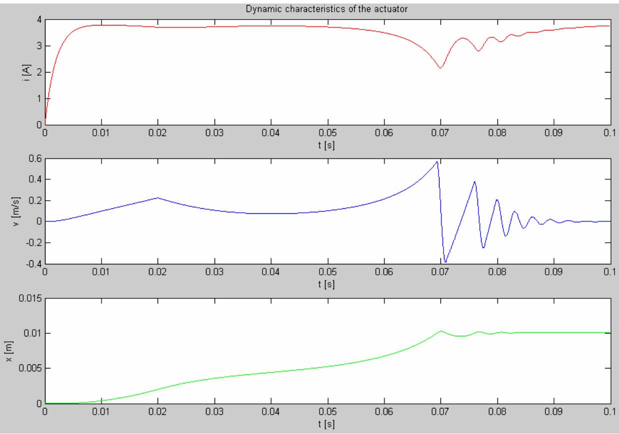

Fig. 6: Dynamic characteristics for DC feeding

6 Conclusion

The aim of this work was to design, PC implement and test in some examples the new method of analysis of a model of mobile component of a robotic system. New method used in this work considers the influence of the non-linearity of a magnetic circuit to specify gained results, but we could more particularize the gained results by deliberation for example of magnetic hysteresis or of leakage fluxes. For these calculations it is necessary to use a proper sophisticated PC program – for example QuickField by using the finite element method.

By designing of a mechatronic electromechanical system it is important to design such a system, whose behaviour will accomplish all given requests, i.e. it is necessary to make a synthesis of this system – for example to determine the values of constants of springs (K_1, K_2, B_1, B_2), in order to minimalize the time of transients and reduce the close time of the magnetic circuit. The most suitable optimalization tool for these calculations is the library „Optimalization“ in Matlab 7.0.

7 References

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