Local Monte Carlo estimation methods in the solution of global illumination equation

Victor S. Zheltov
Light Engineering Department Moscow Power Engineering Institute, Moscow, Russia
zheltov@list.ru

Vladimir P. Budak,
Light Engineering Department Moscow Power Engineering Institute, Moscow, Russia
budakvp@mpei.ru

ABSTRACT

In this article, we consider local estimations of the Monte Carlo method for solving the equation of the global illumination. The local estimations allow directly calculating the luminance at a predetermined point, in a given direction for an arbitrary bidirectional reflectance distribution function (BRDF). Thus, there is no need to construct the map of the illumination. Thereby it is much more effective than direct modeling or the method of finite element. The use in lighting calculations of the object described by the spherical harmonics is also discussed in the article.

Keywords

Local Estimation, Double Local Estimation, Global Illumination, Monte Carlo

1. INTRODUCTION

Visualization of 3D scenes is produced on the basis of solving the global illumination equation, which represents Fredholm integral equation of the second kind [Bud00]:

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{\pi}\int L(\mathbf{r},\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}},\hat{\mathbf{l}}')\left|(\hat{\mathbf{N}},\hat{\mathbf{l}}')\right| d\hat{\mathbf{l}}', \quad (1)$$

where $L(\mathbf{r},\hat{\mathbf{l}})$ is the radiance at the point $\mathbf{r}$ in the direction $\hat{\mathbf{l}}$, $\sigma(\mathbf{r};\hat{\mathbf{l}},\hat{\mathbf{l}}')$ is the bidirectional scattering distribution function (reflectance or transmittance), L_0 is the radiance of the direct radiation straight near the sources, $\hat{\mathbf{N}}$ is the normal at the point $\mathbf{r}$ to the surface of the scene. The equation (1) in a slightly different form called the rendering equation was originally obtained by J.Kajiya, but further we will use it in the form (1).

The global illumination equation (1) does not have the analytical solution, and the numerical simulation methods are used for its solutions. It is possible to pick out some guidelines among them: ray tracing, direct Monte Carlo simulation and the finite element method.

Ray tracing has been widespread and included in such well-known simulation program like 3D Studio Max and Maya. One can differentiate between forward and backward tracing, where, respectively, the rays are traced from a light source, or from the receivers (the camera).

In the case of direct simulation, the scene is divided into elements in which photons are counted. As a result, the illumination map is constructed. This approach is associated with the complexity of the grid and the formation of large memory consumption.

Solving the equation of global illumination the finite element method is also used, which got its name radiosity in the theory of the global illumination. The method is based on the assumption that all elements of the scene are diffuse reflection, and then the equation (1) can be written as

$$M(\mathbf{r}) = M_0(\mathbf{r}) + \frac{\sigma}{\pi}\int_{\Sigma} M(\mathbf{r}')F(\mathbf{r},\mathbf{r}')\Theta(\mathbf{r},\mathbf{r}')d^2r', \quad (2)$$

where $M(\mathbf{r})$ is the radiant exitance at the surface point $\mathbf{r}$, $M_0(\mathbf{r})$ is radiant exitance at the point $\mathbf{r}$, emitted straight from the light source, $F = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}),(\mathbf{r}-\mathbf{r}'))\right|\left|(\hat{\mathbf{N}}(\mathbf{r}'),(\mathbf{r}-\mathbf{r}'))\right|}{(\mathbf{r}-\mathbf{r}')^4}$ is the elementary form-factor, $\Theta(\mathbf{r},\mathbf{r}')$ is the visibility function of element $d^2\mathbf{r}'$ from point $\mathbf{r}$, $\hat{\mathbf{N}}(\mathbf{r})$ is a normal at the point $\mathbf{r}$ to the surface of the scene.

In this paper, we propose to use the local Monte Carlo estimation method, well known for solving the radiative transfer equation in the atmospheric optics [Mar80] [EM76]. In addition, the ability of visualizing three-dimensional (3D) objects defined in the spectrum of spherical harmonics is also analyzed in the paper.

2. LOCAL ESTIMATION

Equation (1) is not suitable for statistical modeling, as a required function is in the integrand at the point $\mathbf{r}'$, but the solution is found at the point $\mathbf{r}$. Besides, the variables $\mathbf{r}'$ and $\hat{\mathbf{l}}'$ are not independent, but related by

 ISBN 978-80-86943-71-8

$$\hat{\mathbf{l}}' = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}. \quad (3)$$

Accordingly, we can rewrite the equation (1) in the form of the integral over the volume

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \\ + \frac{1}{\pi}\int L(\mathbf{r}',\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}},\hat{\mathbf{l}}')\delta\left(\hat{\mathbf{l}}' - \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\right)\frac{|(\hat{\mathbf{N}},\hat{\mathbf{l}})||(\hat{\mathbf{N}}',\hat{\mathbf{l}}')|}{(\mathbf{r}-\mathbf{r}')^2}d^3r'. \quad (4)$$

The kernel of the global illumination equation (4) contains a δ-function, which determines the singularity of the radiance angular distribution, and makes it impossible to simulate the radiance by Monte Carlo methods. The singularity can be eliminated by the integration over the space that for the diffuse reflection is equivalent to equation (2). As a result, the estimation I for $L(\mathbf{r},\hat{\mathbf{l}})$ takes the form

$$I = \mathsf{M}\sum_{n=0}^{\infty} Q_n k(\mathbf{r},\mathbf{r}_n), \quad (5)$$

where $k(\mathbf{r},\mathbf{r}_n)$ is the kernel of equation (4), Q_n is the statistical weight of the Markov chain, and M is the expectation operator of the random ray trajectories.

Markov chain models the sequential of random rays, wandering around the scene. Statistical weight of the first ray Q_1 is determined by the initial radiance of the light source $L_0(\mathbf{r},\hat{\mathbf{l}})$. In this case $k(\mathbf{r},\mathbf{r}_n)$ determines the probability of transition from the point of the Markov chain $\mathbf{r}_n$ to the given point **r.** [Mar80] [EM76]. From all the nodes of the trajectory $\mathbf{r}_n$, where its intersection with the surface of the scene occurred, the contribution to the illumination at the point **r** is calculated on the basis of the kernel $k(\mathbf{r},\mathbf{r}_n)$ in equation (5). Thus, all the probability distribution must satisfy the normality condition.

The values $L_0(\mathbf{r},\hat{\mathbf{l}})$ and $k(\mathbf{r},\mathbf{r}_n)$ will determine the statistical weights of the Markov chain Q_n. In the case of the diffuse reflectance model, each subsequent statistical weight will be multiplied by the coefficient of reflection.

Equation (5) has been called the local estimate of the Monte-Carlo [Mar80] [EM76]. It allows evaluating the illumination at a given position on the scene. Thus, to calculate the illumination at a given point **r** it is necessary to construct the Markov chain and for every act of reflection to calculate the kernel $k(\mathbf{r},\mathbf{r}_n)$ for all given points. The mathematical expectation of the obtained value is equal to the illumination. Figure 1 shows the general scheme for constructing the Markov chain and illumination calculations using a local estimation at a given point.

The Markov chain is a sequence of random events with a finite or countable number of outcomes, characterized by the property that, loosely speaking, for the fixed present the future is independent on the past. In this case, the chain is constructed from a light source. Furthermore, the initial weight of the ray is set to the light source radiance, taking into account the normalization by the flow. After that, the point of intersection of a ray with an element of the scene is founded, and the weight is multiplied by the coefficient of reflection. Then the kernel of equation (4) is calculated for each of the observable points, multiplied by the weight of the ray and added to previous values. Whereupon the statistical sampling of the ray is performed in accordance with the diffuse reflection, and then the following intersection is sought. The process is repeated until either the ray leaves the scene or its weight falls below a predetermined threshold. Sampling and averaging the number of rays, one obtains values of the illuminance at the observable points. We emphasize that the local estimation allows calculating the value of the illumination at several points by one ray. This is a fundamental difference of the local estimation from the direct simulation and ray tracing.

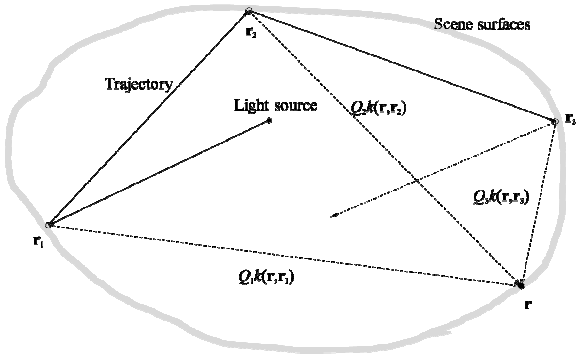

Figure 1. The scheme for constructing a Markov chain and local estimation calculation: solid line – the ray trajectory, dashed line – the local estimation, dashed-dot line – the trajectory continuation.

3. DOUBLE LOCAL ESTIMATION

Mathematics of Double Local Estimation

Global illumination equation can be written in the operator form

$$L = L_0 + \mathbf{K}L. \quad (6)$$

The solution of this equation can be represented in the form of a Neumann series, which allows performing the following transformations

$$L = \sum_{n=0}^{\infty}\mathbf{K}^n L_0 = L_0 + \mathbf{K}L_0 + \sum_{n=2}^{\infty}\mathbf{K}^n L_0 = \\ = L_0 + \mathbf{K}L_0 + \mathbf{K}^2\sum_{n=0}^{\infty}\mathbf{K}^n L_0 = L_0 + \mathbf{K}L_0 + \mathbf{K}^2 L, \quad (7)$$

that in an analytical form that takes the view

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \mathbf{K}L_0 + \\ + \frac{1}{\pi^2}\int_{(\Sigma)} \sigma(\mathbf{r};\hat{\mathbf{l}}',\hat{\mathbf{l}})\delta\left(\hat{\mathbf{l}}' - \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\right)F(\mathbf{r},\mathbf{r}')\times \\ \times\int_{(\Sigma)} L(\mathbf{r}'',\hat{\mathbf{l}}'')\sigma(\mathbf{r}';\hat{\mathbf{l}}'',\hat{\mathbf{l}}')\delta\left(\hat{\mathbf{l}}'' - \frac{\mathbf{r}'-\mathbf{r}''}{|\mathbf{r}'-\mathbf{r}''|}\right)F(\mathbf{r}',\mathbf{r}'')d^3r''d^3r' \quad (8)$$

Local estimation I corresponding to (8) may be called a double local estimation [Mar80] [EM76] and will have the form

$$I = \mathsf{M}\sum_{n=0}^{\infty} Q_n k(\mathbf{r},\hat{\mathbf{l}},\mathbf{r}_n,\hat{\mathbf{l}}_n)\,, \quad (9)$$

where

$$k(\mathbf{r},\hat{\mathbf{l}},\mathbf{r}_n,\hat{\mathbf{l}}_n) = \frac{1}{\pi^2}\sigma(\mathbf{r}_n;\hat{\mathbf{l}}',\hat{\mathbf{l}}_n)\sigma(\mathbf{r};\hat{\mathbf{l}}_n,\hat{\mathbf{l}})F(\mathbf{r}_n,\mathbf{r}'). \quad (10)$$

In the expression (10) the angular singularity is lost as a result of integration, and the independent variables $\mathbf{r}$, $\hat{\mathbf{l}}$, $\mathbf{r}_n$, $\hat{\mathbf{l}}_n$, $\mathbf{r}'$, $\hat{\mathbf{l}}'$ correspond to the geometry of the ray propagation [EM76].

Therefore, the double local estimation allows straightly simulating the global illumination equation (4) and calculating the radiance at a given point in a given direction for the reflection order greater than one.

First Order of Double Local Estimation

The first order of reflection is contained in the item $\mathbf{K}L_0$ of the equation (7) and can be calculated straightly. Let's consider the calculation of the first order of the reflection brightness. In our implementation, we considered the isotropic spherical light sources, and Phong reflection model [Ph75] was used as a model for the reflection. The general scheme for calculating the radiance of the first order of reflection is shown in Figure 2.

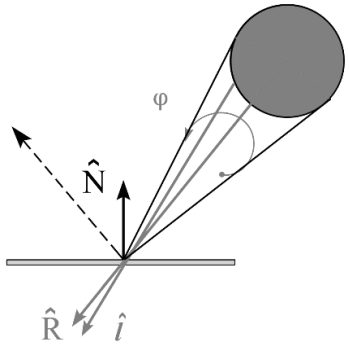

Figure 2. The scheme of sampling the ray from the source in the first order of reflection.

For the calculation of the first order, it is necessary to integrate over the source from the point where the reflection occurs. To evaluate the integral the Monte Carlo method was used. In the case of an isotropic spherical source, one can write the formula for the spherical triangle

$$\left|(\hat{\mathbf{l}},\hat{\mathbf{N}})\right| = \mu\nu + \sqrt{(1-\mu^2)(1-\nu^2)}\cos\varphi\,, \quad (11)$$

where $\mu = (\hat{\mathbf{l}},\hat{\mathbf{R}}) \equiv \cos\vartheta$, $\nu = (\hat{\mathbf{R}},\hat{\mathbf{N}})$.

The ray sampling may be performed in a solid angle Ω, circumscribed around the sphere. Besides φ is sampled equiprobably from 0 to 2π, and the angle ϑ equiprobably from 0 to $\sin\vartheta_{\max} = a/R$, where a is a sphere radius, R is a distance from its center to the target point.

Then the normalizing condition of the probability density by ϑ takes the form:

$$C_0\int_0^{\vartheta_{\max}} \sin\vartheta d\vartheta = 1 \Rightarrow C_0 = \frac{1}{1-\cos\vartheta_{\max}} = \frac{1}{1-\sqrt{1-\frac{a^2}{R^2}}}\,, \quad (12)$$

and the formula for μ sampling will be

$$\frac{1}{1-\cos\vartheta_{\max}}\int_{\mu}^{1} dx = \alpha \Rightarrow \mu = 1-\alpha(1-\cos\vartheta_{\max})\,. \quad (13)$$

Accordingly, the integral for the normalization condition, multiplied by the inverse C_0:

$$C_0^{-1} = 1-\sqrt{1-\frac{a^2}{R^2}}\,. \quad (14)$$

Thus, sampling the ray in the direction to the source, one obtains the value of the radiance at the point of source. Then using the Phong formula one obtains the reflection coefficient in the direction of interest. Multiplying it by the radiance and taking into account the cosine in the kernel of equation (1) one accumulates statistics. After that, one averages and multiplies result by the normalization coefficient (14). The resulting value is the radiance of the first order of reflection. [BZK11]

Highest Orders of Double Local Estimation

Consider the calculation of the higher orders radiance by the method of the double local Monte Carlo estimation. In general, it is similar to the simulation of local estimation, but there are some differences. Figure 3 shows a schematic diagram of the calculation of the radiance by the double local estimation.

To calculate the radiance at a given point in a given direction $(\mathbf{r},\hat{\mathbf{l}})$ one finds a point of intersection with an element of the scene in the opposite direction $\mathbf{r}'$. This point is called sub-point.

Sampling the ray emission from the source that in case of an isotropic spherical source is not complicated, one finds a point of its intersection with an element of the scene. The initial weight of the ray

(Markov chain) will be equal to the value of the radiance at the sampling point on the source. In the double local estimation, similar to a local estimation, it is necessary to compute the two kernels of the global illumination equation. The first kernel describes the transition from the point of intersection of the ray into the sub-point, and the second one describes the transition from the sub-point to the observed point. Therefore, it is necessary to take into account the normalization by Phong reflection model.

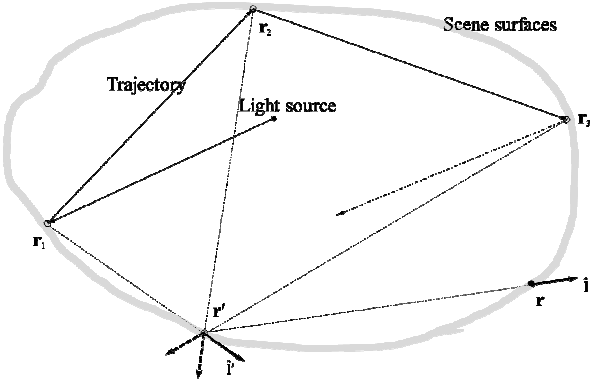

Figure 3. The scheme for constructing the Markov chain and the calculation of the double local estimation. Legend is the same as on Figure 1. Direction fan of $\hat{\mathbf{l}}'$ at the point $\mathbf{r}'$ corresponds to different points $\mathbf{r}_n$.

Similarly with the local estimation one accumulates statistics. After averaging, one obtains the radiance at the observed point in a given direction from the order of reflection greater than one.

4. VALIDATION

The ideal option for the comparison of any numerical method is the presence of exact analytical solutions for the special cases. There are just two analytical solutions for the equation (2). The first of them is the photometric sphere. However, to compare the accuracy of the method it is poorly suited due to the full symmetry. The second special case is the illuminance distribution in the scene of two infinite parallel planes and a point light source between them has been named the Sobolev problem [Sob44]. The solution deriving in Sobolev's article is not convenient to the direct calculations. Let's consider the solution of Sobolev problem that allows obtaining the result in a more acceptable analytical form.

Analyzing the Sobolev problem one transforms the equation (2) into a system of two integral equations. Each of the equations describes the distribution of illumination on one of the planes. The corresponding equation for the first plane takes the form (for the second one will be a similar expression with other indices):

$$E_1(\mathbf{r}) = \frac{\rho_2}{\pi}\int \frac{E_2(\mathbf{r}')d^2r'}{\left[1+(\mathbf{r}-\mathbf{r}')^2\right]^2} + \frac{h_1}{(h_1^2+\mathbf{r}^2)^{3/2}}, \quad (15)$$

where $E_i(\mathbf{r})$ is the illuminance of the i-th plane (i=1,2), $\mathbf{r}$ is the radius-vector of the investigated point from the projection of a point source in the i-th plane, ρ_i is the reflection coefficient of the i-th plane, h_i is the distance from the source to the i-th plane. Let's suppose the source intensity equal to 1 and $h_1+h_2=1$.

Equations form a system of integral equations of convolution type. To solve the system of equations one should perform the Fourier transform. After some analytical transformations and performing the inverse Fourier transform one can obtain the final expression for the illumination distribution on each plane:

$$E_1(r) = \frac{h_1}{(h_1^2+r^2)^{3/2}} + \\ +\rho_2\int_0^\infty \frac{\mathrm{e}^{-h_1 k}\rho_1 k\,\mathrm{K}_1(k) + \mathrm{e}^{-h_2 k}}{1-\rho_1\rho_2 k^2\,\mathrm{K}_1^2(k)}\mathrm{K}_1(k)\mathrm{J}_0(kr)k^2dk, \quad (16)$$

where $\mathrm{K}_1(k)$ is the modified Bessel function of imaginary argument or the MacDonald function of the first order, J_0 is the zero-order Bessel function.

Equation (16) is convenient for computer calculations and allows comparing the mathematical modeling techniques. Figure 4 shows the comparison of the illumination distribution received by the method of the local Monte Carlo estimation, by the double local estimation and the exact solution of Sobolev problem (16).

The figure clearly shows a surge in the bottom of the chart of the double local estimation. It is connected with the fact that in our implementation for each calculated point in the double local estimation different packages rays are used. By increasing the ray number, these artifacts disappear.

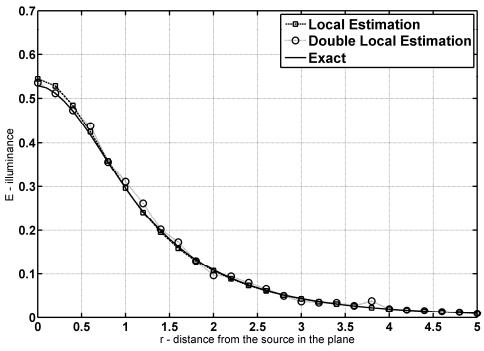

Figure 4. Illuminance distribution in Sobolev problem: $h_1=h_2=0.5$, and coefficients of reflection $\rho_1 = \rho_2 = 0.5$.

In the calculation by the local estimation of 2000 rays were used, and the computation time was less

 ISBN 978-80-86943-71-8

than 1 second by a computer with the processor AMD Athlon 64 X2 5200.

5. SPECTRAL REPRESENTATION OF 3D OBJECTS

Spherical Harmonics

Standard representation of 3D objects is a mesh representation in which objects are described by the set of vertices connected in the faces. This representation is universal and can describe any object. However, it can't precisely reproduce precisely many of the objects, and in the case of significant impact of error on the result the solid modeling of objects can be used, when objects are described analytically. SolidWorks and TracePro are examples of the most popular programs of such an approach.

One of the promising directions in the representation of 3D objects is the usage of object expansion in the basis of spherical functions [MCA06]. Such a description of the objects allows controlling the visualization quality of objects. So objects that are far away from the camera can be rendered at the visualization with low quality. From the standpoint of photometry, the significant advantage of this approach is the continuous reproduction of normal without approximation. Let's consider the mathematical formalism underlying the spherical harmonics.

Spherical harmonic $\left\{ \mathrm{Y}_k^m(\theta,\varphi) : |m| \le k \right\}$ is the special function, defined on the unit sphere

$$\mathrm{Y}_k^m(\theta,\varphi) = \sqrt{\frac{2k+1}{4\pi}}\,\mathrm{Q}_k^m(\cos\theta)\,\mathrm{e}^{im\varphi}, \qquad (17)$$

where θ is the zenith angle [0 π], φ is the azimuthal angle [0 2π],

$$\mathrm{Q}_k^m(\mu) = \sqrt{\frac{(k-m)!}{(k+m)!}}\,\mathrm{P}_k^m(\mu). \qquad (18)$$

are the semi-normalized Schmidt polynomials, $\mathrm{P}_k^m(\cos\theta)$ are the associated Legendre polynomials

Spherical harmonics are orthogonal on the unit sphere:

$$\int_0^{2\pi}\int_0^{\pi} \mathrm{Y}_k^m(\theta,\varphi)\,\mathrm{Y}_{k'}^{m'}(\theta,\varphi)\sin\theta d\theta\varphi = \delta_{k,k'}\delta_{m,m'}, \qquad (19)$$

where δ is Kronecker delta symbol.

The system of spherical functions is complete and any twice continuously differentiable function defined on the sphere can be expanded in the spectrum of spherical harmonics [TS76]

$$f(\theta,\varphi) = \sum_{k=0}^{\infty}\sum_{m=-k}^{k} A_{km}\,\mathrm{P}_k^m(\cos\theta)\,\mathrm{e}^{im\varphi}, \qquad (20)$$

where A_{km} are Fourier coefficients defined as

$$A_{km} = \int_0^{2\pi}\int_0^{\pi} f(\theta,\varphi)\,\mathrm{Y}_k^m(\cos\theta)\sin\theta d\theta d\varphi .$$

Note that the use of the semi-normalized Schmidt polynomials eliminates the need to calculate factorials in the spectral representation of the objects that significantly improves computing performance. Therefore, any object uniquely defined on the sphere relative to a certain point can be expanded in spherical harmonics.

Visualization Spectral Objects

For rendering 3D scenes by any methods, it is always necessary to solve two basic problems: finding the intersection point of the ray with the object and the normal at a given point on the 3D object. In the case of the object representation by a grid, these tasks are successfully solved and optimized. Let's consider these problems in the case of object representation in the basis of spherical functions.

Normal to the surface described by the function $U(r,\theta,\varphi)$, given in spherical coordinates, is equal to the gradient at this point

$$gradU = \frac{\partial U}{\partial r}\hat{\mathbf{e}}_r + \frac{1}{r}\frac{\partial U}{\partial\theta}\hat{\mathbf{e}}_0 + \frac{1}{r\sin\theta}\frac{\partial U}{\partial\varphi}\hat{\mathbf{e}}_\varphi. \qquad (21)$$

Therefore, it is necessary to find the partial derivatives, and then to transfer to Cartesian coordinate system. Finding derivatives with respect to r and φ is not difficult. In turn, the derivative with respect to θ requires differentiating the polynomials Schmidt. Using the known relations one can eventually obtain the following expression:

$$gradU =$$
$$= \hat{\mathbf{i}}\left(-\rho\sin^2\theta\cos\varphi + \sin\theta\cos\theta\cos\varphi\frac{\partial U}{\partial\theta} - \sin\varphi\frac{\partial U}{\partial\varphi}\right) +$$
$$+\hat{\mathbf{j}}\left(-\rho\sin^2\theta\sin\varphi + \sin\theta\cos\theta\sin\varphi\frac{\partial U}{\partial\theta} + \cos\varphi\frac{\partial U}{\partial\varphi}\right) +$$
$$+\hat{\mathbf{k}}\left(-\rho\sin\theta\cos\theta - \sin^2\theta\frac{\partial U}{\partial\theta}\right). \qquad (22)$$

Finding the intersection of the ray with the object specified by spherical harmonics is also not a trivial task. Consider an object defined by the spherical harmonics and located in the center of a Cartesian coordinate system, and the ray from point $\mathbf{r}_0$ in the direction $\hat{\mathbf{l}}$ defined in a vector form

$$\mathbf{r}(\xi) = \mathbf{r}_0 + \xi\hat{\mathbf{l}}. \qquad (23)$$

Cosine of the angle θ at the point of intersection of the surface and the ray relative to the center of the object can be expressed as

$$\cos\theta = \frac{(\hat{\mathbf{k}},\mathbf{r})}{|\mathbf{r}|} = \frac{z_0 + l_z\xi}{\sqrt{r_0^2 + \xi^2 + 2\xi(\hat{\mathbf{l}}, r_0)}}. \qquad (24)$$

 ISBN 978-80-86943-71-8

Vector **ρ** is equal consequentially

$$\boldsymbol{\rho} = \mathbf{r} - \hat{\mathbf{k}}(\hat{\mathbf{k}}, \mathbf{r}) = \mathbf{r}_0 + \xi\hat{\mathbf{l}} - \hat{\mathbf{k}}(z_0 - l_z\xi)\,. \qquad (25)$$

Cosine and sine of the angle φ take the form

$$\cos\varphi = \frac{(\hat{\mathbf{i}}, \boldsymbol{\rho})}{|\boldsymbol{\rho}|} = \frac{x_0 + l_x\xi}{\sqrt{\left(\mathbf{r}_0 + \xi\hat{\mathbf{l}} - \hat{\mathbf{k}}z_0 - \hat{\mathbf{k}}l_z\xi\right)^2}},$$

$$\sin\varphi = \frac{(\hat{\mathbf{j}}, \boldsymbol{\rho})}{|\boldsymbol{\rho}|} = \frac{y_0 + l_y\xi}{\sqrt{\left(\mathbf{r}_0 + \xi\hat{\mathbf{l}} - \hat{\mathbf{k}}z_0 - \hat{\mathbf{k}}l_z\xi\right)^2}}\,. \qquad (26)$$

As a result, one get dependences of the angles θ and φ from one variable ξ. At the point of intersection the equality takes place

$$U(|\mathbf{r}_0 - \xi\hat{\mathbf{l}}|, \theta(\xi), \varphi(\xi)) = 0\,. \qquad (27)$$

Equation (27) contains a set of solutions, but only the first point of intersection is interested in this case. It can be found through various methods, ranging from low efficiency, such as successive approximation, as well as more sophisticated algorithms such as genetic. The use of a particular algorithm depends on the requirements of performance and accuracy. In our implementation, we calculated values successively with a fixed interval and localized the position of the first intersection point, and then using the bisection method refined it.

6. PRACTICE

In our work, we implemented the algorithm of the double local estimation in the MATLAB environment. Double local estimation allows directly calculating the radiance at any point in 3D scene. Figure 5 shows the luminance angular distribution of multiply reflected light at the point of the lower plane in Sobolev problem. The uniform spherical light source is used as a luminaire. The Phong model describes the reflection from the surface.

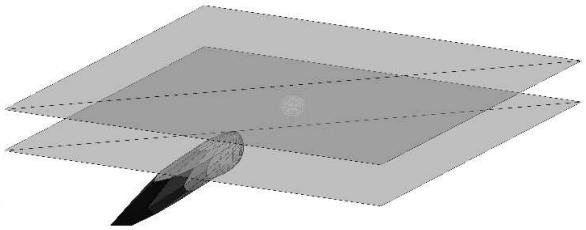

Figure 5. The radiance angular distribution of multiple reflected light in the Sobolev problem.

7. SUMMARY

Nowadays we do not know any method or software application that allows calculating directly the radiance at rendering 3D scene. The double local estimation is a method that allows obtaining the radiance values at any point of 3D scene. Its application allows reviewing newly the complete regulatory framework for the illuminating engineering that was created due to the ability of calculation only the illumination. The double local estimation allows calculating the value of the radiance at several points by one ray.

Note that the convergence of the local estimation is significantly higher than the traditional radiosity and the direct simulation, because it allows evaluating light immediately at all the points in the scene. At the same time, it is significant that the local estimation does not require constructing the mesh, which greatly reduces the amount of RAM.

Methods of local estimation could find its use not only in lighting calculations, but also in computer graphics.

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