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APLICABILITY OF A NEW METHOD OF TRANSFORMATION OF MATERIAL PARAMETERS FOR BETTER DISCRETISATION OF VERY THIN SKIN DEPTH

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Abstract: *The paper deals with tests of a new suggested method for an easier discretisation of thin layers of electromagnetic field penetrating into a conducting material. First the principle of the new method and its derivation is described. Comparison of computation with and without the transformation follows. The comparison is done on several very simple arrangements of induction heating of a ferromagnetic material. The computation were carried out by the FEM based professional code of FEMLAB supplemented with single purpose scripts written by the author in MATLAB and MATHEMATICA.*

Key words: *transformation of material parameters, thin skin depth, finite element method*

INTRODUCTION

The paper deals with a novel method developed for better discretisation of very thin surface layers in which an electromagnetic field of higher frequency is concentrated due to the skin effect.

The method was developed to solve problems with thin layers using professional codes with no possibility to use an impedance boundary method.

1 DERIVATION OF THE METHODOLOGY

The method is based on a simple idea of modifying the material parameters of the conductive material, (its conductivity and permeability) in such a way that the thickness of the skin layer increases. This generally leads, of course, to differences with respect to the original solution, and the principal requirement is that the selected quantities should remain unchanged. And now there arises the main question which results should be considered the most important so that their variation due to the transformation of a model is as small as possible. The answer to this question should arise from the goal for which the method was developed.

The proposed method was developed for three main purposes. First, it should be usable for calculation of the Joule losses in a very thin skin layer and the result should be applicable for subsequent calculation of the heat transfer. The second goal was the possibility of calculation of inductances from energy of electromagnetic field. The third goal is that one should be able to get a correct distribution of the electromagnetic field everywhere except for the transformed domain. From these goals that the method should achieve, it is possible to deduce specifications that the method should satisfy.

There are two principal requirements on the method. The first one is that the total Joule losses under the surface of the conductive material with very thin skin depth should be unaffected by the transformation. That means that the real part of the Poyting vector should not change on the surface of the volume with transformed material parameters. The second requirement says that the total magnetic field energy under the surface and this way also the imaginary part of the Poyting vector on the surface should not change.

The simplest 1D arrangement (a planar electromagnetic wave impacting an infinitely large and infinitely thick conductive wall) was used for derivation of the transformation. The particular way of the transformation of material parameters was developed to exactly achieve the described specifications for this arrangement.

From the analytical solution of the arrangement it was found that such a transformation of material parameters is very simple and reads

$$\mu_{transf} = k_{transf}\,\mu \tag{1}$$

$$\gamma_{transf} = k_{transf}\,\gamma \tag{2}$$

where μ and μ_{transf} are permeabilities of the transformed material before and after the transformation, respectively. Analogously γ and γ_{transf} are conductivities of the material before and after the transformation. The k_{transf} is a coefficient determining how many times the skin depth after transformation is less or greater than the one before transformation. To achieve a bigger skin depth after transformation, the coefficient k_{transf} must be less than one.

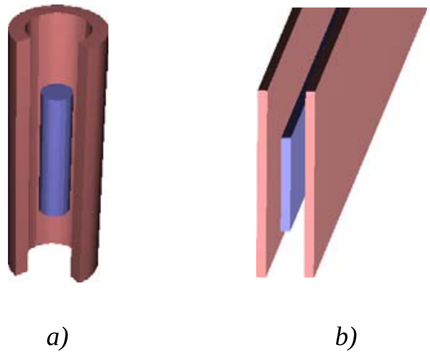
The relation between the skin depths before and after the transformation reads

$$a_{transf} = \frac{1}{k_{transf}}\,a \tag{3}$$

2 CASE EXAMPLES OF APPLICATION OF THE TRANSFORMATION

The transformation was tested on two simple case examples: an induction heating of a ferromagnetic cylinder and an induction heating of a ferromagnetic strip. These arrangements were chosen because they are relatively simple, have the same cross-section, an influence of curvature of the cylindrical arrangements can be rated by comparison of the arrangements and finally also because understanding of behavior of the transformation on these arrangements helps to estimate the behavior on more complex ones.

An arrangement of a ferromagnetic cylinder in a cylindrical inductor is depicted in Fig. 1a and the arrangement of a long ferromagnetic plate between two planar conductors with forward and backward currents in Fig. 1b.



a) b)

Fig.1: a) and b) solved arrangements

Both arrangements can be solved as 2D by the finite element method. It is because the first one is axi-symmetrical and the other one is planar. Moreover, both of them have the same cross-section, so they can be computed on the same mesh. Employing all the symmetries, only one quarter of the geometry is needed to be computed. The scheme of the computed area is in Fig. 2.

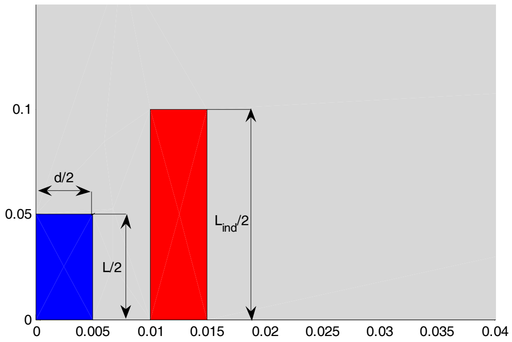


Fig.2: Cross-section of both the axi-symmetrical and planar arrangement (Depicted is the variant with the longer inductor).

Both arrangements were computed in two variants: with an inductor twice longer then the length of the charge and with an inductor much shorter than the length of a charge.

Parameters of the heating are following

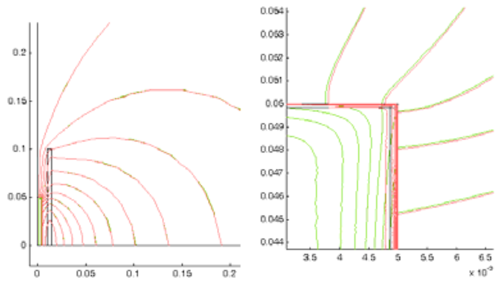
- length of inductor L_{ind} : 0.2 m and 0.03 m
- other geometrical dimensions : are the same as in fig. 2
- coefficient of transformation k_{transf} : 15.4
- skin depth before transformation a : 0.000065 m
- skin depth after transformation a_{transf} : 0.001 m
- specific mass of the strip : 7870 kg/m3
- heat capacity of the strip : 600 J/(kg.K)
- thermal conductivity of the strip : 52 W/(m.K)
- source current * number of turns : 5000 A for planar arrangement and 2500 A for axi-symmetrical arrangement
- ambient temperature : 293 K
- initial temperature : 293 K
- time of heating : 60 s
- emissivity : 0.5
- convective heat transfer coefficient : 3 W/(m2.K)

Electromagnetic field was first computed by both methods that means with and without the transformation.

For the computation without the transformation about 64000 elements were needed, however, for the computation with the transformation, 6400 elements was enough.

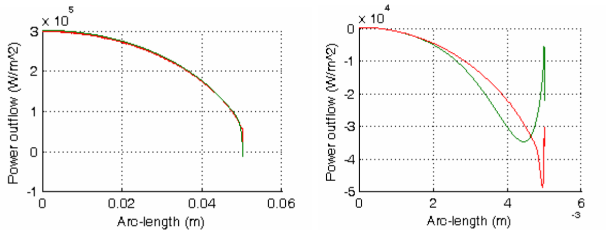
Consequently a heating of the charge by the Joule losses computed from the electromagnetic field was computed. An error after 60s of heating is studied.

Results of planar arrangement L_{ind} = 0.2 m



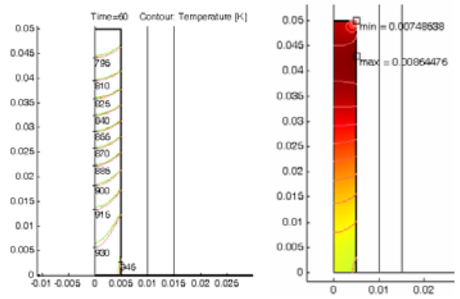
a) *b)*

Fig.3: a) and b) magnetic field distribution (red lines – computation without transformation, green lines – computation with transformation)



a) *b)*

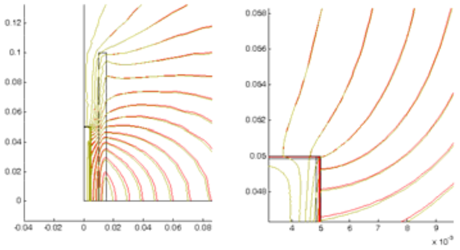
Fig.4: Power flow through the a) vertical and b) horizontal surface of the charge (red lines – computation without transformation, green lines – computation with transformation)



a) *b)*

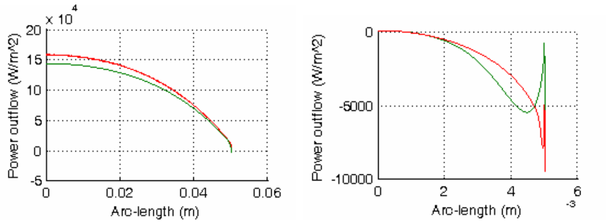
Fig.5: a) Temperature distribution after 60s of heating (red lines – computation without transformation, green lines – computation with transformation)
b) relative error of temperature after 60s of heating (The computation without transformation is taken as the correct value and the difference between the correct final value and the initial value of local temperature is taken as the base for the calculation of relative error)

Results of axi-symmetrical arrangement L_{ind} = 0.2 m



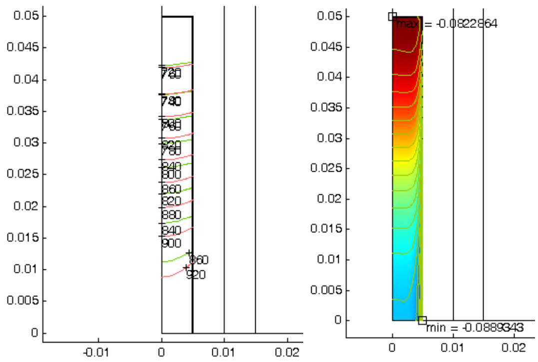
a) *b)*

Fig.6: a) and b) magnetic field distribution (red lines – computation without transformation, green lines – computation with transformation)



a) *b)*

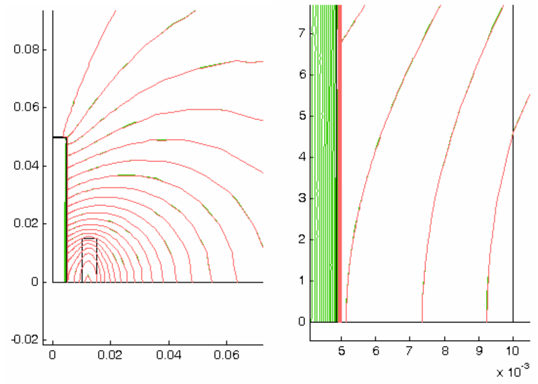
Fig.7: Power flow through the a) vertical and b) horizontal surface of the charge (red lines – computation without transformation, green lines – computation with transformation)



a) *b)*

Fig.8: a) Temperature distribution after 60s of heating (red lines – computation without transformation, green lines – computation with transformation)
b) relative error of temperature after 60s of heating (The computation without transformation is taken as the correct value and the difference between the correct final value and the initial value of local temperature is taken as the base for the calculation of relative error)

Results of planar arrangement L_{ind} = 0.03 m



a) *b)*

Fig.9: a) and b) magnetic field distribution (red lines – computation without transformation, green lines – computation with transformation)

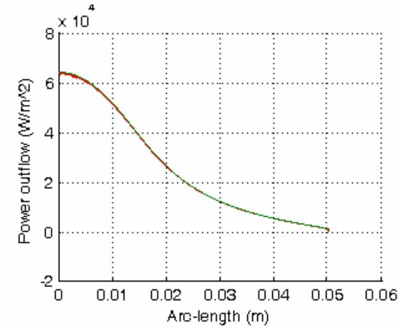
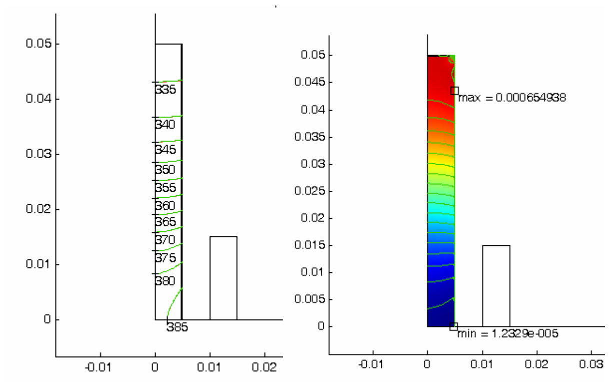


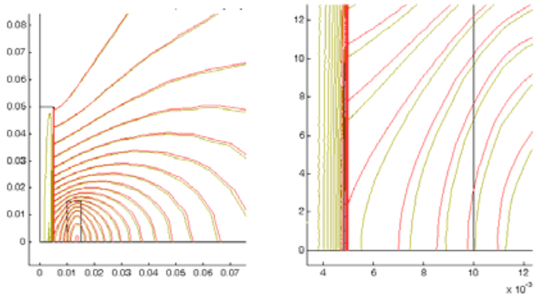
Fig.10: Power flow through the vertical surface of the charge (red lines – computation without transformation, green lines – computation with transformation)



a) *b)*

Fig.11: a) Temperature distribution after 60s of heating (red lines – computation without transformation, green lines – computation with transformation)
b) relative error of temperature after 60s of heating (The computation without transformation is taken as the correct value and the difference between the correct final value and the initial value of local temperature is taken as the base for the calculation of relative error)

Results of axi-symmetrical arrangement L_{ind} = 0.03 m



a) *b)*

Fig.12: a) and b) magnetic field distribution (red lines – computation without transformation, green lines – computation with transformation)

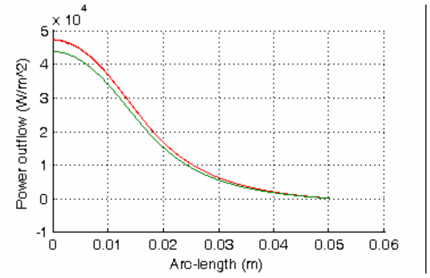
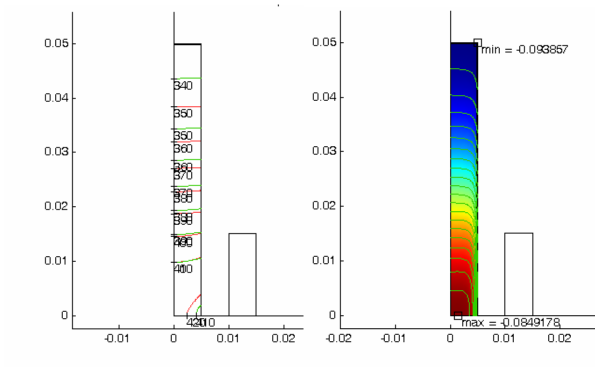


Fig.13: Power flow through the vertical surface of the charge (red lines – computation without transformation, green lines – computation with transformation)



a) *b)*

Fig.14: a) Temperature distribution after 60s of heating (red lines – computation without transformation, green lines – computation with transformation)
b) relative error of temperature after 60s of heating (The computation without transformation is taken as the correct value and the difference between the correct final value and the initial value of local temperature is taken as the base for the calculation of relative error)

3 CONCLUSION

A method for better discretisation of skin layers of electromagnetic fields in a conductive material was suggested. The methodology is intended for tasks comprising very thin skin depths and that are for any purpose solved with use of codes that do not implement an impedance boundary condition.

The suggested method was tested on several case studies of induction heating. Two basic arrangements were investigated: an induction heating of a ferromagnetic cylinder and an induction heating of a ferromagnetic metal plate.

Computed magnetic and temperature fields are compared with fields computed without a transformation. This way the error of the method can be rated.

From the results, it can be seen that results are very good for the planar arrangements, however the error of the method are much higher on the cylindrical arrangements. It can be deduced that small radius of curvature of the surface of a charge can cause either a significant error or a limit to the applicable value of the transformation coefficient.

4 ACKNOWLEDGEMENT

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5 REFERENCES

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