APPLIED MECHANICS 2018
April 9 - 11, 2018, Myslovice, Czech republic

PARAMETER IDENTIFICATION OF LINEAR MECHANICAL SYSTEM

V. Goga[1], J. Murín[2], V. Kutiš[3], J. Paulech[4], G. Gálik[5]

Abstract: This paper presents discrete mathematical model for linear mechanical system composed of a steel cantilever beam with added mass at the beam's free end and further mass suspended on the beam by means of a tension spring. System parameters were first obtained by their direct measurement and then by identifying the input-output transfer function model. Dynamics of the model with various acquired parameters is compared with measurement on the real mechanical system.

Keywords: modelling of mechanical system; parameter identification; transfer function

1 Introduction

Dynamics of the real mechanical system with infinite degrees of freedom (DOF) is in engineering practice described by continuous or discrete mathematical models. Continuous model has large number of DOF and its mathematical representation is a system of partial differential equations (PDE). Solution of this model gives accurate results but is time and computationally consuming. In many cases it is possible to describe the dynamics of real system by a simple discrete mathematical model with several dominant DOF. Moreover, if we assume the linear behaviour of the mechanical system the mathematical model is simplified to a few 2^{nd} order linear ordinary differential equations (ODE) with constant coefficients.

This paper presents discrete mathematical model for mechanical system shown in Figure 1. System is composed of a steel cantilever beam with added mass at the beam's free end and further mass suspended on the beam by means of a tension spring.

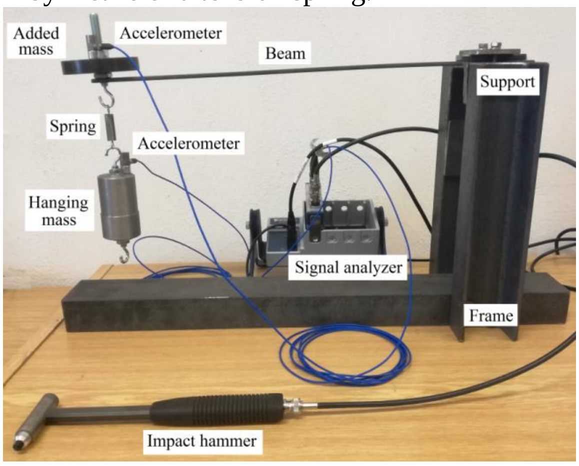

Figure1: Mechanical system and measuring equipment.

[1]Vladimír Goga; Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology STU in Bratislava. Slovak Republic; vladimir.goga@stuba.sk
[2]Justín Murín; Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology STU in Bratislava. Slovak Republic; justin.murin@stuba.sk
[3]Vladimír Kutiš; Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology STU in Bratislava. Slovak Republic; vladimir.kutis@stuba.sk
[4]Juraj Paulech; Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology STU in Bratislava. Slovak Republic; juraj.paulech@stuba.sk
[5]Gabriel Gálik; Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology STU in Bratislava. Slovak Republic; gabriel.galik@stuba.sk

System parameters for the model were directly measured and also identified from experimental testing. Dynamics of the model with various acquired parameters is compared with measurement on the real mechanical system.

2 Discrete model of the mechanical system

Schematic diagram of the mechanical system is shown in Figure 2. If we are interested in vibration of the beam free end and hanging mass, then the system can be represented by a linear discrete 2 DOF model under certain assumptions:

- tension spring is massless with linear stiffness k_1,
- small beam's deflections caused by lateral forces, therefore the beam is replaced by spring with equivalent linear bending stiffness k_2 (1^{st} bending vibration mode),
- the mass m_1 hanging on the tension spring k_1 moves only in vertical direction (1^{st} DOF: coordinate y_1),
- mass m_2 is sum of the added mass M and the equivalent beam mass m_{beam} and moves only in vertical direction (2^{nd} DOF: coordinate y_2),
- it should be note that every mechanical component has an internal structural damping but for simplicity the damping properties of the spring and the beam are described by linear viscous damping c_1 and c_2, respectively.

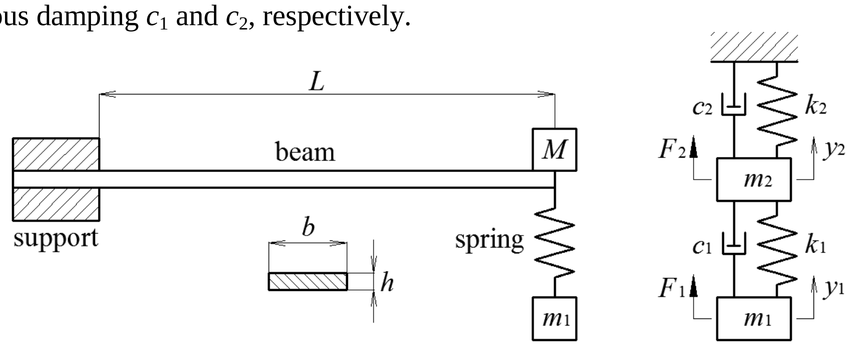

Figure2: Mechanical system and its discrete 2 DOF model.

Differential equations of motion for a discrete 2 DOF model are defined as follows:

$$m_1\ddot{y}_1(t) + c_1\dot{y}_1(t) + k_1 y_1(t) = F_1(t) + c_1\dot{y}_2(t) + k_1 y_2(t) \tag{1}$$

$$m_2\ddot{y}_2(t) + (c_1 + c_2)\dot{y}_2(t) + (k_1 + k_2)y_2(t) = F_2(t) + c_1\dot{y}_1(t) + k_1 y_1(t) \tag{2}$$

where $\ddot{y}$, $\dot{y}$ and y is acceleration, velocity and displacement, respectively and F_1 and F_2 are excitation forces acting on mass m_1 and m_2, respectively.

In control system theory, transfer functions are a standard form for describing the dynamics of linear systems instead differential equations in time domain. Transfer function is generally defined as the ratio of the system output to system input in Laplace *s*-domain. Transfer functions derived for differential equations (1 and 2) are:

$$G_{F1\to y1}(s) = \frac{\frac{1}{m_1}s^2 + \frac{c_1+c_2}{m_1 m_2}s + \frac{k_1+k_2}{m_1 m_2}}{s^4 + \left(\frac{c_1}{m_1} + \frac{c_1+c_2}{m_2}\right)s^3 + \left(\frac{c_1 c_2}{m_1 m_2} + \frac{k_1}{m_1} + \frac{k_1+k_2}{m_2}\right)s^2 + \frac{c_1 k_2 + c_2 k_1}{m_1 m_2}s + \frac{k_1 k_2}{m_1 m_2}} \tag{3}$$

$$G_{F1\to y2}(s) = \frac{\frac{c_1}{m_1 m_2}s + \frac{k_1}{m_1 m_2}}{s^4 + \left(\frac{c_1}{m_1} + \frac{c_1+c_2}{m_2}\right)s^3 + \left(\frac{c_1 c_2}{m_1 m_2} + \frac{k_1}{m_1} + \frac{k_1+k_2}{m_2}\right)s^2 + \frac{c_1 k_2 + c_2 k_1}{m_1 m_2}s + \frac{k_1 k_2}{m_1 m_2}} \tag{4}$$

$$G_{F2\to y1}(s) = \frac{\frac{c_1}{m_1 m_2}s + \frac{k_1}{m_1 m_2}}{s^4 + \left(\frac{c_1}{m_1} + \frac{c_1+c_2}{m_2}\right)s^3 + \left(\frac{c_1 c_2}{m_1 m_2} + \frac{k_1}{m_1} + \frac{k_1+k_2}{m_2}\right)s^2 + \frac{c_1 k_2 + c_2 k_1}{m_1 m_2}s + \frac{k_1 k_2}{m_1 m_2}} \tag{5}$$

$$G_{F2\to y2}(s) = \frac{\frac{1}{m_2}s^2 + \frac{c_1}{m_1 m_2}s + \frac{k_1}{m_1 m_2}}{s^4 + \left(\frac{c_1}{m_1} + \frac{c_1+c_2}{m_2}\right)s^3 + \left(\frac{c_1 c_2}{m_1 m_2} + \frac{k_1}{m_1} + \frac{k_1+k_2}{m_2}\right)s^2 + \frac{c_1 k_2 + c_2 k_1}{m_1 m_2}s + \frac{k_1 k_2}{m_1 m_2}} \tag{6}$$

where indexing $F_i \to y_j$ $(i, j = 1,2)$ means: excitation on mass *i* causes displacement of mass *j*.

3 Determining system parameters by direct measurement

Mechanical system was divided into two separate subsystems:

- 1st subsystem → mass m_1 hanging on tension spring k_1,
- 2nd subsystem → cantilever beam with added mass M.

Parameters of both subsystems were determined by direct measurement or calculation based on physical principles and they are indexed "*real*".

Parameters of 1st subsystem (Table 2):

- mass m_{1_real} → weighting the mass,
- stiffness k_{1_real} → tensile test of the spring,
- damping c_{1_real} → free vibration test (Figure 3): logarithmic decrement method [1],
- eigenfrequency f_{1_real} → free vibration test (Figure 5): spectral analysis.

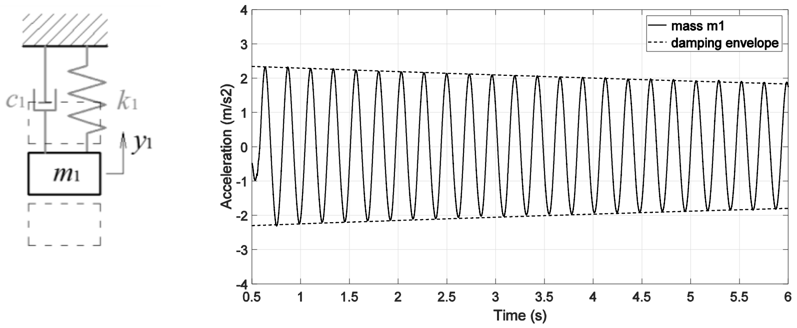

Figure3: Free vibration test - 1st subsystem.

width	*height*	*length*	*density*	*beam mass*	*added mass*	*Young's modulus*
b [m]	h [m]	L [m]	ρ [kgm^{-3}]	m_{beam} [kg]	M [kg]	E [GPa]
0.025	0.0043	0.35	7609	0.286	0.56	200

Table1: Beam dimensions and physical properties.

Parameters of 2nd subsystem (Table 2):

- mass m_{2_real} → added mass plus equivalent beam mass [2]: $m_2 = M + \frac{33}{140} m_{\text{beam}}$,
- stiffness k_{2_real} → beam bending stiffness [3]: $k_2 = \frac{Ebh^3}{4L^3}$
- damping c_{2_real} → free vibration test (Figure 4): logarithmic decrement method [1],
- eigenfrequency f_{2_real} → free vibration test (Figure 5): spectral analysis.

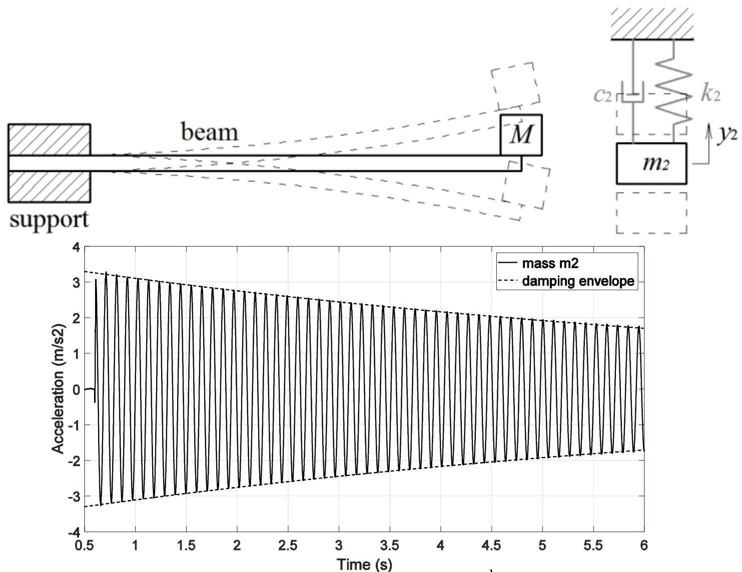

Figure4: Free vibration test - 2nd subsystem.

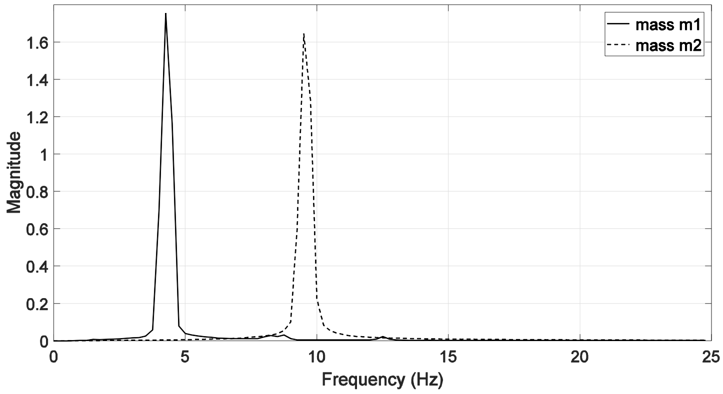

Figure5: Frequency spectrum of individual subsystems.

1st subsystem				*2nd subsystem*			
m_{1_real} [kg]	c_{1_real} [Nsm^{-1}]	k_{1_real} [Nm^{-1}]	f_{1_real} [Hz]	m_{2_real} [kg]	c_{2_real} [Nsm^{-1}]	k_{2_real} [Nm^{-1}]	f_{2_real} [Hz]
0.8	0.07	580	4.29	0.63	0.15	2318	9.55

Table2: Real system parameters.

4 System parameters identification

Real mechanical system was subjected to experimental test (Figure 6). The excitation force (input signal - Figure 7) was applied on the beam's free end and accelerations of the masses m_1 and M were measured, filtered and integrated to displacements y_1 and y_2 (output signals). Piezoelectric accelerometers and impact hammer were used for the measurement. Measured data was processed in System Identification Toolbox (SIT) in software Matlab. Identified parameters are indexed "*ident*".

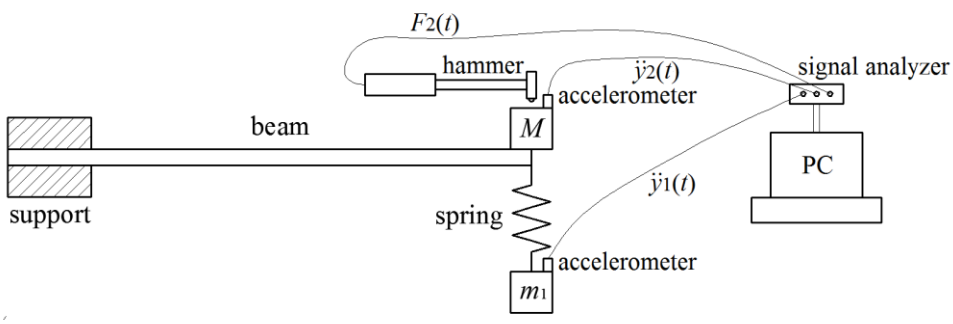

Figure6: Testing the mechanical system to identify parameters.

Identified transfer functions derived from measured input/output signals are:

$$G_{ident\ \ F2\to y1}(s) = \frac{2.331s+1192}{s^4+0.6069s^3+5331s^2+756.1s+2.709\times 10^6} \quad (7)$$

$$G_{ident\ \ F2\to y2}(s) = \frac{1.561s^2+5.772s+1136}{s^4+0.6956s^3+5328s^2+942.5s+2.717\times 10^6} \quad (8)$$

System parameters were then calculated using equations (6 and 8) and are presented in Table 3.

1st subsystem			*2nd subsystem*		
m_{1_ident} [kg]	c_{1_ident} [Nsm^{-1}]	k_{1_ident} [Nm^{-1}]	m_{2_ident} [kg]	c_{2_ident} [Nsm^{-1}]	k_{2_ident} [Nm^{-1}]
0.76	0.16	555	0.64	0.16	2392

Table3: Identified system parameters.

Differences of *real* and *ident* parameters are less than 7 % except the spring damping c_1. Comparisons of the displacements y_1 and y_2 obtained from measured accelerations of real system and from model simulations with real and identified parameters are shown in Figure 8 and 9. Figure 10 shows frequency spectrums of the system. Eigenfrequencies of the system obtained by measurement and simulations are identical: 1st eigenfrequency is 3.77 Hz and 2nd eigenfrequency is 10.76 Hz.

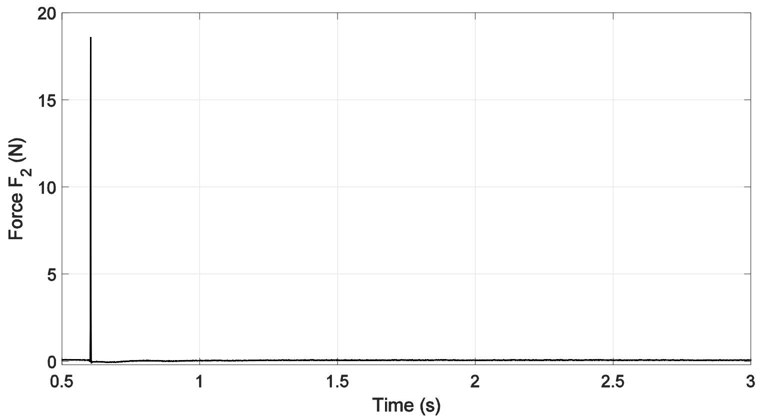

Figure7: Input signal - force: F_2.

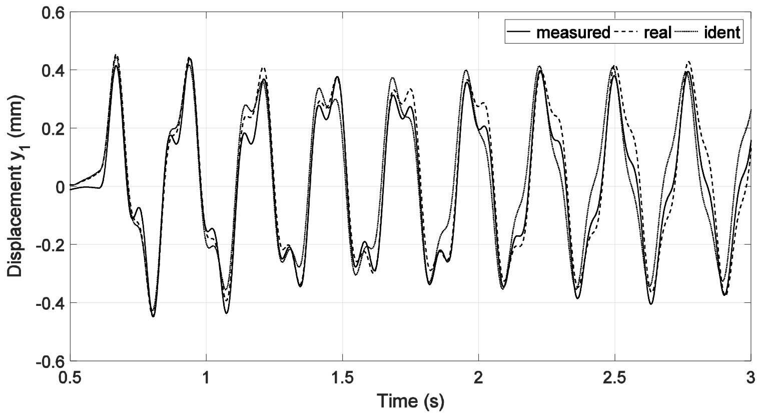

Figure8: Output signal - displacement: y_1.

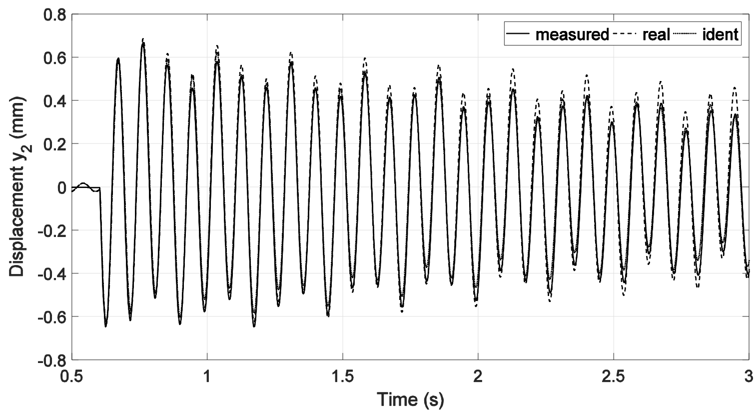

Figure9: Output signal - displacement: y_2.

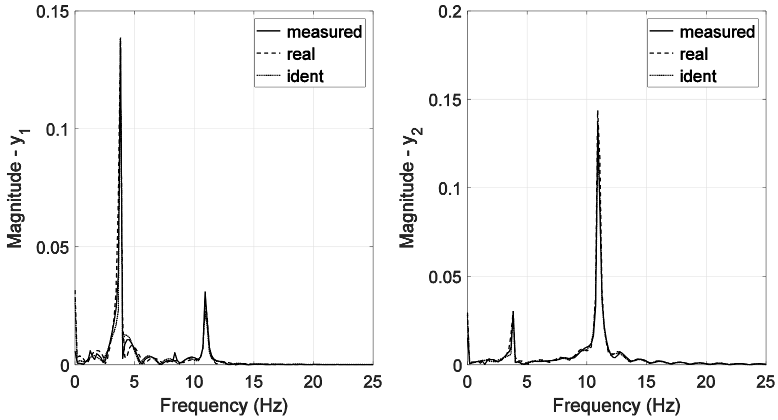

Figure10: System frequency spectrum derived from displacements y_1 and y_2.

5 Conclusion

Dynamics of the real mechanical system was described by linear discrete 2 DOF model in the form of differential equations of motion as well as in the form of transfer functions. System parameters were obtained first by direct measurements and by calculations based on the physical principles. The other way of finding system parameters was based on the identification test performed on the real system. Parameters were identified from transfer functions which were compiled from measured input and output signals. Differences in parameters obtained through different approaches are less than 7 %. The only significant deviation is for the spring damping coefficient. This deviation is caused by considering over simplified viscous damping model for the spring as well as the beam. Nevertheless, the dynamics of the discrete model is in good agreement with measurement on the real mechanical system.

Acknowledgement

This work was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0613 and APVV-0246-12, by Grant Agency VEGA, grant No. 1/0102/18 and 1/0081/18. Authors are also grateful to the HPC Centre at the Slovak University of Technology in Bratislava, which is a part of the Slovak Infrastructure of High Performance Computing (SIVVP project, ITMS code 26230120002, funded by the European Regional Development Funds), for the computational time and resources made available.

References

[1] INMAN, D.J. *Vibration with Control*. England: John Wiley & Sons, 2006. ISBN 0-470-01051-7.
[2] BEARDS, C.F. *Engineering Vibration Analysis with Application to Control Systems*. London: Edward Arnold, 1995. ISBN 0-340-63183-6.
[3] BEER, F.P. et al. *Statics and Mechanics of Materials*. New York: McGraw-Hill, 2006. ISBN 0-07-338015-5.