High Voltage Insulator for Outdoor Use Solved by Adaptive Finite Element Method

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Abstract **Problems of automatic adaptivity in finite element method of higher orders of accuracy are discussed. Particular attention is paid to *hp*-adaptivity that exhibits the highest level of flexibility and extremely fast convergence. The theoretical aspects are illustrated by an example of a high voltage insulator for outdoor use solved numerically by our own codes Hermes and Agros2D. The results are compared with data obtained by two commercial codes Comsol Multiphysics and Quickfield.**

Keywords **curvilinear elements, automatic adaptivity, *hp*-FEM, high voltage insulator.**

I. INTRODUCTION

Insulators are devices that are used for effective separation of the power lines and ground structures. Another purpose of these devices is the mechanical support for power lines. Insulators used for high-voltage power transmission are mostly made of glass, ceramic or polymer materials and are covered with a smooth glaze that allows sheding water [1]-[2].

Modeling of distribution of electric field is not a common practice in the design of high voltage insulators. But knowledge of the electric field distribution on the surface of the insulator is crucial for the design of these devices. Generally used threshold value for the maximum electric field is 4.5 kV/cm.

II. MATHEMATICAL MODEL

From the physical viewpoint, the task is quite simple. Its continuous mathematical model is described by the equation in the form

$$\mathrm{div}\ (\varepsilon\ \mathrm{grad}\ \varphi) = 0\,, \tag{1}$$

where ε denotes the relative permittivity of the material. The boundary conditions are given by the prescribed potentials φ of the electrodes, Dirichlet conditions according to Fig. 1 and Neumann condition $D_\mathrm{n} = 0$ at a sufficient distance of the system elsewhere.

III. NUMERICAL SOLUTION

The numerical solution of the problem is realized by a fully adaptive higher-order finite element method whose algorithms are implemented into code Agros2D [4] based on the Hermes library [3]. Both codes have been developed in our group for almost ten years.

The codes written in C++ are intended for the monolithic numerical solution of systems of generally nonlinear and nonstationary second-order partial differential equations whose principal purpose is hard-coupled modeling of complex physical problems. While Hermes is a library containing the most advanced procedures and algorithms for the numerical processing of the task solved, Agros2D represents a powerful preprocessor and postprocessor. Both codes are freely distributable under the GNU General Public License.

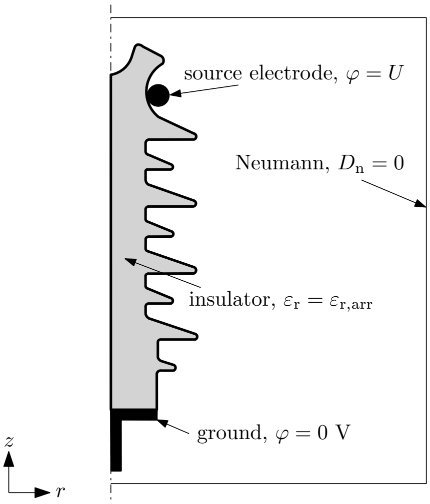

Fig. 1. Basic arrangement of the considered insulator

The numerical solution of this model is then carried out by a fully adaptive higher-order finite element method (*hp*-FEM) [5] and [6]. It represents a modern version of the finite element method, which allows combining finite elements of variable size (*h*) and polynomial degree (*p*) in order to obtain fast exponential convergence of the solution.

IV. ILLUSTRATIVE EXAMPLE

Consider a typical high-voltage insulator in Fig. 2 with 8 sheds of length 36 cm with two electrodes. The upper one carries potential $\varphi = 36$ kV, the lower one is grounded, so that its potential $\varphi = 0$ kV. High-voltage insulator is made of ceramic with relative permittivity $\varepsilon_\mathrm{r} = 6$ and is located in the air. The arrangement is considered axisymmetric and the task is to find the distribution of electric potential φ and electric field $\boldsymbol{E}$ in the system.

The problem was solved using several codes: while Comsol Multiphysics 4.2 and QuickField are professional commercial codes, FEMM 4.2 and Agros2D are freely distributable applications.

Fig. 3 shows some results obtained using Agros2D. Its left part depicts the mesh (grey lines show the original

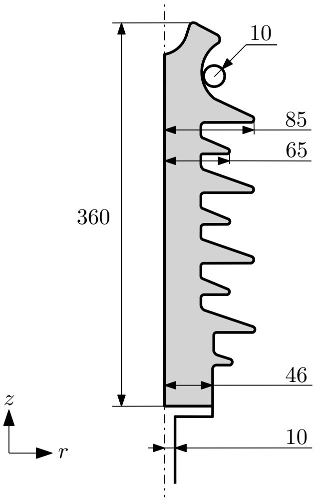

Fig. 2. The investigated high-voltage insulator

mesh while the dark lines the final mesh after 12 steps of adaptivity and the numbers give the orders of the corresponding polynomials) and right part shows the distribution of potential φ in the vicinity of the voltage insulator.

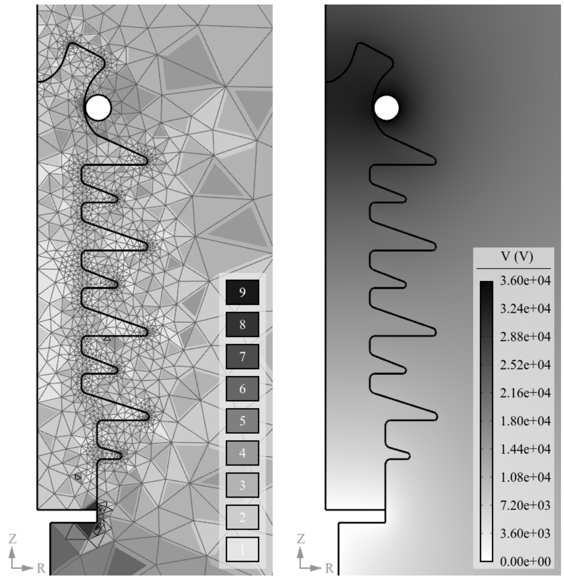

Fig. 3. Left – original rough mesh (grey lines) and final mesh (dark lines), right – distribution of potential (code Agros2D – *hp*-adaptivity, p = 1, number of DOFs 5475, relative error of solution η = 0.852 %)

Fig. 4 shows the comparison of cases using curvilinear and standard elements. To achieve the desired accuracy $\eta = 0.1$ % we need 8600 number of DOFs for normal elements while only 5100 number of DOFs with curvilinear elements.

Important charts obtained from the mentioned commercial and freely distributable codes are depicted in Fig. 5. Compared is the value of the total electrostatic energy W_e in the system as a function of the number of DOFs. Applications FEMM and QuickField only work with linear elements without adaptivity and the results obviously converge very slowly. Faster is the convergence in Comsol Multiphysics. On the other hand, this code does not support the hanging nodes, so that much more elements are needed.

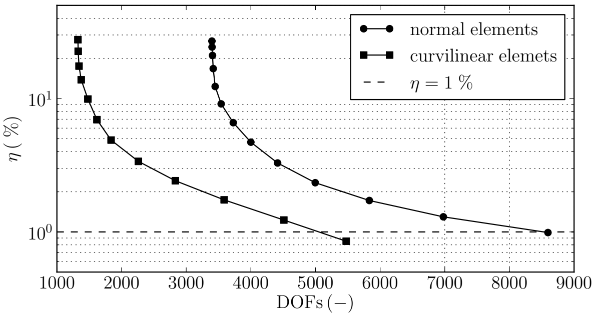

Fig. 4. Dependence of relative error η of the system on the number of DOFs

Finally, Agros2D with adaptivity starting from a rough mesh converges extremely fast, with a substantially lower number of DOFs. Thus, usage of Agros2D is much more effective.

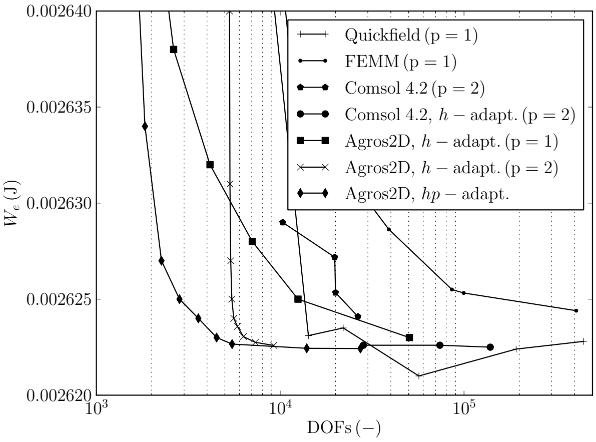

Fig. 5. Dependence of total energy W_{e} of the system on the number of DOFs

V. ACKNOWLEDGEMENTS

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