September 7. - 9.9.2009
Cheb, Czech Republic

# ELECTROMAGNETIC ACCELERATION OF FERROMAGNETIC BODIES

ING. KAREL LEUBNER

**Abstract**: *Acceleration of ferromagnetic bodies is used in a lot of various industrial and other applications. Its mathematical model consists of two ordinary differential equations describing the time evolution of the current in the field circuit and motion of the accelerated body and a partial differential equation describing the distribution of electromagnetic field (whose knowledge is necessary for determination of the electromagnetic force acting on the body and inductance of the field circuit). In specific cases it is also necessary to evaluate the steady-state temperature field in order to check the most exposed parts of the device with respect to their temperature rise. The model is then solved numerically. The methodology is illustrated by a typical example whose results are discussed.*

**Key words**: *Electromagnetic acceleration, electromagnetic field, electromagnetic force, numerical modeling.*

## INTRODUCTION

Devices working on the principle of electromagnetic acceleration are known more than 150 years. We can mention, for example, electromagnetic actuators of various constructions, parts of switching gears, electromagnetic guns, and many others.

But despite their simple construction, their design and modeling is not an easy business due to the presence of parts that are nonlinear and some of them also move. And only the progress in the computational engineering and capabilities of existing HW after 1980s allowed the experts in the domain sufficiently accurate and reliable numerical modeling of such systems.

Since then, the development in the field is relatively fast, mainly in the domain of electromagnetic improvement and optimization of the devices. But with their ever growing parameters, another aspect of the problem became important, namely the temperature rise of the most exposed parts and in specific cases also their possible thermal deformations.

Within the framework of my doctoral studies I started studying the problem intensively two years ago, taking into account all the mentioned viewpoints. The paper presents the complete model of the device, its mathematical solution realized by commercial code QuickField in combination with a lot of own procedures and scripts and also two illustrative examples.

## 1 FORMULATION OF THE PROBLEM

Consider the arrangement in Fig. 1. The system consists of a field coil carrying current of an arbitrary time evolution, a ceramic or plastic leading pipe and a ferromagnetic body that is to be accelerated. Geometry and physical properties of these parts are supposed to be known.

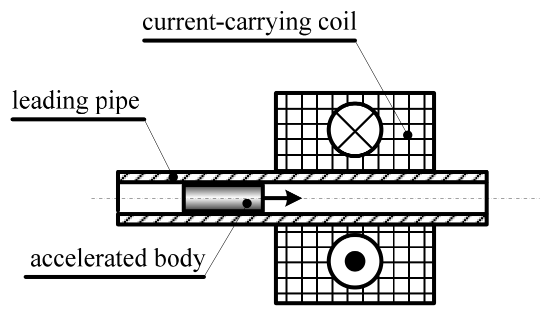


*Fig. 1: Basic arrangement of the investigated system*

The time variable magnetic field generated by the field coil produces magnetic forces acting on the ferromagnetic body that starts moving in the direction indicated by the arrow (it is pulled into the coil).

The aim of this work is to model the movement of the body (its acceleration and velocity) under the conditions specific for the particular device.

## 2 TWO PRINCIPAL VERSIONS OF THE DEVICE

As indicated above, there exist several fundamental kinds of devices making use of the principle. The first and presumably the largest group is created by *electro-*

*magnetic actuators*. A typical device of this type is depicted in Fig. 2. The movable ferromagnetic core acts as a mechanical switching element.

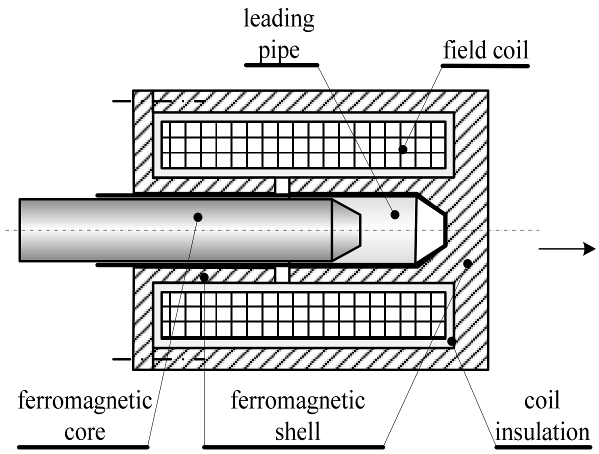


*Fig. 2: Schematic view of an electromagnetic actuator*

Another group consists of *electromagnetic guns*. Their function may be described referring to Fig. 1. The ferromagnetic body (projectile) is accelerated by the magnetic field produced by the field current. But as far as it gets approximately to the middle of the coil, the field current must be switched off (otherwise it would start to be decelerated as it would be attracted back into the coil).

This paper is confined to investigation of the static and dynamic characteristics of electromagnetic actuators fed from DC voltage sources.

## 3 CONTINUOUS MATHEMATICAL MODEL

Consider the device in Fig. 2. The field coil is supposed to be connected to a source of DC voltage $U$, see Fig. 3.

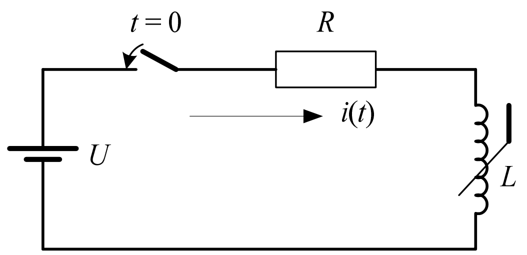


*Fig. 3: Electric circuit of the actuator*

The complete mathematical model of the actuator consists of two ordinary differential equations (ODEs) describing the electric and mechanical circuits and one partial differential equations (PDEs) describing the distribution of magnetic field (but sometimes it is also important to calculate the distribution of the coupled temperature field). All these equations are discussed in the following text.

- **Electric circuit:**

$$Ri + \frac{\mathrm{d}}{\mathrm{d}t}(Li) = U, \quad i(0) = 0, \tag{1}$$

where $R$ is the total resistance of the feeding conductors and field coil and $L$ is the inductance of the field coil that is a nonlinear function of the instantaneous position $x$ of the core and instantaneous value $i$ of the field current.

- **Movement of the core:** (is realized only in the $x$-direction)

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = F_{\mathrm{em}} - F_{\mathrm{dr}}, \qquad v(0) = 0,$$
$$v = \frac{\mathrm{d}x}{\mathrm{d}t}, \qquad x(0) = x_{\mathrm{start}}, \tag{2}$$

- where $m$ is the mass of the ferromagnetic core, $v$ is the component of its velocity in the $x$ direction, $F_{\mathrm{em}}$ is the $x$-component of the electromagnetic force acting on it and $F_{\mathrm{fr}}$ denotes the sum of the drag forces, for example due to friction or aerodynamic resistances. The quantity $F_{\mathrm{em}}$, analogous to inductance $L$, is also a nonlinear function of the instantaneous position $x$ of the core and instantaneous value $i$ of the field current. Both the above quantities have to be calculated from the distribution of magnetic field in the system.

- **Magnetic field in the system:** its distribution is best described by the magnetic vector potential $\boldsymbol{A}$. As the time variations of magnetic field are rather slow, the field may be considered quasistatic and the governing equation reads

$$\mathrm{curl}\left(\frac{1}{\mu}\mathrm{curl}\,\boldsymbol{A}\right) = \boldsymbol{J}, \tag{3}$$

where $\mu$ denotes the magnetic permeability and $\boldsymbol{J}$ is the field current density. If the actuator may be considered axisymmetric, both vectors $\boldsymbol{A}$ and $\boldsymbol{J}$ have only the circumferential component. Equation (3) has to be supplemented with correct boundary conditions for $\boldsymbol{A}$.

- **Inductance** $L$ occurring in (1) can be calculated from formula

$$L = \frac{2W_{\mathrm{m}}}{i^2}, \tag{4}$$

where $W_{\mathrm{m}}$ is the energy of magnetic field of the system that may be calculated using formula

$$W_{\mathrm{m}} = \frac{1}{2}\int_V \boldsymbol{J}\cdot\boldsymbol{A}\,\mathrm{d}V, \tag{5}$$

where $V$ is the volume of the definition area.

- **Electromagnetic force** $F_{\mathrm{em}}$ acting on the core must be determined from the Maxwell tensor, which is a relatively complicated business. Nevertheless, for axisymmetric arrangements this force may be expressed by integral

$$\boldsymbol{F}_{\mathrm{em}} = \frac{1}{2}\oint_{S_{\mathrm{c}}}\left[\boldsymbol{H}(\boldsymbol{n}\cdot\boldsymbol{B}) + \boldsymbol{B}(\boldsymbol{n}\cdot\boldsymbol{H}) - \boldsymbol{n}(\boldsymbol{B}\cdot\boldsymbol{H})\right]\mathrm{d}\boldsymbol{S}. \tag{6}$$

The integration has to be performed over the whole boundary $S_{\mathrm{c}}$ of the core, $\boldsymbol{B}$ and $\boldsymbol{H}$ are the field vectors and $\boldsymbol{n}$ is the unit vector of the outward normal to boundary $S_{\mathrm{c}}$.

Several more remarks should be added concerning the computation of the above equations and expressions.

- After the core reaches the end of the tube, it stops. At this moment we stop solving equation (2).
- On the other hand, current $i(t)$ in the field circuit described by (1) can continue varying. It depends, of course, on the further operation regime of the actuator (for example, the core may remain it its final position or after a while it may return to its original position).
- Of great advantage is to calculate the dependencies of the inductance $L$ and force $F_{\mathrm{em}}$ on the field current $i$ and position $x$. In this way we will obtain two nomograms in which we can easily find the values of both above quantities for particular values $i$ and $x$ (and, where necessary, also their derivatives).

## 4 NUMERICAL MODEL AND ITS SOLUTION

The task was solved numerically by means of the FEM-based code QuickField (physical fields) and a lot of own procedures and scripts for the time integration of relevant quantities (description of the time evolution of current in the field circuit and velocity or trajectory in the mechanical circuit). The algorithm consists of the following steps:

1. Computation of the nomograms $L = L(i,x)$ and $F_{\mathrm{em}}(i,x)$. This may advantageously be realized by cooperation of QuickField and Excel. In order to obtain sufficiently correct maps, the density of the grid should be as dense as possible.
2. Simultaneous solution of equations (1) or (7) (according to the task solved) and (2). But first it is necessary to modify the term $\mathrm{d}(Li)/\mathrm{d}t$ occurring in (1). It is clear that

$$\frac{\mathrm{d}}{\mathrm{d}t}(Li) = L\frac{\mathrm{d}i}{\mathrm{d}t} + i\frac{\mathrm{d}L}{\mathrm{d}t} = = L\frac{\mathrm{d}i}{\mathrm{d}t} + i\frac{\mathrm{d}L}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = L\frac{\mathrm{d}i}{\mathrm{d}t} + iv\frac{\mathrm{d}L}{\mathrm{d}x}. \tag{8}$$

3. Selection of the starting position of the core $x_{\mathrm{start}}$ and time step $\Delta t$. The initial conditions for the field current and velocity of the core are $v(0) = i(0) = 0$.
4. Suppose that we know the values of $x_k, v_k$, and $i_k$ at the $k$ th time level. Now it is necessary to deter mine all these quantities at the $k+1$th time level.
5. Equation (1) may be modified (using (8)) to the form

$$Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + iv\frac{\mathrm{d}L}{\mathrm{d}x} = U$$

and, hence,

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{U - Ri - iv(\mathrm{d}L/\mathrm{d}x)}{L}.$$

Consider, for an illustration, the simplest Euler method of the time integration. Discretization of this equation with respect to time provides

$$\frac{i_{k+1} - i_k}{\Delta t} = \frac{U - Ri_k - i_k v_k \left(\frac{\mathrm{d}L}{\mathrm{d}x}\right)_k}{L_k}.$$

The values of $L_k$ and $(\mathrm{d}L/\mathrm{d}x)_k$ can easily calculated from the corresponding nomogram $L(i,x)$. Finally,

$$i_{k+1} = \frac{U - Ri_k - i_k v_k \left(\frac{\mathrm{d}L}{\mathrm{d}x}\right)_k}{L_k} \cdot \Delta t + i_k.$$

6. Equations (2) are discretized in the following way:

$$m\frac{v_{k+1} - v_k}{\Delta t} = F_{\mathrm{em},k} - F_{\mathrm{dr},k},$$

$$\frac{x_{k+1} - x_k}{\Delta t} = v_k,$$

and, hence,

$$v_{k+1} = \frac{F_{\mathrm{em},k} - F_{\mathrm{dr},k}}{m}\Delta t + v_k,$$

$$x_{k+1} = v_k \Delta t + x_k.$$

7. While the value $F_{\mathrm{em},k}$ can easily be determined from nomogram $F_{\mathrm{em}}(i,x)$, the value of $F_{\mathrm{dr},k}$ depends on the particular situation in the leading tube. For example, its value may depend on the second power of velocity, i.e., $F_{\mathrm{dr},k} = kv_k^2$. From the moment when the core reaches the end of the leading tube, $v_{k+1} = 0,\ x_{k+1} = x_{\max}$.
8. If the condition of finishing the calculation is satisfied, the solution is stopped.
9. Return to point 4.

## 5 ILLUSTRATIVE EXAMPLE

Consider an actuator according to Fig. 2. Its dimensions are given in Fig. 4

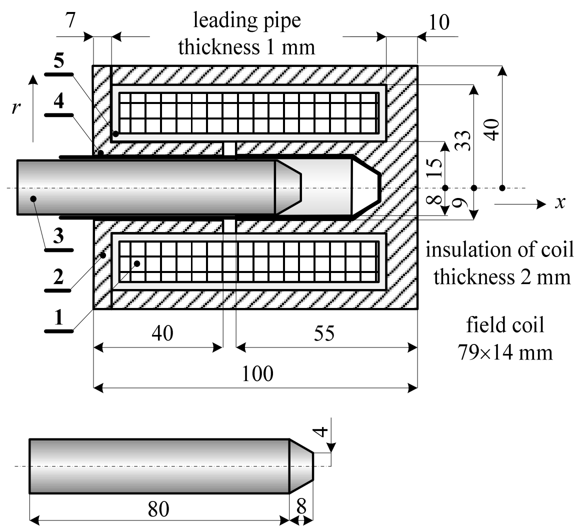


*Fig. 4: The investigated actuator*
**1** – *field coil (copper),* **2** – *ferromagnetic shell (carbon steel 12 040),* **3** – *plunger (carbon steel 12 040),* **4** – *leading pipe (plastic),* **5** – *coil insulation (Teflon)*

The voltage $U$ of the source is 12 V, the total resistance of the circuit $R = 0.15\,\Omega$. The coefficient of the aerodynamic resistance $k = 0.05$ and the time step was chosen $\Delta t = 0.00005$ s.

Both ferromagnetic shell and plunger are made from carbon steel CSN 12 040, whose magnetization characteristic is shown in Fig. 5.

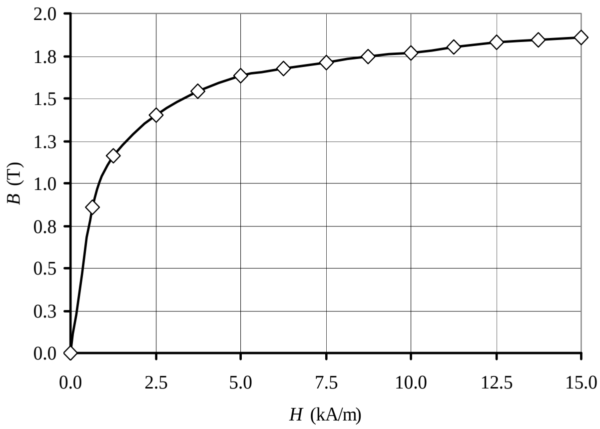


*Fig. 5: Magnetization characteristic of carbon steel CSN 12 040*

The dependence of electromagnetic force $F_{em}(i, x)$ for this actuator is in Fig. 6 and inductance $L(i, x)$ in Fig. 7.

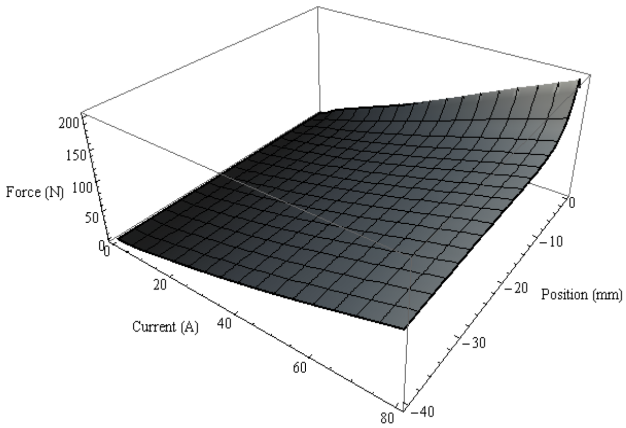


*Fig. 6: The dependence of electromagnetic force $F_{em}$ acting on the plunger on current $i$ and position $x$*

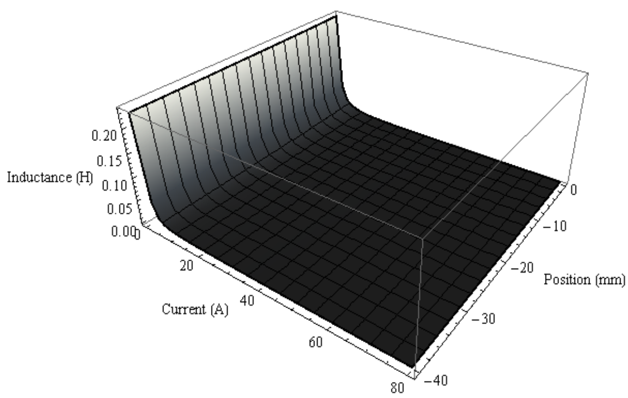


*Fig. 7: The dependence of inductance $L$ of the system on current $i$ and position $x$*

For small currents the relative permeability $\mu$ of ferromagnetic parts is rather high (ferromagnetics is not saturated) and that is why the inductance of the system is high. In case of higher currents the ferromagnetics starts to be saturated or even oversaturated, which leads to very fast decrease of permeability and consequent decrease of the inductance.

For an illustration I calculated the characteristics of the actuator for the case when the front of the plunger is 40 mm from the end of the hole in the electromagnet. It means that its starting position $x_0 = -40$ mm.

Figure 8 shows the time evolution of the field current. At the beginning of the process its value is relatively small and grows slowly. But at the moment when the larger part of the plunger gets into the coil, it starts growing very steeply, up to the asymptotic value $I_a = U / R = 80$ A. It is worth noting that the field current may change even after the moment when the plunger hits the end of the hole and stops moving.

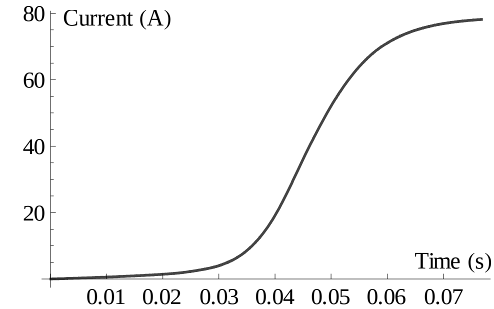


*Fig. 8: Time evolution of the field current*

Figure 9 shows the corresponding time evolution of the Maxwell force acting on the plunger. At the beginning it copies the square of the field current in Fig. 8. But then (due to steep changes of energy) it grows quickly until the front of the plunger hits the end of the hole. Due to the field current that increases even after this moment the force grows as well.

A visible peak at time $t = 0.055$ s (time when the plunger gets to the end of the hole) is caused by different equations describing the system before and after impact.

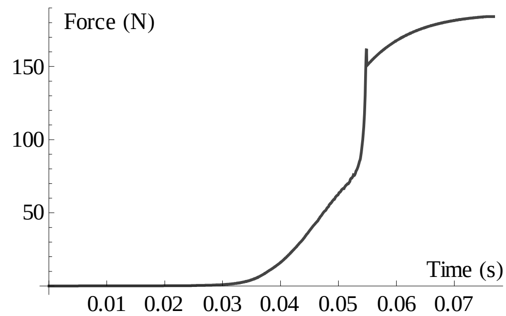


*Fig. 9: Time evolution of the Maxwell force*

Figure 10 depicts the time evolution of the aerodynamic resistance that is considered proportional to the square of velocity of the plunger.

Figure 11 contains the time evolution of inductance of the field coil. In the period where the field current is still small and a greater part of the plunger is still out of the body of the magnet (so that the ferromagnetic parts of the system are undersaturated), its value is high and exceeds 200 mH. But with higher field current the saturation grows, magnetic permeability of the magnetic circuit and plunger decreases and so does the inductance. Thereby we obtain a positive feedback because the smaller inductance, the higher the field current (the voltage of the source remaining unchanged).

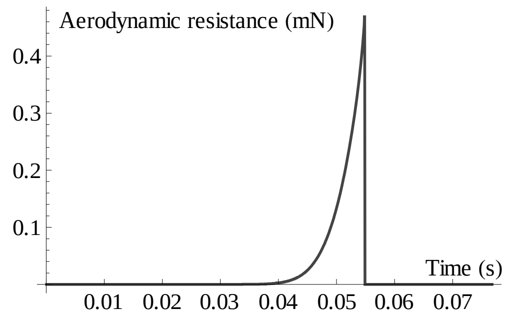


*Fig. 10: Time evolution of the aerodynamic resistance*

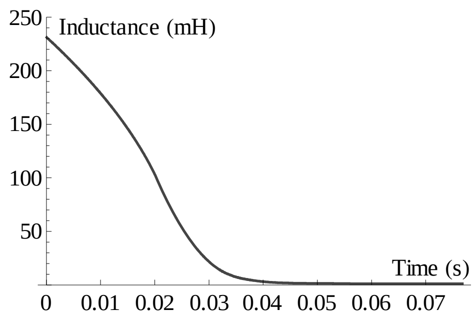


*Fig. 11: Time evolution of the system inductance*

Another important quantity is the magnetic flux $\Phi$ produced by the field coil that is given as a product of the inductance and field current. Its time evolution is depicted in Fig. 12.

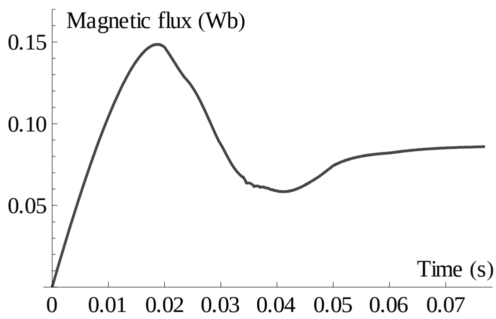


*Fig. 12: Time evolution of magnetic flux in the system produced by the field coil*

Finally, the last three figures show the kinematic quantities characterizing the movement of the plunger.

Figure 13 shows the time evolution of its acceleration calculated from the total force acting on the plunger (given by the difference between the magnetic and drag forces). It can be seen that in about 0.055 s the plunger gets to the end of the hole.

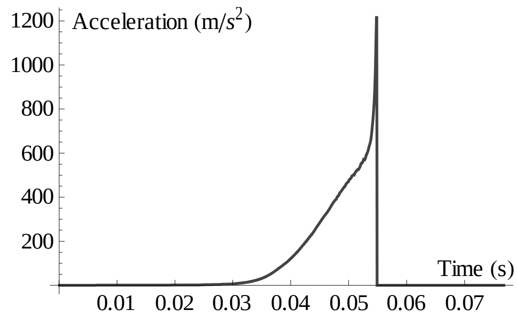


*Fig. 13: Time evolution of acceleration of the plunger*

Speed of the plunger exhibits the same features. Its time evolution is shown in Fig. 14. At the end of the process the speed reaches almost 6 m/s, which can be quite dangerous from the viewpoint of the lifetime of the device.

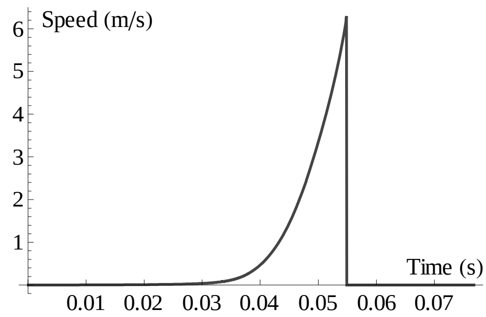


*Fig. 14: Time evolution of speed of the plunger*

The dependence of the trajectory of the plunger is depicted in Fig. 15.

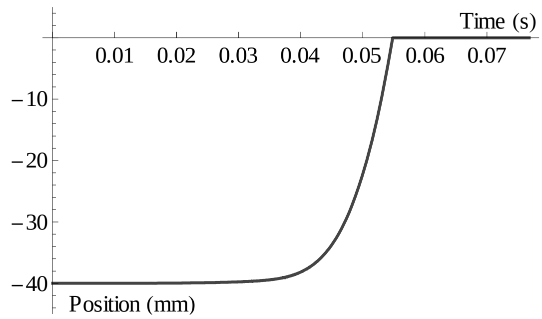


*Fig. 15: Trajectory of the plunger as a function of time*

## 6 CONCLUSION

The algorithm provides physically realistic results and seems to be sufficiently fast. Next work in the domain will be aimed at more detailed determination of the time evolution of temperature field in the system and mechanical strains and stresses in the most exposed parts of the device.

**Author**: Ing. Karel Leubner, Czech Technical University, Faculty of Electrical Engineering, Technicka 2, 166 27 Praha, CR, E-mail: karel.leubner@gmail.com.