September 7. - 9.9.2009
Cheb, Czech Republic

THE MULTILEVEL SETS IDEA TO SOLVE THE INVERSE PROBLEM IN ELECTRICAL IMPEDANCE TOMOGRAPHY

TOMASZ RYMARCZYK, STEFAN F. FILIPOWICZ, JAN SIKORA

Abstract: *This paper presents the applications of the level set function for identification the unknown shape of an interface motivated by Electrical Impedance Tomography (EIT). The Mumford-Shah method was proposed in the iterative algorithm. A new approach was adopted based on a continuous approximation of material coefficient distribution using modified level set methods and the finite element method. A model problem in electrical impedance tomography for the identification of unknown shapes from data in a narrow strip along the boundary of the domain is investigated.*

Key words: *Level Set Method, Inverse Problem, Electrical Impedance Tomography, Finite Element Method, MumFord-Shah model*

INTRODUCTION

The presenting method was based a numerical scheme for the identification of piecewise constant conductivity coefficient for a problem arising from electrical impedance tomography. The inverse problem is nonlinear and highly ill-posed. Several of numerical techniques with different advantages have been proposed to solve the problem. The level set idea was proposed here. The representation of the shape of the boundary and its evolution during an iterative reconstruction process is achieved by the level set method [4,5,7]. The shape derivatives of this problem involve the normal derivative of the potential along the unknown boundary. The conductivity values in different regions are determined by the finite element method [2]. Given the boundary, the potential is obtained by solving Laplace’s equation for the potential in the entire domain. The advection diffusion equation is then solved with the given boundary conditions. The extension methodology discussed earlier is used to build a velocity field through the narrow band, which is then used to update the level set function which advances the void boundary.

The idea has also been used successfully in the context of inverse problem. The pioneering work of Osher and Santosa [5] uses the level set method for an inverse problem associated with shape optimization.

Numerical algorithm is a combination of the level set method for following the evolving step edges and the finite element method for computing the velocity. The Mumford-Shah functional was extended to the electrical impedance tomography problem [3,8,9]. In addition to minimizing the objection function of the difference between the potential due to the applied current and the measured potential.

1 LEVEL SET METHOD

The level set idea, devised in Osher and Sethian [5], is known to be a powerful and versatile tool to model evolution of interfaces. The original idea behind the level set method was a simple one. Given an interface Γ in R^n of dimension one, bounding an open region Ω. It was analyzed and computed its subsequent motion under a velocity field $\vec{v}$. This velocity can depend on position, time, the geometry of the interface (e.g. its normal or its mean curvature) and the external physical conditions.

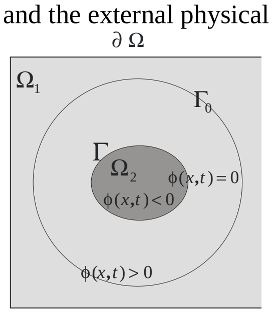


Fig.1. The representation of the level set function.

The idea is merely to define a smooth function ϕ. The motion is analyzed by the convection the ϕ values (levels) with the velocity field $\vec{V}$.

$$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = 0 \qquad (1)$$

Here $\vec{V}$ is the desired velocity on the interface, and is arbitrary elsewhere. Actually, only the normal component of v is needed $v_N = \vec{v} \cdot \frac{\nabla \phi}{|\nabla \phi|}$, so (2) becomes

$$\frac{\partial \phi}{\partial t} + v_N \cdot |\nabla \phi| = 0 \qquad (2)$$

In the level set representation, the interface, which is the set of points (x,y) satisfying $\phi(x,t) = 0$ is not explicitly given. There is only information $\phi(x_i, y_i)$ at each grid point.
When flat or steep regions complicate the determination of the contour, reinitialization is necessary. The reinitialization procedure is based by replacing by another function that has the same zero level set but behaves better. This is based on following partial differential equation:

$$\frac{\partial}{\partial t}\phi + S(\phi)(\nabla \phi - 1) = 0 \qquad (3)$$

Figure 2 presents the images segmentation by using the level set method.

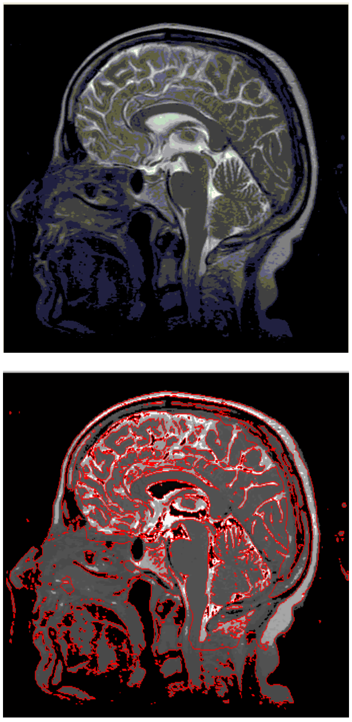


Fig.2. The image reconstruction by the level set function

2 MUMFORD-SHAH MODEL

For more than two phases was introduced the multiple level sets idea by Vese and Chan [9]. The algorithm set formulation and algorithm for the general Mumford-Shah minimization problem in image processing, to compute piecewise-smooth optimal approximations of a given image. The proposed model follows and fully generalizes works [8,9], where there was proposed an active contour model without edges based on a 2-phase segmentation and level sets. The piecewise-constant segmentation of images allows for more then two segments using a new multi-phase level set formulation and partition of the image domain.

In this paper was assume two different materials with piecewise constant conductivities γ_1 and γ_2. The problem can be easily generalized to the case where the domain contains more than two materials.
Then γ is representing following:

$$\gamma = \gamma_1 H(\phi) + \gamma_2 (1 - H(\phi)) \qquad (4)$$

where H is the Heaviside function
$H = 1 \;\; for \;\; x \geq 0 \qquad H = 0 \;\; for \;\; x < 0$

3 ELECTRICAL IMPEDANCE TOMOGRAPHY

Electrical impedance tomography is a widely investigated problem with many applications in physical and biological sciences [1,2,6]. It is well known that the inverse problem is nonlinear and highly ill-posed. Various of numerical techniques with different advantages have been proposed to solve the problem. First of all, it is give a precise mathematical model for electrical impedance tomography.

Forward problem solution in EIT consist in determining potential distribution inside the region Ω under given boundary conditions and full information about region under consideration; that is in solving Laplace's equation:

$$div(\gamma \, grad \, \varphi) = 0 \qquad (5)$$

where φ - electric potential, γ - conductivity.
Under Dirichlet boundary conditions in points adjacent to electrodes and Neumann boundary conditions on remaining part of the boundary.
Problem reduces to determine functional minimum:

$$I(\varphi) = \frac{1}{2} \int_{\Omega} \gamma |grad \, \varphi|^2 \, d\Omega \qquad (6)$$

The following functional is minimized:

$$F = 0.5 \sum_{j=1}^{p} (\mathbf{\Phi} - \mathbf{V_0})^T (\mathbf{\Phi} - \mathbf{V_0}) \qquad (7)$$

where p is the number of the projection angles.
The derivative of F with respect to γ is given by

$$\frac{\partial F}{\partial \gamma} = - \sum_{j=1}^{p} \nabla \varphi_j \nabla \overline{\varphi}_j \qquad (8)$$

Level set function is updated the following iterative scheme:

$$\phi^{k+1} = \phi^{k} - \mu \frac{\partial F}{\partial \phi} \tag{9}$$

where coefficient $\mu>0$ and

$$\frac{\partial F}{\partial \phi} = \frac{\partial F}{\partial \gamma}\frac{\partial \gamma}{\partial \phi} = \frac{\partial F}{\partial \gamma}(\gamma_1 - \gamma_2)\delta(\phi) \tag{10}$$

where δ is the Dirac delta function.
Conductivities are calculated as:

$$\gamma_1^{k+1} = \gamma_1^{k} - \beta \frac{\partial F}{\partial \gamma_1} \tag{11}$$

$$\gamma_2^{k+1} = \gamma_2^{k} - \beta \frac{\partial F}{\partial \gamma_2} \tag{12}$$

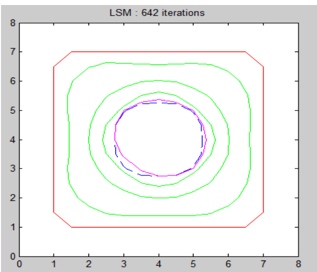
where coefficient $\beta>0$.
The delta function $\delta(\phi)$ and Heaviside function $H(\phi)$ are calculated as:

$$\delta_{\varepsilon}(\phi) = \frac{\varepsilon}{\pi(\phi^2 + \varepsilon^2)} \tag{13}$$

$$H_{\varepsilon}(\phi) = \frac{1}{\pi}tan^{-1}(\frac{\phi}{\varepsilon}) + \frac{1}{2} \tag{14}$$

where $\varepsilon>0$.

a)



b)

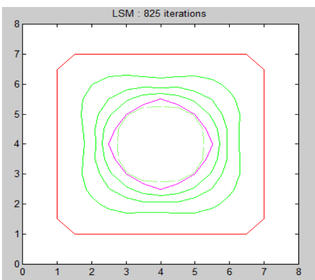


Fig.3. The image reconstruction in EIT (discretisation 16x16): a) eps=0, alfa=0.9, b) eps=0.0001, alfa=0.5.

The numerical model was inserted in the inside of the examined object (fig.3). The grid was used by16x16 elements solution, the number of those variables was equal only to 289. It can be observed that a better quality image is obtained for the higher spatial image resolution.

The level set function with the MumShah-Ford algorithm for identifying the unknown shape of an interface was used. Level set methods and the finite element method were chose for electrical impedance tomography. It is sometimes more important to recover the shape of the domains containing different materials than to recover the values for the materials. Level set methods can produce good results in identifying the sharp interfaces. In order use this approach for practical problems. The presented techniques was shown to be successful to identify the unknown boundary shapes.

4 References

[1] Filipowicz S.F., Rymarczyk T., Sikora J.: Level Set Method for Inverse Problem Solution In Electrical Impedance Tomography. Proceedings of the XII International Conference on Electrical Bioimpedance & V Electrical Impedance Tomography, p.519-522, Gdańsk, 2004.

[2] Filipowicz S.F., Rymarczyk T.: Tomografia Impedancyjna, pomiary, konstrukcje i metody tworzenia obrazu. BelStudio, Warsaw, 2003.

[3] Mumford D., Shah J.: *Optimal approximation by piecewise smooth functions and associated variational problems*. Comm. Pure Appl. Math., (42):577–685, 1989.

[4] Osher S., Fedkiw R.: *Level Set Methods and Dynamic Implicit Surfaces*. Springer, New York, 2003.

[5] Osher S., Sethian J.A.: *Fronts Propagating with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations*. Journal of Computational Physics, 79, 12-49, 1988.

[6] Rymarczyk T., Filipowicz S.F., Sikora J.: *Level set methods for an inverse problem in electrical impedance tomography*. 5th International Symposium on Process Tomography In Poland, Zakopane, 2008.

[7] Sethian J.A.: *Level Set Methods and Fast Marching Methods*. Cambridge University Press, 1999.

[8] Tai C., Chung E., Chan T.: *Electrical impedance tomography using level set representation and total variational regularization*. Journal of Computational Physics, vol. 205, no. 1, pp. 357–372, 2005.

[9] Vese L. Chan T.: *A new multiphase level set framework for image segmentation via the Mumford and Shah model*. CAM Report 01-25, UCLA Math. Dept., 2001.

MSc Tomasz Rymarczyk[2]
Prof. Stefan F. Filipowicz[1,2]
Prof. Jan Sikora[2,3]

[1] Warsaw University of Technology, Institute of the Theory of Electrical Engineering, Measurement and Information Systems, Pl. Politechniki 1, 00-661 Warszawa,
e-mail: 2xf@nov.iem.pw.edu.pl
[2] Electrotechnical Institute ul. Pożaryskiego 28, 04-703 Warszawa,
e-mail: tomasz@rymarczyk.com
[3] Lublin University of Technology, Department of Electronics, Nadbystrzycka 38d, 20-618 Lublin