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SELECTED PROBLEMS OF THE USE OF THREE-PHASE INDUCTION MOTORS WITH ONE CONDENSER IN SINGLE-PHASE AC NETS

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Abstract: *In many economic units where only single-phase voltage is available arises the need of using the induction motors of the power in the range to 5kW. Standard single-phase motors of such a power in practice do not exist. Therefore, the paper presents and analyses the possibility of replacing the typical single-phase engine with a standard three-phase induction motor for purposes of driving the devices. The three-phase motors of large power and speed range are commonly available. Their application with single-phase supply is practically feasible and usually remunerative, as a three-phase supply system with small value of ordered power is more expensive and its wiring more complicated. Three-phase motors provided with stator winding and capacitor may be used under single-phase supply for driving small-power devices (ranging to watts) and high-power ones (e.g. in railway vehicles supplied from AC contact line).*

Key words: electrical machines, induction motors, asymmetry of currents.

1. INTRODUCTION

The induction (asynchronous) machines provided with a three-phase stator winding are the most versatile in the applications and, moreover, under the operational conditions are distinguished by maximal reliability and durability. Such machines are used mainly as engines, although recently they are also used as generators, among others in combustion-engine driven generator sets. Principal fault of such machines used as the engines supplied directly from a constant frequency mains consists in small ability to control their rotational speed. Growing use of inverters enabled the application of the such engines supplied with three-phase voltage of adjusted frequency to driving various devices, often of highly complicated structure and large range of frequency variations. Unfortunately, the inverters, in spite of their common use, remain relatively expensive. Therefore, they are not used in many commonly utilized devices that, at the same time, must be inexpensive. This is the reason why in such household equipment as e.g. washing machines the formerly used induction single-phase motors are replaced with commutator motors in which stepless adjustment of rotational speed in a very large range becomes possible thanks to the use of thirystor voltage control. Such type of driving system is commonly used in washing machines and, despite its simple operating principles, it becomes a source of electromagnetic disturbances harmful for a low-voltage mains network. A thirystor including driving system introduces so-called higher harmonics to the mains, of the frequencies that must be controlled up to the 40th range, i.e. up to 2000Hz. A commutator occurring in the motor is also another source of the electromagnetic disturbance, commonly referred to as radio-electric disturbance, due to its frequency ranges. The electromagnetic disturbance caused by the commutator should be tested in the range from 0.15 to 30MHz and from 30 to 1000MHz. The disturbance of the lower band are transmitted by the mains, while in the upper band – by air. The electric power network is not used for transmitting internet, first of all due to the lower band, i.e. network transmitted electromagnetic disturbances. On the other hand, an important advantage of the induction motors supplied directly from mains consists mainly in the fact that they cause no electromagnetic disturbance and, in consequence, should be recommended as more ecological.

The induction motors with three-phase stator winding, usually of smaller power, may be easily adapted to single-phase supply which is particularly advantageous in case of small customers, e.g. household users, due to small expense related to the device implementation and electric power consumption. In many applications the cost of a motor with three-phase stator winding is lower as compared to a typical single-phase engine. Moreover, available power range of single-phase motors is usually small, not exceeding 2.5kW. The induction motors with three-phase stator winding are available from small power, e.g. 0.06kW up to very large ones. Allowable power of an engine to be used under a single-phase supply is constrained by proper regulations.

2. MOTOR OPERATION CONDITIONS UNDER A SINGLE-PHASE SUPPLY

In order to consider the motor operation conditions under a single-phase supply the method of symmetric components must be used. The same method is also used for analyzing operation of the motors with three-phase voltage supply, when the system of three-phase voltage is asymmetrical or the system includes some modifications as, for example, a single-phase braking or periodical voltage decay in one of the phases. Important advantage of the method of symmetric components consists in the fact that the equations and the way of their formulating remains unchanged irrespective of the connecting and supply systems. Another important feature consists in possible omitting of the zero-sequence symmetrical component while analyzing operation of the machine with three-phase stator winding supplied with single-phase voltage.

The idea of the operation of a motor with three-phase stator winding under single-phase voltage supply is so formulated as to impose a three-phase voltage system of the stator winding despite the single-phase supply. The three-phase voltage system in spite of single-phase supply may be simply achieved by connecting two additional impedances to the motor circuit. One of them plays a role of capacitance reactance in the whole range of the motor operation, while the other is an induction or capacitance reactance, according to the rotating velocity (slip). It may be easily shown that adjustment of these impedances enables obtaining the three-phase voltage system at the motor terminals in the whole expected load range of the engine. In such a case the motor behaves as the one supplied from a symmetrical three-phase network. At present such an artificial symmetrization of the motor voltages is no more required. On the other hand, such a system should be recommended that, in the simplest way, allows for motor operation approaching the three-phase supply, in spite of a single-phase supply. Such a condition is achieved by connecting to the stator circuit a single additional impedance of a capacitance reactance type, i.e. a single capacitor. In order to obtain possibly the simplest system, capacitance of the capacitor should be constant in the entire expected range of the motor load and so chosen as to achieve maximal factor of the motor rated power utilization. In such a system the motor behaves as a machine supplied with non-symmetrical three-phase voltage. It should be noticed that, in spite of only one capacitor, under a single load corresponding to a definite slip value for the angle $\varphi=60^o$ a symmetrical three-phase voltage system occurs at the stator winding terminals of the single-phase supplied motor.

It is known that the windings of three-phase induction motors are delta- or star-connected. Under a single-phase supply such a system of the winding connection should be used that corresponds to the rated supply voltage and rated voltages of particular connections. In case of the single-phase rated voltage of 230 or 220V the best variant would be a motor of rated voltage equal to 230V (or 220V) with delta connection. The rating plate should read 230V (delta)/400V(star). Under rated mains voltage of 400V the motor under a three-phase supply should operate in star connection. At present there are still many motors of rated voltage 220/380V. It is assumed in the present paper that the stator windings are delta-connected, i.e. the motor is supplied with the voltage of 230V or 220V. The motor is provided with a single additionally connected capacitor (Z_k) as shown in Fig.1.

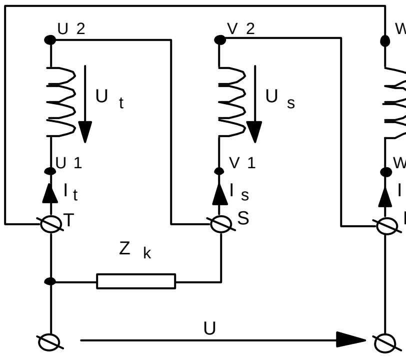

Fig.1 Diagram of the motor connection

The essence of the problem presented in the paper is selection of such capacitance of the capacitor as to obtain maximal factor of the motor rated power utilization, with the definite rated parameters of the motor remaining under the allowable limit. The capacitance may be chosen based on the optimization calculation or practically, by proper charging of the motor and finding the capacitance for which current intensities in two phase windings are equal to the rated level with the minimal level of the third current intensity. Such an approach is important, as the motor in such a system might be used in various drive systems for which the choice of capacitance is not necessarily required. If capacitance cannot be chosen in the above mentioned manner, the capacitor may be selected according to the principle specified in the conclusions from the present paper.

The optimization calculation of the capacitance selection is based on the input assumption of the criterion of possible utilization of the motor rated power. One criterion is based on the principle of such charging of the motor as to prevent the current intensity from exceeding its rated level. In case of the other criterion it is assumed that the currents of particular phases may exceed their rated levels but the sum of power loss of the windings should remain below the rated power loss. The first criterion is more severe and should be applied when the motor is used with maximal power for a longer period of time. The other admits larger use of the motor power leading to excessive temperature growth above the allowable value in the windings in which the currents exceed their rated levels. The criterion may be applied during occasional or intermittent operation of the motor. In order to determine an optimal capacitor capacitance in case of the first criterion the use of the formulas for the currents in particular phases is sufficient. Once are drawn the plots of the currents in the phases as functions of the slips, the slip value for which the currents in two phases are equal may be easily found. This is shown in Fig. 2. The current of the third phase is remarkably lower.

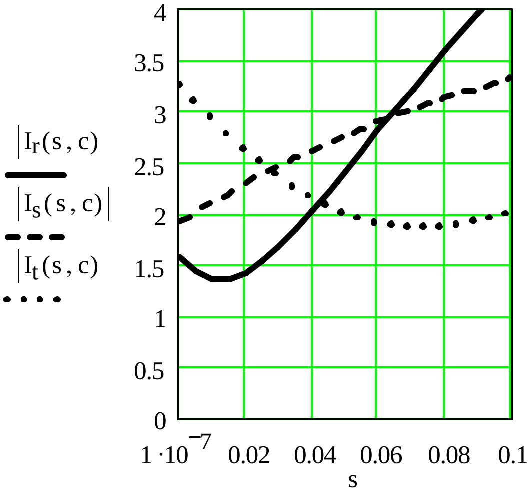

Fig. 2 Patterns of phase currents as functions of the slip for a chosen capacitance

Variation of the capacitance enables finding a variant for which the equal (rated) currents occur at maximal slip. In case of assumption of the first criterion the capacitance value may be considered as an optimal one. On the other hand, in case of the second criterion the optimal capacitance allowing for maximal utilization of the motor power should be found by the optimization calculation described in the paper.

The formulas used in the paper, resulting from the method of symmetrical components, have a so universal character that may be applied to calculation of a single-phase supplied motor, e.g. with or without working capacitor or during a single-phase braking. The case of single-phase braking is inasmuch interesting as in the entire motor operation range the positive and negative-sequence symmetric components are equal. In the other cases the values of the voltage symmetric components vary with the slip changes.

Another interesting feature of operation of a motor provided with three-phase stator winding under single-phase supply consists in occurrence of higher harmonics and their effect on the motor and capacitor operation. In this case one should answer the question whether the higher harmonic components lead to capacitor overload and whether the capacitor in the single-phase motor supply gives rise to significant growth in transmission of the higher harmonics to the network

3. BASIC RELATIONSHIPS DESCRIBING MOTOR OPERATION

While analyzing the induction machines with the method of symmetric components the basic relationships of the method should be used, including, first of all, the equations of voltage, current, and moments. The voltages and currents of particular symmetric components are interrelated by the following relationships:

$$U_1 = I_1 Z_1 \quad \text{and} \quad U_2 = I_2 Z_2$$

where Z_1 and Z_2 are machine impedances for the symmetric components, to be calculated based on an equivalent scheme including its characteristic parameters. The impedances are functions of the slip *s* (1):

$$Z_1(s) = R_s + jXr_s + \frac{Z_0\left(\frac{R'_w}{s} + jXr'_w\right)}{\frac{R'_w}{s} + Z_0 + jXr'_w}$$

$$(1)$$

$$Z_2(s) = R_s + jXr_s + \frac{jX_\mu\left(\frac{R'_w}{2-s} + jXr'_w\right)}{\frac{R'_w}{2-s} + j(X_\mu Xr'_w)}$$

where

$$Z_0 = \frac{jX_\mu R_{Fe}}{R_{Fe} + jX_\mu}$$

Example variations of both impedances as functions of the slip are shown in Fig. 3.

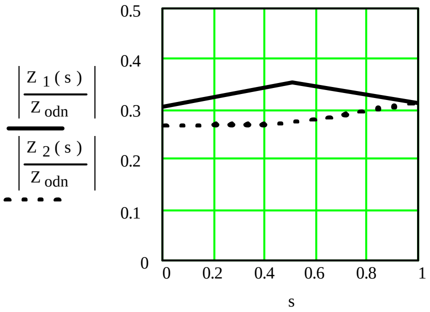

Fig. 3. Example patterns of the impedance as a function of the slip

For purposes of converting the phase values to the symmetric components and vice versa the transformation matrices are used.

In order to calculate particular components of electromagnetic power and moment the following formulas may be used (2):

$$P_{em1} = 3(|I'_{w1}|)^2 R'_w \frac{1-s}{s} \qquad M_{em1} = \frac{pP_{em1}}{2\pi f(1-s)}$$

$$(2)$$

$$P_{em2} = 3(|I'_{w2}|)^2 R'_w \frac{1-s}{s-2} \qquad M_{em2} = \frac{pP_{em2}}{2\pi f(1-s)}$$

where: p – the number of pole pairs, I_{w1} and I_{w1} – the symmetric components of the rotor current, calculated from the formula (3):

$$I'_{w1}=\frac{U_1-I_{s1}(R_s+jXr_s)}{\frac{R'_w}{s}+jXr'_w}$$

(3)

$$I'_{w2}=\frac{U_2-I_{s2}(R_s+jXr_s)}{\frac{R'_w}{2-s}+jXr'_w}$$

The resultant moment M of the motor is a difference between the positive and negative sequence components, i.e.

$$M(s)=M_{em1}(s)-M_{em2}(s)\,. \tag{4}$$

The $M_{em1}(s)$ component is proportional to $(U_1)^2$, while the $M_{em2}(2\text{-}s)$ component is proportional to $(U_2)^2$, where U_1 and U_2 are symmetric voltage components. The effective mechanical power is determined by the equation

$$P_{mu}=M\omega-\Delta p_m \tag{5}$$

where Δp_m is for mechanical losses, ω - angular velocity corresponding to s slip.

In order to analyze the motor operation at least the winding current and voltage equations should be used. The scheme shown in Fig. 1. allows for formulating the following Kirchhoff system of equations.

$$U=U_r \tag{6}$$

$$U_t(s,C)=Z_k\cdot\left(I_s(s,C)-I_t(s,C)\right) \tag{7}$$

Application of basic relationships resulting from the method of symmetrical components enables the following formulation of the equation (6)

$$U=U_1(s,C)+U_2(s,C) \tag{6'}$$

The equation (7) has been transformed with the use of the following relationships:

$$U_t(s,C)=a\cdot U_1(s,C)+a^2\cdot U_2(s,C)$$

$$U_s(s,C)=a^2U_1(s,C)+aU_2(s,C)$$

$$Z_k=-j\cdot\frac{1}{\omega\cdot C} \tag{8}$$

$$I_r(s,C)=I_{s1}(s,C)+I_{s2}(s,C)$$

$$I_s(s,C)=a^2\cdot I_1(s,C)+a\cdot I_2(s,C)$$

$$I_t(s,C)=a\cdot I_1(s,C)+a^2\cdot I_2(s,C)$$

Figure 4 presents an example pattern of the phase voltages as functions of the slip. It may be easily found that in the range of small slip values corresponding to the motor load the phase voltage are nearly equal, while for the zero approaching slip two voltages exceed the rated levels.

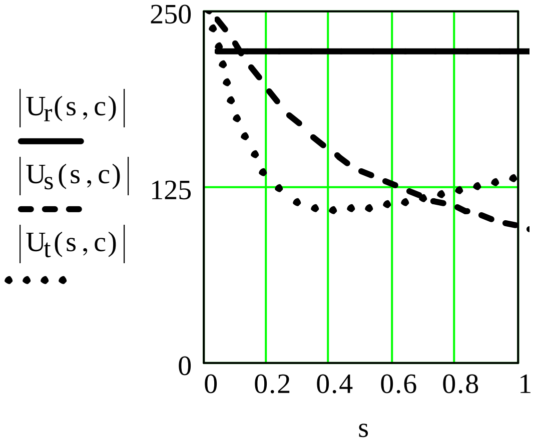

Fig. 4. The pattern of phase voltages as a function of the slip for an example capacitance value

Symmetrical components of stator currents are determined by the relationships: for the positive: and negative-sequence components

$$I_1(s,C)=\frac{U_1(s,C)}{Z_1(s)}$$

$$I_2(s,C)=\frac{U_2(s,C)}{Z_2(s)}$$

here: $Z_1(s)$ and $Z_1(s)$ are motor impedances at the slip s, for symmetrical positive- and negative sequence components, Z_k being impedance of the capacitor of C capacitance. The current intensities in particular phases are calculated with consideration of a transformation matrix of the symmetric components to the phase values. Similar procedure is performed in case of the voltage values.

Once the system of equations (6') and (7) is solved in terms of the positive sequence $U_1(s,C)$ and negative sequence $U_2(s,C)$ phase voltages, their values may be determined by the following expressions (9):

$$U_1(s,C)=\frac{U\left[-a-\frac{j}{\omega CZ_2(s)}(1-a)\right]}{(1-a)\left(1-\frac{j}{\omega CZ_2(s)}-\frac{j}{\omega CZ_1(s)}\right)}$$

(9)

$$U_2(s,C)=\frac{U\left[1-\frac{j}{\omega CZ_1(s)}(1-a)\right]}{(1-a)\left(1-\frac{j}{\omega CZ_2(s)}-\frac{j}{\omega CZ_1(s)}\right)}$$

Figure 5 presents an example pattern of the voltage symmetric components as a function of the slip.

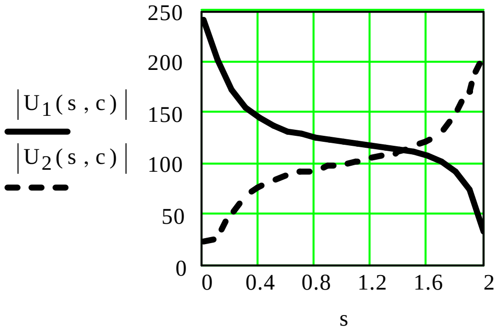

Fig. 5. Example pattern of the voltage symmetric components as a function of the slip

In case of single-phase braking neither U_1 nor U_2 depend on the slip and keep constant values in the entire frequency range.

Based on the mathematical model describing the basic values characterizing operation of the three-phase induction motor provided with a capacitor in a single-phase mains an algorithm has been developed serving for determining capacitance for two optimization criteria. Relative power values at the motor shaft and total power loss in the windings result from the following relationships (10) and (11):

$$\varepsilon(s,C)=\frac{P_m(s,C)}{P_n} \qquad (10)$$

$$\delta P(s,C)=\frac{\Delta P_S(s,C)+\Delta P_W(s,C)}{3\left[\left(\left|I_n\right|\right)^2 R_s+\left(\left|I'_{wn}\right|\right)^2 R'_w\right]} \qquad (11)$$

where:

$$\Delta P_s(s,C) = \left[\left(\left|I_{s1}(s,C)\right|\right)^2+\left(\left|I_{s2}(s,C)\right|\right)^2\right]\cdot 3\cdot R_s$$

$$\Delta P_w(s,C) = 3\cdot R'_w\cdot\left[\left(\left|I'_{w1}(s,C)\right|\right)^2+\left(\left|I'_{w2}(s,C)\right|\right)^2\right]$$

$$I'_{wn} = \frac{U-\dfrac{U}{Z_1(s_n)}\cdot\left(R_s+j\cdot Xr_s\right)}{\dfrac{R'_w}{s_n}+j\cdot Xr'_w}$$

The capacitance value ensuring optimization of motor operation under two assumptions discussed above has been determined with the use of the algorithms seeking the result in the range of possible solutions defined by the capacitance and slip variations. Each of the values is characterized by an assumed searching step decisive for the accuracy of the optimal solution and the duration of the process. The system of defining the search range of the optimal solution consists in determining maximal number of the capacitance values, the searching step, and the slip variation range. In case of the example optimization process presented in the next section these parameters are as follows (12):

$$C_j = C_{pocz} + j\cdot\Delta C$$

$$C_{pocz} = 3\cdot 10^{-5}\ \mathrm{F} \qquad \Delta C = 0.5\cdot 10^{-6}\ \mathrm{F}$$

$$j = 0..jmax \qquad jmax = 100 \qquad (12)$$

$$s_i = 10^{-6} + i\cdot 0.001$$

$$i = 0..imax \qquad imax = 100$$

Initial slip and capacitance values should be non-zero in order to avoid singularity in the calculation process.

The algorithms of the optimization processes formulated in MathCAD 6.0 PLUS take the following forms, according to the assumed optimization criteria:

- without excess of the rated current in any stator winding phase - the **Algorithm 1**.
- without excess of the rated value of total power loss in the windings- the **Algorithm 2**.

The above algorithms determine the value of the "j" index defining the capacitance value meeting the assumed optimization criterion. Values of the optimal capacitances are equal to the variables C_{joptI} and $C_{jop\delta P}$.

Algorithm 1

```
$j_{optI}$ :=
  $\varepsilon opt \leftarrow 0$
  $j_{opt} \leftarrow 0$
  for $j \in 0..jmax$
      $smax \leftarrow 10^{-6}$
      for $i \in 0..imax$
          $wyr \leftarrow 1$
          $wyr \leftarrow 0$  if  $\left|I_r(s_i, C_j)\right| \ge \left|I_n\right|$
          $wyr \leftarrow 0$  if  $\left|I_s(s_i, C_j)\right| \ge \left|I_n\right|$
          $wyr \leftarrow 0$  if  $\left|I_t(s_i, C_j)\right| \ge \left|I_n\right|$
          $smax \leftarrow s_i$  if  $wyr = 1$
      $j_{opt} \leftarrow j$  if  $\varepsilon(smax, C_j) > \varepsilon opt$
      $\varepsilon opt \leftarrow \varepsilon(smax, C_j)$  if  $\varepsilon(smax, C_j) > \varepsilon opt$
  $j_{opt}$
```

Algorithm 2

```
$j_{opt\delta P}$ :=
  $\varepsilon opt \leftarrow 0$
  $j_{opt} \leftarrow 0$
  for $j \in 0..jmax$
      $smax \leftarrow 10^{-6}$
      for $i \in 0..imax$
          $smax \leftarrow s_i$  if  $\delta P(s_i, C_j) \le 1$
      $j_{opt} \leftarrow j$  if  $\varepsilon(smax, C_j) > \varepsilon opt$
      $\varepsilon opt \leftarrow \varepsilon(smax, C_j)$  if  $\varepsilon(smax, C_j) > \varepsilon opt$
  $j_{opt}$
```

4. RESULTS OF THE EXAMPLE OPTIMIZATION PROCESS

In order to verify the method the optimal capacitance values of the capacitor have been determined for the cases presented in item No 3, for the induction motor of SZJe14b type, the power P_n=1100W, voltage U=220V, with delta-connection, rated speed n_n=1390 r.p.m., connected to single-phase 220V network according to the diagram of Fig. 1.

Parameters of an equivalent scheme determined with conventional methods amounted to: R_s=6.7 Ω, R'_w=6.2 Ω, Xr_s=Xr'_w=8.7 Ω, X_μ=103.3Ω, R_{Fe}=1320Ω.

Moreover, the value of mechanical loss has been determined, that for idle run amounts to ΔP_m=10.5W.

The following values of optimal stator capacitance and limit values of power and slip have been obtained:

- without excess of the rated current in any stator winding phase (13):

$$C_{joptI} = 6.2\cdot 10^{-5}\,\mathrm{F},\ P_m(s\max I_{joptI}, C_{joptI}) = 781.3\,\mathrm{W}$$

$$s\max I_{joptI} = 0.048 \qquad (13)$$

- without excess of the rated value of total power loss in the windings (14):

$$C_{jopt\delta P} = 7.65\cdot 10^{-5}\,\mathrm{F}$$

$$P_m(s\max \delta P_{jopt\delta P}, C_{jopt\delta P}) = 975.8\,\mathrm{W} \qquad (14)$$

$$s\max \delta P_{jopt\delta P} = 0.059$$

The courses of the optimization processes in case of both criteria are shown in Fig. 6 by the plots of the capacitor capacitance, the relative power at the motor shaft for maximal slip causing no excess of the assumed optimization criterion (the variable smax of the algorithm).

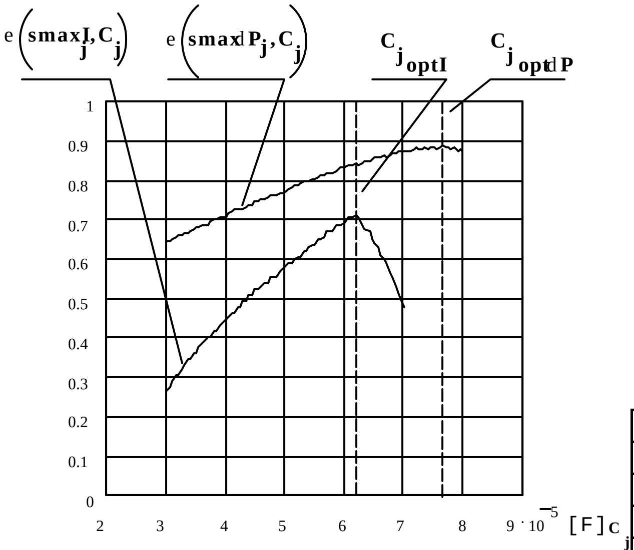

Fig. 6. The plot of relative variation of the shaft power for the slip causing no excess of the assumed optimization criterion, as a function of capacitance

5. EXPERIMENTAL VERIFICATION

The optimization results have been experimentally verified based on the results of charging test of the motor referred to in item No 4 during its cooperation with the capacitors of the capacitance values approaching the ones obtained in the optimization process: C = 62 μF, nearly equal to optimal capacitance for the case without excess of the rated current in any stator winding phase, and C = 70 μF, slightly differing from the optimal capacitance for the case without excess of the rated value of total power loss in the windings.

In result of the tests the limit powers at smaxI and smaxδP slips have been obtained, amounting to 749W and 936W, respectively. Results of the measurements are shown in Fig. 7 by the circles connected with thin lines. On the other hand, the calculation results achieved based on the model applied for the optimization purposes are denoted with thick lines, without points.

According to the assumed variant the optimal capacitor capacity is contained in the range from 50 to 72 μF per 1kW of the motor power. The rated power utilization factor is usually below 68 to 72 per cent.

The selected measurement results of the voltage and current harmonic components are specified in Table 1, where U_z is the supply voltage; U_{bk} – the capacitor voltage, I_{za} – the supply current, and I_k – the capacitor current. All the values are in percent scale.

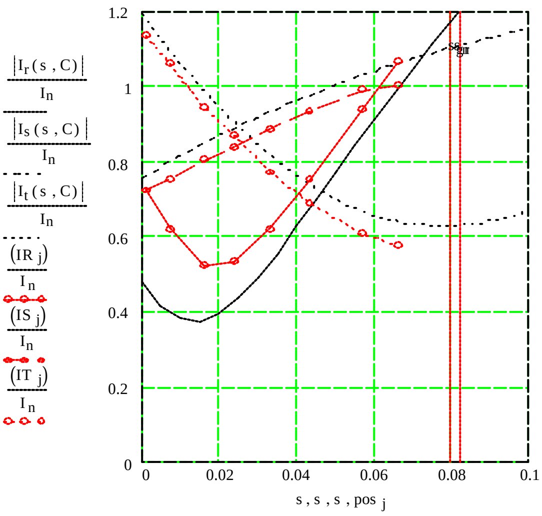

Fig. 7. The phase currents and motor shaft power as functions of the slip during the loading test

Table 1. Voltage and current harmonic components

v	U_z	U_{hk}	I_{za}	I_k
1	1.0	1.0	1.0	1.0
3	-	0.017	0.040	0.051
5	0.015	0.004	0.025	0.028
7	0.004	-	0.013	0.006
9	0.005	-	-	-

6. SUMMARY AND CONCLUSIONS

An induction motor with three-phase stator winding may operate in a single-phase network. Connection of a properly selected capacitor enables its operation even with the power reaching up to 70 percent of the motor rated power. In operational conditions the motor rated power is usually restrained in order to prevent exceeding the current allowable with regard to the applied protection. The capacity of the condenser can be estimated using the rule of (60 – 70) μF for 0.8 of the motors nominal power. In the considered circuit, where the capacitor is connected in series with the motor windings, transmission of higher harmonics through the capacitor is constrained by the motor inductive reactance. Therefore, no risk arises of overload of the capacitor with higher harmonics as in case of a capacitor used for boosting the power factor, when for the 5th harmonic the value of 0.075 and for the 7th harmonic – the value 0.037 are obtained. In the operational practice the capacitor capacitance may be experimentally selected provided that the capacitors of small capacitance are available. As an optimal capacitance may be considered such its value for which further growth of the load gives equal currents in two phases.

5. REFERENCES

[1] Stein Z. Zagadnienia stanów niesymetrycznych trójfazowych maszyn indukcyjnych. WUPP, Rozprawy, Poznan, 1977.

[2] Stein Z. Stolpe M. Zielinska M. Optymalizacja pojemnosci kondensatora przy pracy trójfazowego silnika indukcyjnego z kondensatorem w sieci jednofazowej. Praca wykonana w Politechnice Poznanskiej ramach TB – 42 – 572 / 98 - DS

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