THE APPLICATION OF THE MATLAB AND COMSOL PROGRAMS FOR THE COMPUTATION OF A 3D NON-STATIONARY ELECTRIC FIELD

ING. ROMAN HAMAR, PH.D.

Abstract: *Computations of three-dimensional non-stationary electromagnetic fields described by differential equations of hyperbolic type are much more demanding and more required than computations of stationary electromagnetic fields. The objective of this paper is to describe the aspects of computing the 3D non-stationary inharmonious electric field by means of programs MATLAB and COMSOL. On the basis of the computations performed as an illustrative example of electrical insulation subjected to a pulse voltage, it is possible to regard the combination of MATLAB and the finite difference method as unsuitable for performing the 3D non-stationary electromagnetic fields computations. However, the results proved a good applicability of the COMSOL program together with the finite element method for the purpose mentioned above.*

Key words: *Non-stationary electromagnetic field, mathematical model, pulse stress of insulation, MATLAB, COMSOL*

INTRODUCTION

Nowadays, computations of stationary electromagnetic fields do not seem to be a problem. It is the computation of non-stationary electromagnetic fields that is in demand as well as more complex, especially if all material parameters of a given medium are taken into consideration. If the electromagnetic field varies with time inharmoniously, its computation is rather complicated. Furthermore, a computation of this field in three dimensional space makes the solution even more problematic.

The aim of this paper is to describe the aspects of computing the non-stationary inharmonious electric field in three dimensional space by means of the finite difference method in the MATLAB program and consequently by means of the finite element method in the COMSOL program. The computation of the three-dimensional non-stationary electric field is demonstrated on the illustrative example of insulation subjected to pulse voltage.

1 CONTINUOUS MATHEMATICAL MODEL

The propagation of non-stationary electromagnetic field through a medium of the permittivity ε, permeability μ and the conductivity γ (on condition that these parameters of the medium are constant) is possible to describe by means of differential equations for the electric field strength E and for the magnetic induction B [2]

$$\nabla^2 E - \mu\,\varepsilon\,\frac{\partial^2 E}{\partial t^2} - \mu\,\gamma\,\frac{\partial E}{\partial t} = 0 \qquad (1)$$

$$\nabla^2 B - \mu\,\varepsilon\,\frac{\partial^2 B}{\partial t^2} - \mu\,\gamma\,\frac{\partial B}{\partial t} = 0 \qquad (2)$$

The equations (1) and (2) are called generalized homogeneous wave equations, which describe the propagation of a dumped electromagnetic wave through a medium. These differential equations of hyperbolic type are possible to derive from Maxwell's equations [1].

An easier solution to a non-stationary electromagnetic field can be achieved by means of electromagnetic potentials A and φ. For example, in the solution to the two-dimensional electromagnetic field, just one component of the vector magnetic potential is non-zero. Therefore, only one unknown scalar quantity is computed first. The equations of the electromagnetic potentials A and φ are analogous to the previous equations [2]

$$\nabla^2 A - \mu\,\varepsilon\,\frac{\partial^2 A}{\partial t^2} - \mu\,\gamma\,\frac{\partial A}{\partial t} = 0 \qquad (3)$$

$$\nabla^2 \varphi - \mu \, \varepsilon \, \frac{\partial^2 \varphi}{\partial t^2} - \mu \, \gamma \, \frac{\partial \varphi}{\partial t} = 0 \qquad (4)$$

A differential equation of hyperbolic type is often impossible to solve analytically; therefore, a kind of numerical method and corresponding software are used.

2 ILLUSTRATIVE EXAMPLE

The development of variable frequency drives has placed the issue of insulation materials subjected to pulse voltage at the forefront of attention. Therefore, a three-dimensional non-stationary electric field in electric insulation of a motor winding sample was solved as an illustrative example.

Fig. 1 shows a motor winding sample consisting of an internal electrode, external electrode and insulation. The dimensions of the internal electrode are 5 x 18 x 140 mm and the thickness of the electrical insulation is 0.5 mm. The external electrode is 15 mm long is made of aluminium foil wound on the insulation.
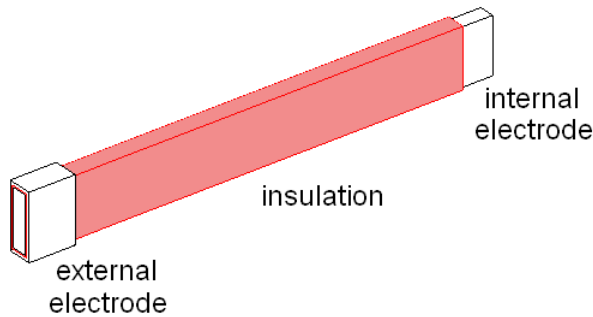

Fig. 1: Motor winding sample

The relative permittivity of the electrical insulation is $\varepsilon_r = 4$ and its conductivity is $\gamma = 10^{-6}$ S/m. The insulation is subjected to a pulse voltage which is supplied to the internal electrode. The external electrode is grounded. The course of this pulse voltage is depicted in fig. 2. The graph clearly indicates the steepness of the 10 kV/µs pulse both at the rising and falling edges of the pulse.
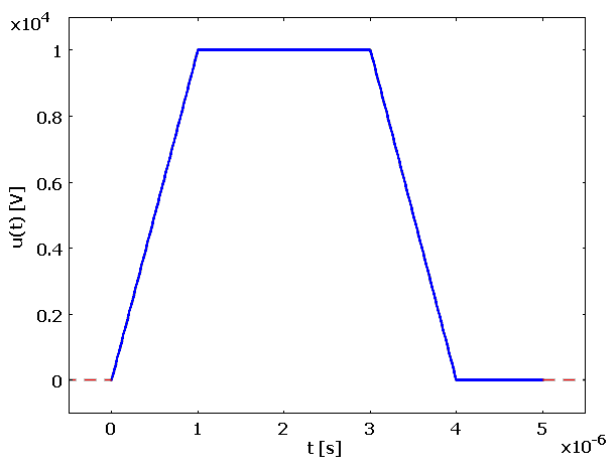

Fig. 2: Pulse course

The cross-section of the motor winding sample is axisymmetric according to both the horizontal and vertical axes, so it was enough to simulate just its symmetric quarter. However, computation of the non-stationary electric field for the entire length of the sample would have been numerically demanding and even useless from the point of view of electric field gradient. Therefore, it was first necessary to analyze which part of the sample would be sufficient for the calculation. For this reason, an undemanding calculation of an electrostatic field was first performed for the entire length of the sample.

The results of this analysis showed that in order to calculate the non-stationary electric field of the given sample it would be enough to simulate a small area at the boundary between the external electrode and the insulation. This small part of the specimen is demarcated by the 2 mm length of the insulation with the external electrode + the 3 mm length of the insulation without the external electrode.

In order to compute the electrostatic field of the entire length of the sample, an older version of the COMSOL program was used. This was because the newer version 3.4 capable of solving the required type of non-stationary field had not been available in the department yet. Fig. 3 depicts the result of the computation of the electrostatic field in the insulation. It displays the electric field strength only in the selected section of the sample, i.e. around the boundary between the external electrode and the insulation. On the boundary there is obvious the highest value of the electric field of 94 kV/mm. Further, on the right side of fig. 3 it is also possible to notice high values of the electric field strength, namely in the bend of the insulation located under the external electrode. The area for further computation was thus determined.
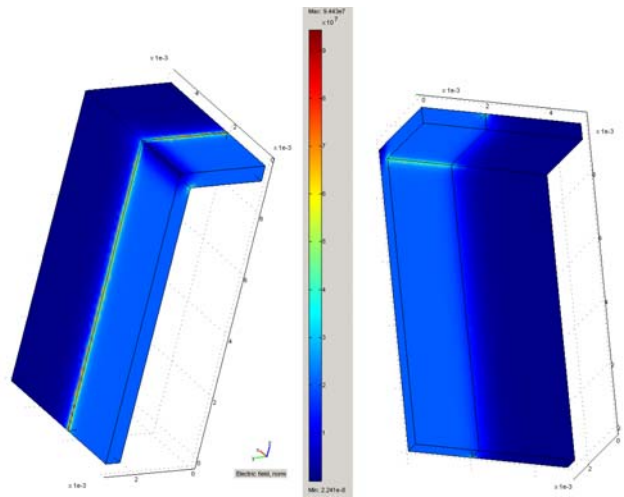

Fig. 3: First analysis - electrostatic field

3 RESULTS OBTAINED FROM PROGRAM MATLAB

First, there was an attempt to solve the given problem by means of the finite difference method in the MATLAB program. As explained above, the reason for this was the absence of a professional program capable of solving

non-stationary field described by the differential equation of hyperbolic type

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = \mu\,\gamma\,\frac{\partial \varphi}{\partial t} + \mu\,\varepsilon\,\frac{\partial^2 \varphi}{\partial t^2} \qquad (5)$$

For the numerical solution using the finite difference method there was created an even mesh with a spatiotemporal domain $\Omega(x, y, z, t)$, which was discretized in the directions of spatial coordinates x, y, z with steps Δx, Δy, Δz and in the direction of the time coordinate t with the step Δt. The difference scheme was chosen according to [2], page 141. In order to achieve a numerical stability of the solution, the implicit form of the difference approximation of equation (5) was used. The derivations in the differential equation were approximated using the equations given in [2], table 3.1. This procedure resulted in achieving the difference approximation of the equation being solved.

$$\frac{\varphi_{i+1,j,k,l+1} - 2\varphi_{i,j,k,l+1} + \varphi_{i-1,j,k,l+1}}{\Delta x^2} +$$

$$+ \frac{\varphi_{i,j+1,k,l+1} - 2\varphi_{i,j,k,l+1} + \varphi_{i,j-1,k,l+1}}{\Delta y^2} +$$

$$+ \frac{\varphi_{i,j,k+1,l+1} - 2\varphi_{i,j,k,l+1} + \varphi_{i,j,k-1,l+1}}{\Delta z^2} =$$

$$= \mu\,\gamma\,\frac{\varphi_{i,j,k,l+1} - \varphi_{i,j,k,l-1}}{2\,\Delta t} + \qquad (6)$$

$$+ \mu\,\varepsilon\,\frac{\varphi_{i,j,k,l+1} - 2\varphi_{i,j,k,l} + \varphi_{i,j,k,l-1}}{\Delta t^2}$$

where i, j, k, l are the particular indexes of nodes of the used difference mesh in directions x, y, z, t. Further, there was performed the difference approximation of boundary conditions in the nodes located on the boundary of the area Ω.

The difference mesh consisted of a set of nodes, some of which represent the electrodes. For these nodes and time $t \geq 0$ the following marginal conditions were set: the condition $\varphi = 0$ V for the nodes of the grounded external electrode and for the nodes of the internal electrode the condition $\varphi = \varphi_0(t)$, where $\varphi_0(t)$ is a given continuous function of time (fig. 2). The initial conditions for all the nodes in the area Ω at time $t = 0$ were $\varphi = 0$ and $\frac{\partial \varphi}{\partial t} = 0$.

The numerical computation was performed using the MATLAB program produced by the MathWorks Corporation (www.mathworks.com). Since the very beginning of the programming, big memory requirements were apparent because the results of two previous time levels were necessary to be stored in the main memory to be able to start each new time step of the computation Unfortunately, after exceeding approx. 5000 unknown quantities these memory requirements resulted in the termination of the program and displaying a shortage of main memory.

In order to present the results from the MATLAB program, the dimensions of the insulation were intentionally changed and the density of nodes was reduced to better distinguish among the individual nodes and vectors of the electric field strength. Fig. 4 depicts the types of the individual nodes, for example, the nodes representing Dirichlet boundary condition or Neumann boundary condition. Before assembling matrices of a set of equations, it was necessary to assign a particular type to each node in the mesh. In fig. 5, electric field vectors come out from the places of the given nodes.
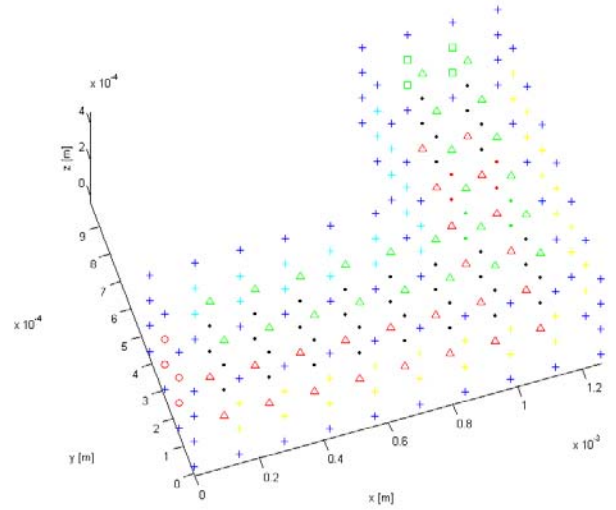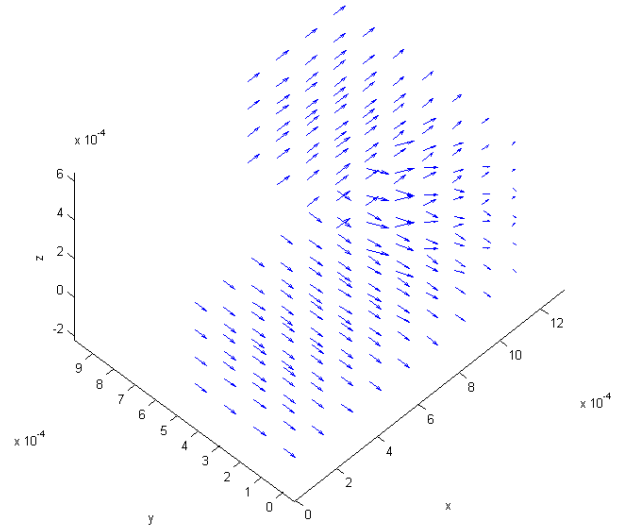

Fig. 4: Nodal types

Fig. 5: Electric field vectors

As explained above, the computation was limited by the number of 5000 unknown quantities; therefore, accurate results could not be expected. With respect to convergence it was not worth presenting the results obtained around this critical limit. Fortunately, there was purchased the professional program COMSOL, version 3.4, which is able to solve a non-stationary field described by the differential equation of hyperbolic type.

4 RESULTS OBTAINED FROM PROGRAM COMSOL VERSION 3.4

The numerical computation based on the finite element method was performed using the program COMSOL, version 3.4 produced by the COMSOL AB Corporation (www.comsol.com). The designed model consisted of 105 thousand elements. In time dependence on the pulse course there were created animations displaying the distribution of electric potential or the electric field strength in the insulation. Figures 6 and 7 were created from the animation of the electric field strength for the time 1 μs, when the rising edge of the pulse ends and when the electric field strength reaches peak values. Fig. 6 shows the electric field intensity in the particular slices of the insulation. The distance between the slides was set at 1 mm. The colour range in the picture indicates that a high value of the electric field strength within the particular slices, i.e. 32 kV/mm, is located in the bend of the insulation under the internal electrode. There follows from fig. 7 that the peak value of the electric field strength, namely 49 kV/mm, is located on the boundary between the external electrode and the insulation. Thus, the places with high values of the electric field strength were proved. These values followed from the preliminary computation of electrostatic field described in section 2; however, this computation was numerically inaccurate as a result of a rough mesh application.
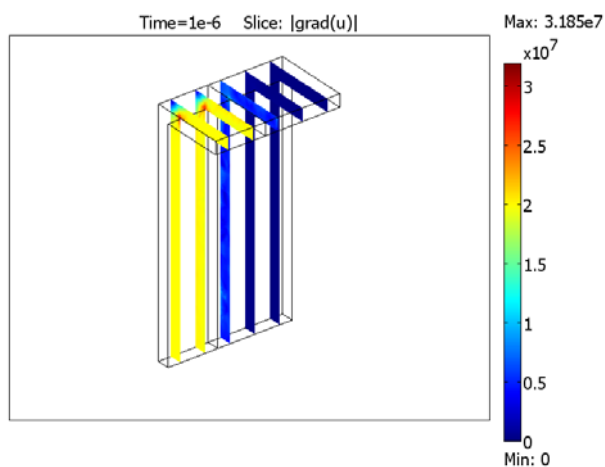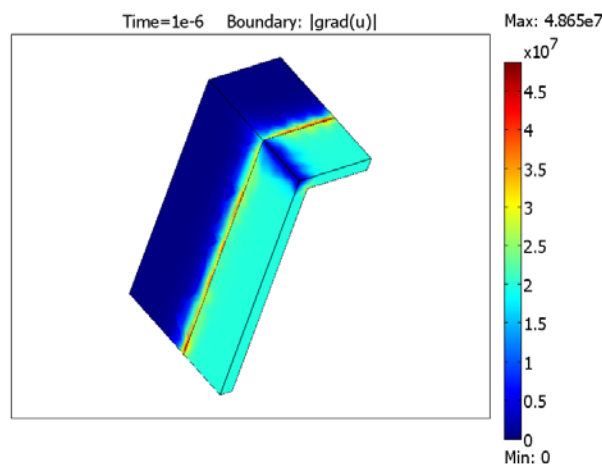

Fig. 6: Electric field – slice plot

Fig. 7: Electric field – boundary plot

5 CONCLUSION

The computation of electrical insulation subjected to a pulse voltage - an illustrative example of a 3D non-stationary electric field – was carried out by means of the programs MATLAB and COMSOL. In case there is not available a professional program for the computation of 3D non-stationary electromagnetic fields specified by differential equations of hyperbolic type, the program MATLAB cannot be recommended because the computation is limited by the number of 5000 unknown quantities. On the contrary, a very efficient tool seems to be the professional program COMSOL applying the finite element method, which was proved to be useful for this type of problem. However, it is necessary to apply version 3.4 of this program as the previous version was not able to solve the presented type of field.

6 ACKNOWLEDGMENT

7 REFERENCES

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Ing. Roman Hamar, Ph.D.
University of West Bohemia
Faculty of Electrical Engineering
Univerzitní 26
306 14 Plzeň
E-mail: hamar@kte.zcu.cz