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MAGNETIC FIELD DECOMPOSITION AND REVERSE APPROXIMATION METHOD

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Abstract: *In this work, we developed a method, which is used for approximation of basic magnetic field B0 inside a specific spherical volume. This method uses Legendre multinominals expansion of magnetic flux density field measured at discrete points on spherical volume. Using optimization technique of Least square method we modeled map of magnetic flux density on the desired surface and compared it to measured values of magnetic flux density. The coefficients of spherical function expansion obtained from approximation process can be used for determination of shimming currents which feed correction coils of tomograph. The results of approximated maps are described and compared to measured maps in the experimental part of this article*

Key words: *NMR, optimization methods, shimming, legendre multinominals, homogeneity.*

INTRODUCTION

Magnetic resonance tomography is an imaging technique used primary in medical setting to produce high quality images of the human body. Magnetic resonance imaging is based on the principles of nuclear magnetic resonance (NMR) and at the present time it is the most developed imaging technique at biomedical imaging [1]. Lately, medical science lays stress on the measuring of exactly defined parts of human body, especially human brain. If we want to obtain the best quality images we have to pay attention to homogeneity of magnetic fields, which are used to scan desired samples inside the NMR tomograph. We should know how to reduce in-homogeneity, which can cause misleading information at the final images of samples. Generally, in-homogeneity of magnetic fields at magnetic resonance imaging cause contour distortion of images. To eliminate this in-homogeneity correctly, we need to know the map of the magnetic field and we also need to have an exact information about parameters of the magnetic field. This paper presents the experimental method, which can easily create the map of electromagnetic flux density at any defined area inside the tomograph. This method uses mathematical theory of Legendre multinominals [3], which are used for approximation of magnetic field, if we know specific coefficients. The coefficients of Legendre multinominals, which are computed using measured values of magnetic flux density at exactly defined discrete points are used for creating map of magnetic field. If we know these coefficients, we are able to compute magnetic flux density at any point of defined area. At the ideal case, there should be no difference between measured data and approximated data. According to analogy between values of coefficients and shimming currents, we will be able to propose an iterative method to make an optimization of basic magnetic field in MR tomography [4].

1 PRINCIPLES OF NMR

In quantum mechanics, spin [6] is important for systems at atomic length scales, such as individual atoms, protons or electrons. One of the most remarkable discoveries associated with quantum physics is the fact, that elementary particles can possess non zero spin. Elementary particles are particles that cannot be divided into any smaller units, such as the photon, the electron and the various quarks. The spin carried by each elementary particle has a fixed value that depends only on the type of particle, and cannot be altered in any known way. Particles with spin can possess a magnetic dipole moment, just like a rotating electrically charged body in classical electrodynamics. The main principle of magnetic resonance spectroscopy and magnetic resonance imagining is, that radiofrequency fields (RF pulses) excite transitions between different spin states in a magnetic field. The information content can be retrieved as resonance frequency, spin to spin couplings and relaxation rates. We can imagine, that protons are rotating along their axes and there is also a wobbling motion called precession, that occurs when a spinning object is the subject of an external force. Thanks to the positive charge of protons and its spin, protons generate a magnetic field and gets a magnetic dipole moment. If the protons are placed in a magnetic field, the magnetic moment will do precessional motion about the direction of magnetic field with specific frequency. This frequency is called Larmor frequency and can be described by the Larmor equation [6]

$$\Omega = \gamma B \tag{1}$$

where Ω is the frequency of precession, γ is the gyro-magnetic ratio and B is strength of external magnetic field. In ordinary materials, the magnetic dipole moments of individual atoms produce magnetic fields that cancel one another, because each dipole points in a random direction. In ferromagnetic materials however, the dipole moments are all lined up with another, producing a macroscopic, non-zero magnetic field. If there is no external magnetic field, magnetic moments of atoms are chaotically spread and there is nearly no resulting magnetization vector M_0. If we place a sample into the stationary magnetic field B_0, we will realize, that there is a vector of magnetization M_0 which is created as a sum of magnetic moments of each atom. The direction of this vector is the same as the direction of external magnetic field B_0. This state is called longitudinal magnetization. Now we apply a high frequency magnetic field of induction B_1, which is vertical to stationary magnetic field B_0. This high-frequency magnetic field causes resonance effect and magnetization vector M_0 starts to rotate with specific angular frequency. To measure vector M_0, we need to drop it into the x-y plain (on condition that B_0 has direction of z axes). This dropping is done by a high-frequency excitation pulse B_1, which has a proper shape. This state is called transversal magnetization. Set of these pulses is called pulse sequence. Pulse sequence is a pre-selected set of defined RF and gradient pulses, usually repeated many times during a scan. Pulse sequences control all hardware aspects of the measurement process. At the x-y plain, there is scanning coil, which is used for scanning of FID signal.

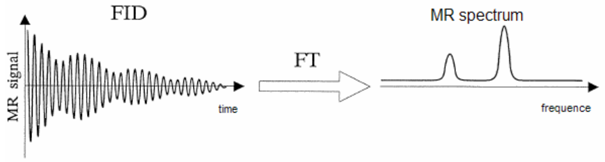

Fig.1: FID signal and MR spectrum

After excitation pulses, the spins has tendency to minimize transverse magnetization and to maximize longitudinal magnetization. The transverse magnetization decays toward zero with characteristic time constant T2 and the longitudinal magnetization returns towards maximum with a characteristic time constant T1

2 BASICS OF LEGENDRE MULTINOMINALS

If we want to determine the magnetic flux density values in the specific points of measured area, we should use Legendre multinominals. The behaviour of first 6 multinominals shows Fig. 2.

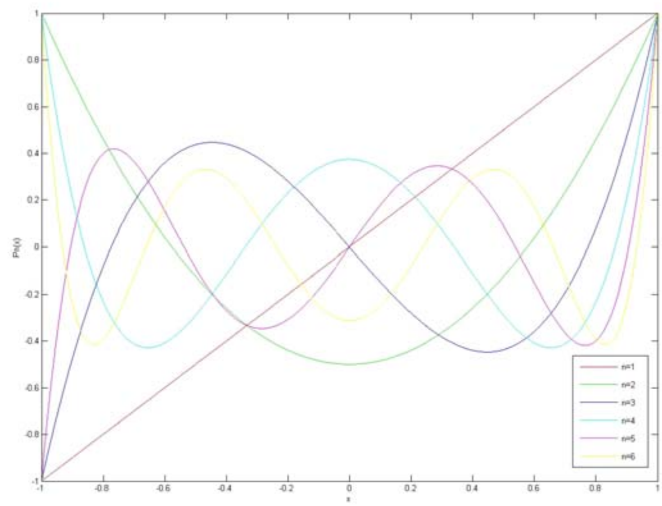

Fig.2: Behavior of Legendre polynoms up to 6th degree

Magnetic field induction can be approximated at any point of measured area. These points can be selected using spherical coordinates [r, θ, φ], so we can define the approximation formula as follows [3]

$$B_a(r,\theta,\varphi) = \sum_{k=0}^{N_K} \cdot \sum_{m=0}^{m=k} \cdot r^k \cdot P_{m,k}(\cos\theta) \cdot \cdot\left[A_{m,k}\cos m\cdot\varphi + B_{m,k}\sin m\cdot\varphi\right] \tag{2}$$

It is possible to find coefficients A_{mk} a B_{mk} like minimum value of this formula

$$\Psi = \min \sum_{i=1}^{N_m} (B_{im} - B_{ia})^2 \tag{3}$$

where B_m are measured values of magnetic induction at the desired area (circle, sphere, cylinder) and B_{ia} are approximated values of magnetic induction. This method is known as Least square method (LSM). Legendre multinominals of zero and first order are defined as

$$P_0(z) = 1 \tag{4}$$

$$P_1(z) = z = \cos\upsilon \tag{5}$$

and Legendre multinominals of higher order are defined according to recursion formula

$$P_{n+1}(z) = \left[(2n+1)\cdot z\cdot P_n(z) - n\cdot P_{n-1}(z)\right]/(n+1) \tag{6}$$

3 PRINCIPLE OF B_0 CORRECTION

Basic principle of non-homogeneous magnetic field is quite simple. If we know, that map of the magnetic flux density on specific volume has certain behavior, we will add another correction magnetic field, which has opposite behavior. The goal is to optimize correction field to desired behavior and the superposition of those two fields will be ideally without any fluctuations. In praxis absolutely homogeneous field is not possible, so we have to specify, which homogeneity is good for our purposes and which volume we will consider. We can define homogeneity on specific area according to this formula

$$Homogeneity = \frac{B_{0,\max} - B_{0,\min}}{B_{0,mean}} \quad (7)$$

where $B_{0,max}$ and $B_{0,min}$ are maximal and minimal values of magnetic flux density and $B_{0,mean}$ is average value of magnetic flux density on specific volume [1]. In literature is mentioned, that the homogeneity of basic magnetic field B_0 should be better than 10 – 50 ppm on volume with diameter 50 – 60 cm. For purposes of spectroscopy we desire much better fields. The correction process is called "shimming" and has to be done nearly for each manufactured magnet, because manufactures are not able to produce magnets with absolutely homogeneous behavior. In fact, there are two types of correction – passive and active. Active gradients are generated by shimming coils. Ideal fields of these coils can be modeled according to Tab. 1.

Name of gradient	Equation for computing	Order
Z^1	z	1
Z^2	$2z^2(x^2+y^2)$	2
Z^3	$z[2z^2-3(x^2+y^2)]$	3
Z^4	$8z^2[z^2-3(x^2+y^2)]+3(x^2+y^2)^2$	4
Z^5	$48z^3[z^2-5(x^2+y^2)]+90z(x^2+y^2)^2$	5
X	x	1
Y	y	1
ZX	zx	2
ZY	zy	2
X^2-Y^2	x^2-y^2	2
XY	xy	2
Z^2X	$x[4z^2-(x^2+y^2)]$	3
Z^2Y	$y[4z^2-(x^2+y^2)]$	3
ZXY	zxy	3
$Z(X^2-Y^2)$	$z(x^2-y^2)])$	3
X^3	$x(x^2-3y^2)$	5
Y^3	$y(3x^2-y^2)$	5

Tab.1. Equations for computing correction gradients

If we are able to generate many behaviors of gradients, it means gradients of higher orders, we are able to correct nearly every in-homogeneity of basic magnetic field. In Tab.1 we discuss only gradients up to 5th order, because it corresponds with expansion of magnetic flux density used for our approximation at experimental part. [5]. Fig. 3 shows contour 3D graph of normalized gradient X^3.

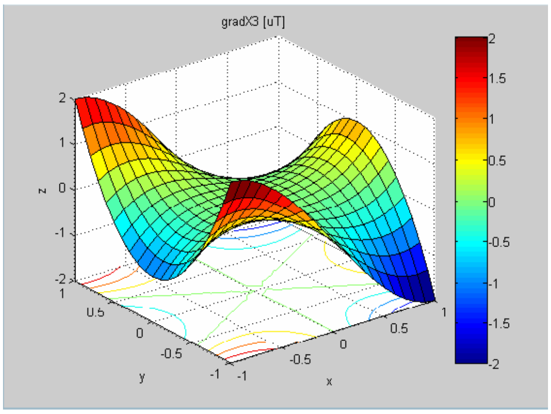

Fig.3: Ideal computed gradient X^3

In our case we will take advantage of analogy between those gradients and coefficients of expansion according to (2).

4 Measurement Process

There is a sophisticated moving mechanism (Fig. 4), which enables staff to control the position of the probe in the magnetic field of tomograph.

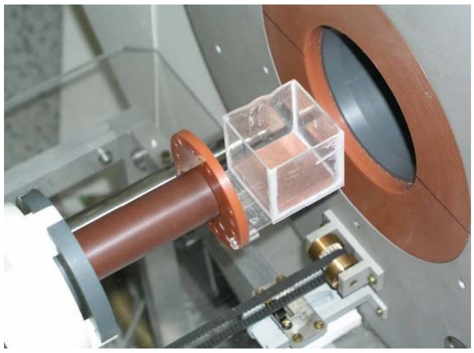

Fig.4: Detailed view of moving mechanism

For mapping of defined sphere is used a very small probe, which consists of a glass sphere, filled with water. This probe also contains scanning coils, which are used to get FID signal. Moving mechanism enables to put the probe into exactly defined points and measure magnetic field at those points , so we can map desired dimension. When the probe is situated at the correct place, we can apply one of the pulse sequences to get the frequency values. Data are sampled during a gradient echo sequence, which is achieved by de-phasing the spins with a negatively pulsed gradient before they are re-phased by an opposite gradient with opposite polarity to generate the echo. The program used to set the parameters of pulse sequences and show the spectral characteristics of measurement is called The NMR 1D .

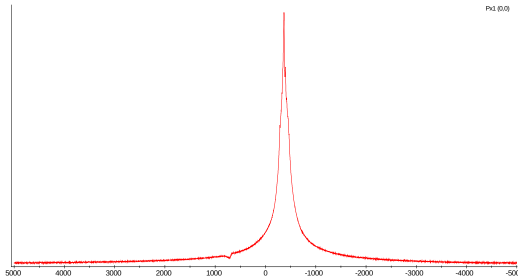

Fig.5: Example of spectral line inside homogeneous field

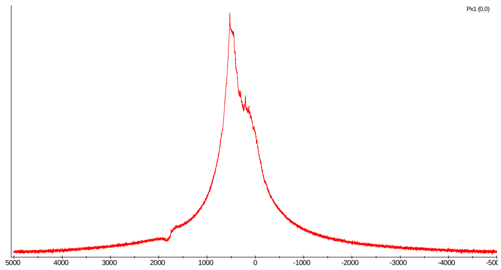

Fig.6: Example of spectral line inside non – homogeneous field

5 APPROXIMATION RESULTS

All values of magnetic induction on following figures are presented at [μT] unit. Fig. 8 shows map of the field on the surface of sphere, which is created only from measured values. Fig. 9 is map created from computed values, it means values which were computed during minimum searching (LSM) in Matlab. If we will consider the errors of whole process, we have to divide them to errors obtained during measuring of magnetic flux density at discrete points and errors obtained during solving equations used for approximation. From those pictures, we are able to discuss only errors of approximation. We used fifth order of Legendre polynoms, which seems to be enough for our purposes. The influence of approximation error on polynom's degree can bee seen at Fig. 7.

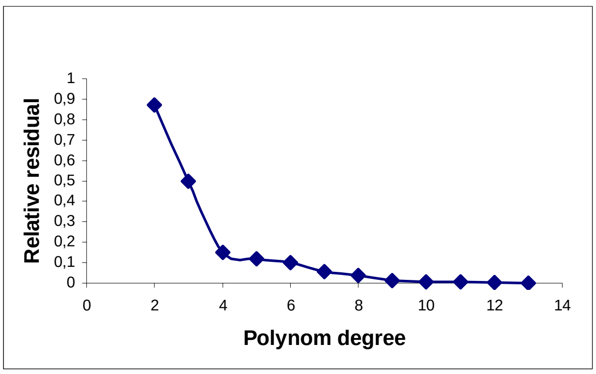

Fig.7: Relative residual versus polynom degree

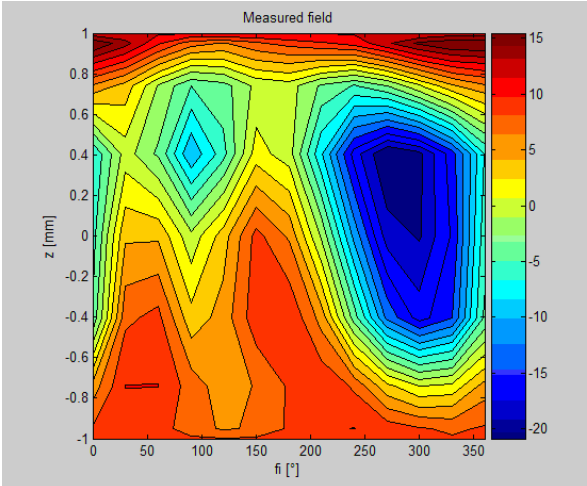

Fig.8: Measured map on the sphere surface

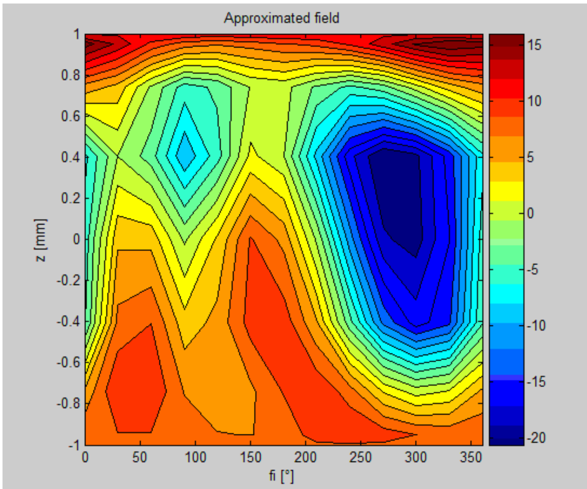

Fig.9: Approximated map on the sphere surface

Now we tried to compute slice through the mapped volume and confront this slice with measured slices. First we had to process measured phase image. It means to do FFT transformation, normalization and phase unwrapping of measured phase image. Fig 10 shows phase image before unwrapping process, but after brightness normalization. Noisy area around region of interest was normalized to zero. Fig. 11 shows unwrapped and colored image.

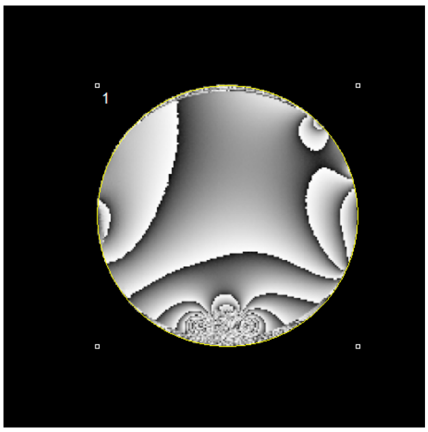

Fig.10: Phase image of slice before unwrapping

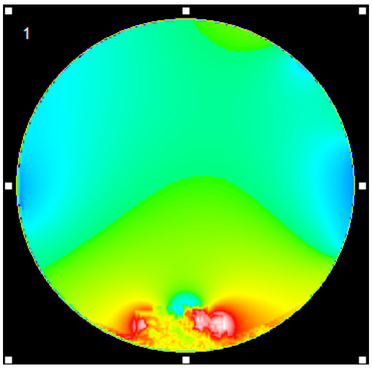

Fig.11: Field after unwrapping and normalization

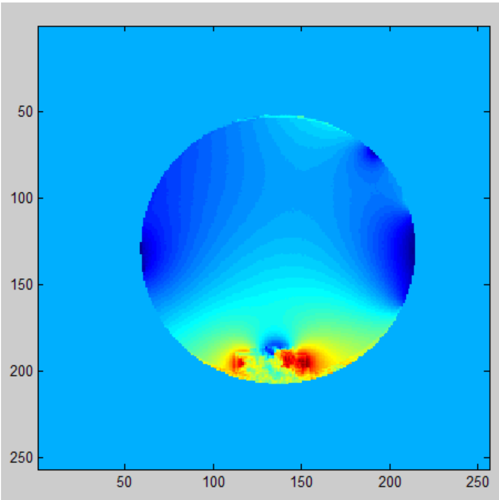

Fig.12: Imported map of the field in Matlab

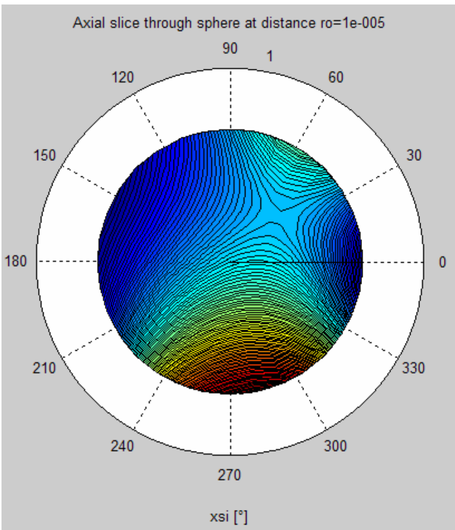

Fig.13: Computed map in Matlab

When we imported processed image into Matlab, we can see contour lines which correspond to equal values of magenetic flux density. On Fig. 12 and Fig. 13 we can see clearly, that the slices are very similar. The noisy area in the bottom of image is caused by air bubble inside glass sphere filled with water.

6 Conclusion

We proposed a method for optimization of basic magnetic field B_0 in tomography. This method is based on results of approximation using spherical functions. Legendre multinominals are suitable for optimization, because of analogy between correction gradients of tomograph and coefficients obtained from expansion of magnetic flux density values. As we can see from results, the accuracy of approximation could be better, so the future work can be directed towards minimization of differences between measured and approximated values of magnetic flux density.

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8 References

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