A Novel Accurate 3D Surfaces Description Using the Arc-Length Reparametrized Level curves of the Three-Polar Representation

Amal Rihani
CRISTAL Laboratory, GRIFT research group
ENSI, La Manouba University
2010, La manouba, Tunisia
amal.rihani@ensi-uma.tn

Majdi Jribi
CRISTAL Laboratory, GRIFT research group
ENSI, La Manouba University
2010, La manouba, Tunisia
majdi.jribi@ensi.rnu.tn

Faouzi Ghorbel
CRISTAL Laboratory, GRIFT research group
ENSI, La Manouba University
2010, La manouba, Tunisia
faouzi.ghorbel@ensi.rnu.tn

ABSTRACT

This paper studies the problem of the 3D surfaces representation. Our starting point is the extraction of the three-polar representation from the 3D shapes. It consists on a level curves set of the superposition of the three geodesic potentials generated from three reference points of the surface. These curves are characterized by their invariance under the $M(3)$ group of $\mathbb{R}^3$ displacements. We intend to make the arc-length reparametrization of each level curve to ensure its independence to the initial parametrization. The novel representation is materialized by the points of the arc-length reparametrization of all the level curves. Therefore, we obtain an invariant representation under the $M(3)$ transformations group and independent to the initial parametrization. In this work, we implement it on 3D faces since this type of surfaces knows actually a growing interest for the identities determination especially after the many terrorist acts occurred around the world. We experiment, in this context, the identification scenario on a part of the BU-3DFE database. The obtained results show the accuracy of the novel representation.

Keywords

Three-polar, geodesic potential, level set, curve, arc-length, shape representation, invariant, approximation, 3D face, identification.

1 INTRODUCTION

3D shape recognition has become an important issue in the pattern recognition field. This is due especially to the growing development of the 3D scanning tools and the good quality of the obtained 3D data. The pattern recognition with three dimensional data was proposed as an alternative to the one with 2D images. In fact, 3D surfaces permit to overcome the problems of pose and illumination often encountered in 2D data.

However, 3D surfaces lack of a canonical parametrization. Indeed, for the same surface, many parameterizations could exist. They depend on the point of view and the orientation of the surface. This fact makes hard the recognition procedure with 3D data. In order to cross as much as possible these difficulties, the extraction of invariant description from 3D surfaces under some geometrical transformations is proposed as an efficient alternative.

We intend in this work to construct a novel representation of 3D surfaces which is invariant under the $M(3)$ group of transformations ($\mathbb{R}^3$ rotations and translations). This novel representation could be applied to all types of 3D objects. We give, here, a special attention to 3D faces. In fact, this type of surfaces is actually of a paramount importance. It is a powerful tool for the persons identities recognition.

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1.1 Related works

We present in this part, an overview of some 3D surfaces description methods including several ones that were implemented on 3D faces. In the literature, the 3D shape description methods can be classified into four main families: the view based methods, the graph based approaches, the global ones and those considered as local.

In the view based approach, a 3D object is characterized by its 2D projections on canonical directions. In fact two objects are assumed to be similar if they are similar from the same point of view. 2D invariant descriptors could be, then, applied on this set of 2D images in order to extract an accurate representation of the 3D object. The 2D Zernike moments [Che03] and the Fourier descriptors [Vra04] are among of the most used descriptors in this context.

For the graph based methods, we try to represent a 3D object as a graph showing how shape components are linked together. These methods can be classified into two major categories: the Reeb graph one [Tun05] and the Skeleton method [Sun03]. The Reeb graph is a topological structure. It is obtained according to the Morse theory [Shi91] that characterizes a closed 3D surface. In the case of the Skeleton graph, a 3D object is represented by its Skeleton often obtained by the median axis of the used 3D surface.

In the construction of a 3D global shape description method, the representation of a 3D surface is obtained by the geometrical characteristics of the whole object. Several 3D global surfaces description methods were performed in the literature. Osada et al. [Osa02] proposed the 3D distribution forms method. It consists on a novel signature of a 3D object obtained by a probability distribution of a shape function. Paquet et al. [Paq99] were among the first who proposed the famous cords histogram method. It is based on the extraction of the statistical characteristics from the cords of the 3D object. Here, we denote by cords, all segments connecting the gravity center of a 3D object and its triangles centers. The famous 3D Hough Descriptor (3DHD) proposed by Zaharia et al. [Zah01a] is a global descriptor that accumulates the parameters of the representative planes defined by the triangles in a given 3D mesh.

In the fourth approaches category, a local 3D representation is extracted from a 3D surface. Several past 3D local shape description methods were proposed. We mention the pioneer work of Faugeras et al. [Fau86] that characterizes a 3D surface by its high curvature values zones. Zaharia et al. [Zah01b] used the high curvature values surface points or the inflexion ones to extract a statistical description from histograms. Their descriptor is called the shape index histogram. Bannour et al. [Ban00] generalized the idea of the 3D surface description by only the high curvature zones to a method that describes a 3D surface by a set of invariant points corresponding to the levels of the curvature values areas. In the context of 3D faces description with curvatures, Shu-wei et al. [Shu12] used the gaussian curvature to characterize the 3D faces. Here, a 3D face is described by a feature vector of gaussian curvature. The distance between pair of 3D faces is obtained via the distance between their feature vectors. Ganguly et al. [Gan14] proposed to describe a 3D face by a two pairwise curvatures analysis. The first one is the mean, and the maximum curvatures and the second pair corresponds to the gaussian and the minimum curvatures. 3D faces are, then, compared and matched using this description. In order to compare between 3D faces, several methods based on a curvature computation were used to to extract interest points from this type of 3D surfaces. De Giorgis et al. [Deg15] identified fiducial points from 3D faces using a multi-scale curvature analysis. Berreti et al. [Ber13] proposed to use the meshDOG algorithm (Difference Of Gaussian) based on a mean curvature computation to determinate accurate interest points from 3D faces. Another kind of local 3D surface description is based on the geodesic computing around feature points. Many authors [Sam06, Sri08, Gad12] proposed to compute the unipolar representation that consists on the geodesic level curves around a reference point of the 3D shape. They impose, therefore, a coordinates system to the 3D surface. They apply these representations in the context of 3D faces description. Other works, used many unipolar representations around many reference points to locally describe a 3D surface [Maa11]. In order to ensure a more stability of the unipolar representation in the case of error on reference point, Ghorbel et al. [Gho13] proposed a novel representation called the bipolar one. It consists on the invariant set of points corresponding to the levels of the sum of the two geodesic potentials generated from two reference points of the surface. Here, the geodesic information coming from each reference point are combined and not used each one lonely like the representation with many unipolar representations [Maa11]. Jribi et al. [Jri13, Jri14] proposed a novel representation qualified by the three-polar one. It is defined by the invariant points of the surface corresponding to the levels of the sum of the three geodesic potentials generated from three reference points. The same authors proposed an ordered version of the three-polar representation obtained by the intersection between the last one and the radial lines levels representation obtained with the same angular separation [Jri15]. The last representations (bipolar and three-polar) were implemented and tested on some 3D faces.

1.2 Our approach

We propose here a novel 3D surfaces representation. The base of this work is the three-polar one. This last one corresponds to a set of invariant curves under the group of $\mathbb{R}^3$ rotations and translations($M(3)$ group). Once a 3D surface is described by these curves, it becomes more easy to extract an accurate representation from a 3D surface. In fact, the problem of 3D surfaces description is transformed to a problem of 3D curves description. In this context, we try to describe these curves independently to their first parametrizations. We, therefore, characterize them by their arclength reparametrization. The obtained points from all the curves consist the proposed novel representation. We apply the proposed approach for the description of

3D faces. We use the Hausdorff shape distance as a similarity metric to compare between different shapes.
The reminder of this paper is organized as follows: We detail in the second section all the steps of the novel representation construction. In the third section, we expose the used similarity metric that corresponds to the Hausdorff Shape distance. We apply, finally, in the fourth section the novel representation for the description of 3D faces. The obtained results for the identification scenario on a part of the BU-3DFE database [Lij06] of 3D faces are exposed.

2 CONSTRUCTION OF THE NOVEL 3D SURFACES REPRESENTATION

We intend in this work to describe a 3D surface by an accurate, finite and invariant set of points under the geometrical transformations of the $M(3)$ group. We suppose here that a 3D object is a continuous surface. It is considered as a 2D-differential manifold that we denote by S. The three-polar representation, known by its stability under the errors on the reference points extraction [Jri14] is used as a starting representation. This last one corresponds to a set of invariant curves under the same group of transformations. Finite and accurate points are obtained by the discretization of each curve. The discretization procedure has a major importance for the construction of the novel representation. In fact, an accurate discrete representation of the level curves leads to an efficient novel representation. Therefore, the steps of the novel 3D representation can be summarized as follows: (i) The first step consists on the three-polar representation construction. (ii) In the second step, an accurate description of each three-polar level curve should be performed.
We use the following mathematical considerations for the construction of the novel representation. Let P_1 and P_2 be two points of S. We denote by:

- $\gamma(P_1,P_2)$: the geodesic curve joining P_1 and P_2. It is the curve having the minimum of distance between P_1 and P_2 and belonging to the surface S.
- $\widetilde{\gamma}(P_1,P_2)$: the length of the geodesic curve computed between P_1 and P_2
- $U_r(P)$: the geodesic potential generated from a point r of S. It is the function that computes for each point P of S the length of the geodesic curve joining it to the point r.
  We describe in the rest of the section the two steps cited above.

2.1 Brief recall of the three-polar representation

The three-polar representation is constructed from three reference points of a 3D surface S. It is built in order to ensure a more stability in the case of extraction errors on the reference points [Jri14]. This 3D representation consists on a set of curves extracted from the 3D object assumed to be, here, a 2D-differential manifold. These curves correspond to the levels of the sum of the three geodesic potentials generated from the used three reference points of the surface. It is easy to see that these level curves are invariant under the geometrical transformations of the $M(3)$ group since the geodesic computation is invariant under the same transformations.
Therefore, let denote by P_1, P_2 and P_3 three reference points of S, U_{P1}, U_{P2} and U_{P3} their corresponding geodesic potential functions and U_3 the sum of these three geodesic potentials.
The three-polar representation composed by a set of K level curves can be formulated as follows:

$$M^k(S) = \{C^{\lambda_i}\}_{i=1..k} \tag{1}$$

where C^{λ_i} is the level curve with the value λ_i of the sum U_3 of the three geodesic potentials generated from the used three reference points. Therefore:

$$C^{\lambda_i} = \{p \in S, U_3(p) = \lambda_i\} \tag{2}$$

We note here that the curves$\{C^{\lambda_i}\}_{i=1..k}$ are extracted from the 3D surface with the same step of the sum of the three geodesic potentials.

2.2 Accurate description of the level curves

A 3D object is assumed to be a 2D-differential manifold. It is represented by a collection of indexed 3D curves $\{C^{\lambda_i}\}_{i=1..k}$ of the three-polar representation. A level curve C^{λ_i} parametrization denoted by $C^{\lambda_i}(t)$ is a 1-periodic function of a continuous parameter t defined by:

$$C^{\lambda_i}(t) : [0,1] \to \mathbb{R}^3 \tag{3}$$
$$t \mapsto [x(t), y(t), z(t)]^t$$

It is important to note that for the same curve we can find many parametrizations. They depend on the position and the orientation of the used curve and the speed we go over it. This fact makes hard the comparison between curves. In order to overcome this problem, we propose to use a $\mathbb{G}$-invariant reparametrization of each curve. $\mathbb{G}$ is group of the geometrical transformations applied to a curve. A reparametrization of $C^{\lambda_i}(t)$, noted $C^{\lambda_i}(\widehat{t})$, is defined as follows :

$$C^{\lambda_i}(\widehat{t}) = C^{\lambda_i}(\tau(t)) = [x(\tau(t)), y(\tau(t)), z(\tau(t))]^t, t \in [0,1] \tag{4}$$

where τ is an increasing function defined on [0,1].
Let consider $C_1^{\lambda_i}(t_1)$ and $C_2^{\lambda_i}(t_2)$ two parameterizations of a curve C^{λ_i} and its image by the geometrical transformation $g \in \mathbb{G}$. After the $\mathbb{G}$-invariant reparametrization,

we obtain:

$$C_2^{\lambda_i}(\hat{t}) = g(C_1^{\lambda_i}(\hat{t}+t_0)) \quad (5)$$

where $t_0 \in \mathbb{Z}$, $g \in \mathbb{G}$ and t_0 is the starting points difference between the curves.
In our context, $\mathbb{G}$ corresponds to the $M(3)$ group formed by the $\mathbb{R}^3$ rotations and translations. This transformations group preserves the length of curves. The speed we go over a curve will affect the parametrization. We perform, therefore, the arc-length reparametrization of this curve. This implies that it is covered with a constant speed. The arc-length reparametrization of a 3D curve C^{λ_i} is defined as follows:

$$S(t) = 1/L \int_0^t \sqrt{x(t)'^2 + y(t)'^2 + z(t)'^2} dt, t \in [0,T] \quad (6)$$

Here, L denotes the length of the level curve C^{λ_i}.

3 SIMILARITY METRIC

In order to compare between 3D shapes, we use the novel 3D representation as a signature. The well known Hausdorff shape distance introduced by Ghorbel et al. [Gho98, Gho12] is used as a similarity metric. Let G be the group of all possible parameterizations of surfaces. It can be the $\mathbb{R}^2$ plane for the open surface or the $\mathbb{S}^2$ for the closed ones. In the context of 3D surfaces pieces diffeomorphic to G, on which act the $M(3)$ group, the Hausdorff shape distance can be defined for two surfaces pieces S_1 and S_2 and two displacements g_1 and g_2 as follows:

$$\triangle(S_1,S_2) = max(\rho(S_1,S_2),\rho(S_2,S_1)) \quad (7)$$

where:

$$\rho(S_1,S_2) = \sup_{g_1 \in M(3)} \inf_{g_2 \in M(3)} \|g_1 S_1 - g_2 S_2\|^2_{L^2} \quad (8)$$

Since the $M(3)$ displacement group preserves this norm, the Hausdorff shape distance can be reduced to the following quantity:

$$\triangle(S_1,S_2) = \inf_{h \in M(3)} \|S_1 - hS_2\|^2_{L^2} \quad (9)$$

In order to compute the Hausdorff shape distance value between two surfaces, the optimal transformation between these two objects should be determined. We use in this context, The Iterative Closest Point (ICP) algorithm [Bes92] to estimate this transformation and thus to reach the real value of this distance.

4 DESCRIPTION OF 3D FACES WITH THE NOVEL REPRESENTATION

Actually, human recognition via biometric traits is of a paramount importance especially with the many terrorist acts occurred around the world. The face is one of the most used biometric traits since it does not require the cooperation of the subjects. The 3D faces description is becoming actually an area of growing interest especially with the rapid development of 3D scanning tools. We try, in this context, to apply the novel representation for the description of 3D faces. We present in the rest of this section the used database and we detail the construction steps of the novel representation on this special type of 3D surface.

4.1 The used database

In order to study the performance of the novel 3D representation for the description of 3D faces, we use the BU-3DFE database [Lij06] which contains 100 subjects (56 females and 44 males) from different ethnicities. Seven facial expressions (neutral, disgust, happiness, angry, surprise, sadness and fear) are available for each subject. Each facial expression is presented by four levels of magnitude.

4.2 The used three reference points

The selection of the reference points is the first step of the three-polar representation construction. The nose tip which is used in the unipolar representation based on only one reference point [Sam06, Sri08, Gad12] will be also used as a reference point for the three-polar representation. The two outer corners of eyes will be selected as candidate reference points. For the automatic extraction of the reference points, we use the approach proposed by Szeptycki et al. [Sze09a] which is based on a curvature analysis of a 3D face.

4.3 Geodesic computation

We have assumed in the construction of the three-polar representation that a 3D surface is a 2D-differential manifold. In practice, this surface is represented by a 3D mesh composed by a set of vertices and edges.
In order to compute the geodesic potentials from the reference points, we should be able to compute the geodesic curves between pairs of points of the 3D discrete mesh. We use in our work the fast marching algorithm [Set96] to compute geodesic paths between pairs of points and subsequently the geodesic potentials.

4.4 Extraction of the level curves of the three-polar representation

Since the 3D face corresponds to a discrete object, its level curves of the three geodesic potentials sum are also discrete. Each level curve will be composed by a set of points from the 3D face. In practice a level curve of value λ can be seen as a trip. It is formulated as follows:

$$C^{\lambda} = \{P \in S, \lambda - \varepsilon \leq U_3(P) \leq \lambda + \varepsilon\} \quad (10)$$

where ε is a real positive value chosen according to the resolution of the mesh to avoid the intersections between successive level curves.

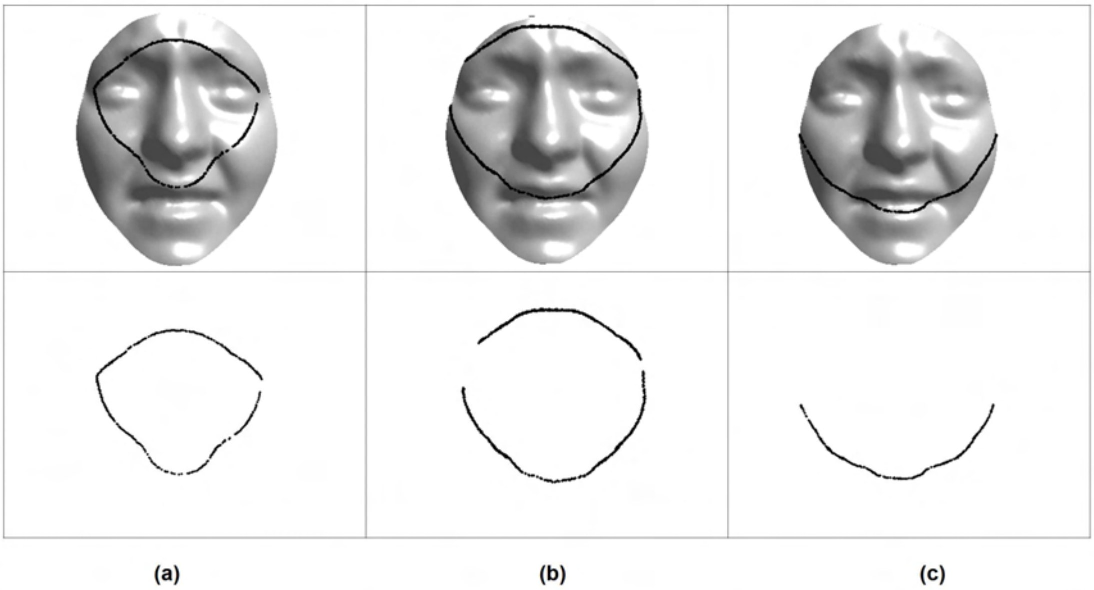

Figure 1: Different kinds of level curves from the three-polar representation. (a): A closed level curve. (b): An open level curve with two separated parts. (c): An open level curve composed by one part.

4.5 Arc-length reparametrization of the discrete level curves

After the extraction of the discrete level curves from a 3D face, we proceed to their arc-length reparametrization. In the context of 3D faces study, the 3D surfaces are open. Therefore, it is naturally to obtain some open level curves since we reach the surface border. Fig. 1 illustrates many kinds of obtained level curves from the three-polar representation. Fig. 1(a) shows an example of a closed level curves. Fig. 1(b) illustrates an open level curves composed by two separated parts. The curve of the Fig. 1(c) corresponds to an open curve with only one part.

In order to make all curves closed for the computation of the arc-length reparametrization, we propose in this work to complete the empty parts of the open level curves by some border points. Fig. 2 illustrates some open curves that were completed by the used border points of the 3D surface. Once all the level curves are closed, before we perform their arc-length reparametrization, we approximate each one of them by the B-spline function. Let $\{p_{i,\lambda}\}_{i=0..N_\lambda}$ be the set of the N_λ discrete points of a curve C^λ of level value equal to λ. The approximated curve by the B-spline function denoted by $C^\lambda(t)$ can be formulated as follows:

$$C^\lambda(t) = \sum_{i=1}^{N_\lambda} B_{i,k-1}(t) \times \left(\left(1 - \frac{t-t_i}{t_{i+k}-t_i}\right)P_{i-1,\lambda} + \frac{t-t_i}{t_{i+k}-t_i}P_{i,\lambda}\right) \quad (11)$$

where $B_{i,k}(t)$ is the B-spline basic functions.
We apply, then, the arc-length reparametrization of each approximated level curve. We obtain equidistant points on each curve since we go over it with the same speed.

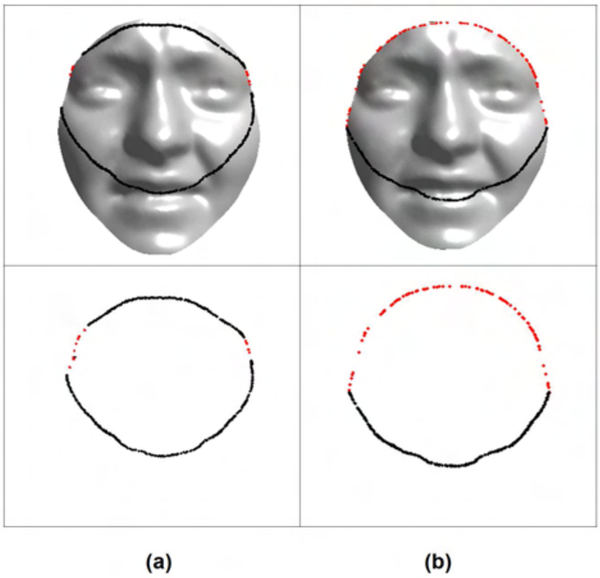

Figure 2: Illustration of two open level curves completed by the border points.(a): A curve with two separated parts. (b): A curve with one part.

Fig. 3(a) represents a level curve of the three-polar representation extracted from a 3D face. Fig. 3(b) shows the approximation of the same curve by the B-spline function and Fig. 3 (c) illustrates the arc-length reparametrization of this curve.

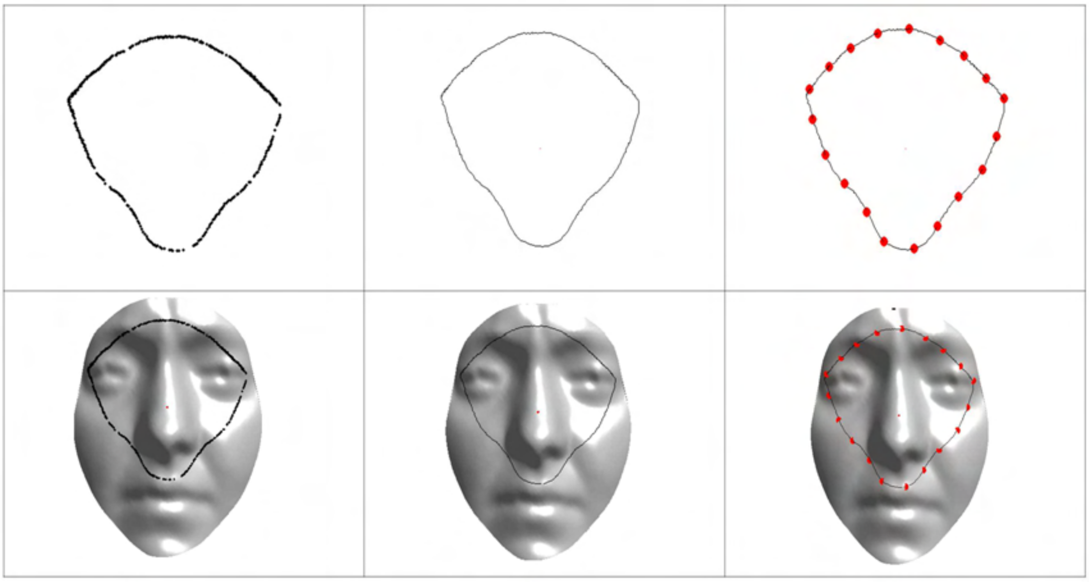

Figure 3: The steps of the description of a level curve from the extraction to the arc-length reparametrization. (a): A discrete level curve of the three-polar representation. (b): Approximation with the B-spline function. (c): The obtained points after the arc-length reparametrization.

4.6 Accuracy of the novel representation for the description of 3D faces

We test, here, the performance of the novel representation for the description of the 3D faces. We use for the experimentation a part of the database BU-3DFE [Lij06]. This portion is composed by the first magnitude level of each facial expression and the neutral face of all the database subjects (100 persons). A total of 700 faces are, then, used for the experimentation.
Fig. 4 presents all the steps of the novel representation construction for two faces with different facial expressions going from the three-polar representation extraction (Fig. 4(a)) to the construction of the novel representation (Fig. 4(d)).
We focus our study on the identification scenario. In this case, a person is compared to all the individuals of the used database and matched to the most similar ones. We run the experiments with the protocol All vs All which consists on the comparison of each face of the database to all the others. Here, the gallery subset and the probes set corresponds to the all 700 faces.

Fig. 5 shows the Cumulative Matching Curve of the proposed 3D representation under the protocol All vs. All. The obtained results are about 92.68% for the rank one recognition rate. This significant recognition rate proves the accuracy of the novel 3D surfaces representation for the description of 3D faces.

5 CONCLUSION

We introduced in this work a novel 3D surfaces description, which is based on the three-polar representation. This last one consists on a set of invariant curves under the $M(3)$ group of $\mathbb{R}^3$ rotations and translations. These curves correspond to the levels of the superposition of the three geodesic potentials generated from three reference points of the surface. The novelty of this work lies on the arc-length reparametrization of each curve of the three-polar representation. This fact makes them independent to the initial parametrization. We apply the novel representation for the description of 3D faces knowing actually a growing interest for the determination of persons identities. To illustrate the performance of the proposed representation for 3D faces description, we implement, in this context, the identification scenario on a part of the BU-3DFE database. We obtain a rank one recognition rate of about 92.68% .
We propose in future work to experiment the novel representation on the standard database of 3D faces FRGC V2. An important perspective is to make a study for the best choice of the three reference points for the three-polar representation. We intend also to compare the proposed representation with some works of the state of the arts using the standard protocols.

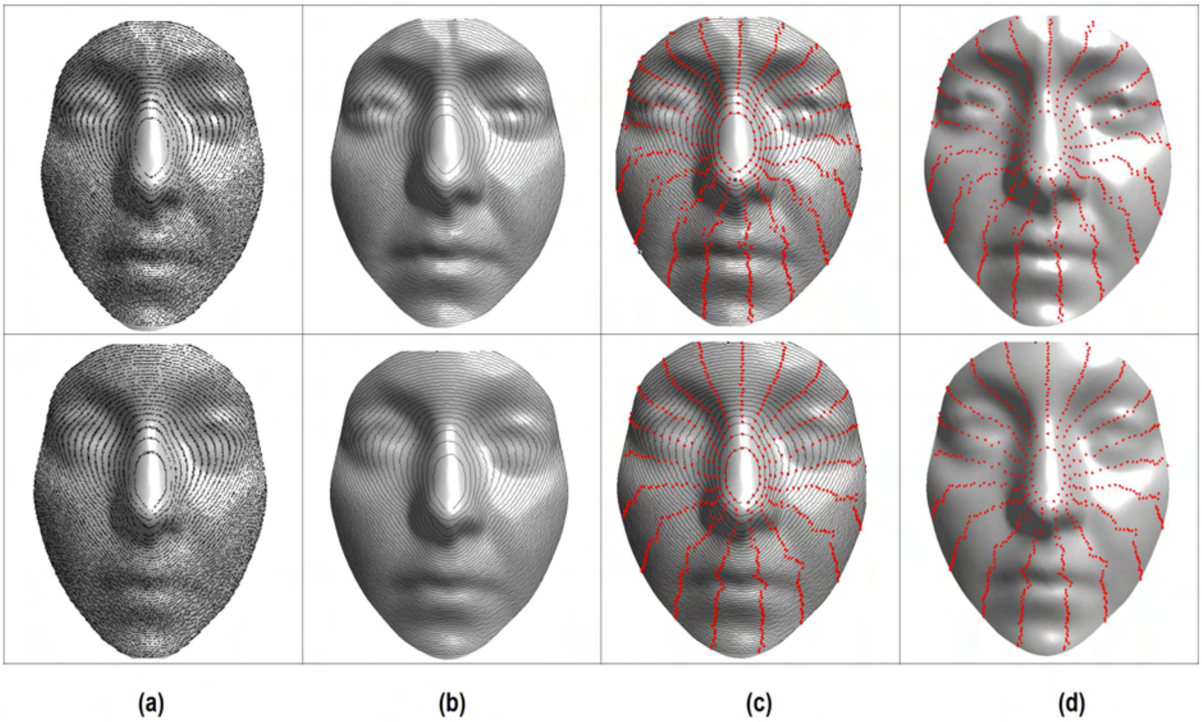

Figure 4: (a): The level curves of the three-polar representation. (b): Approximation of the level curves with the B-spline function. (c): The arc-length reparametrization of the level curves. (d) The obtained points of the novel 3D representation.

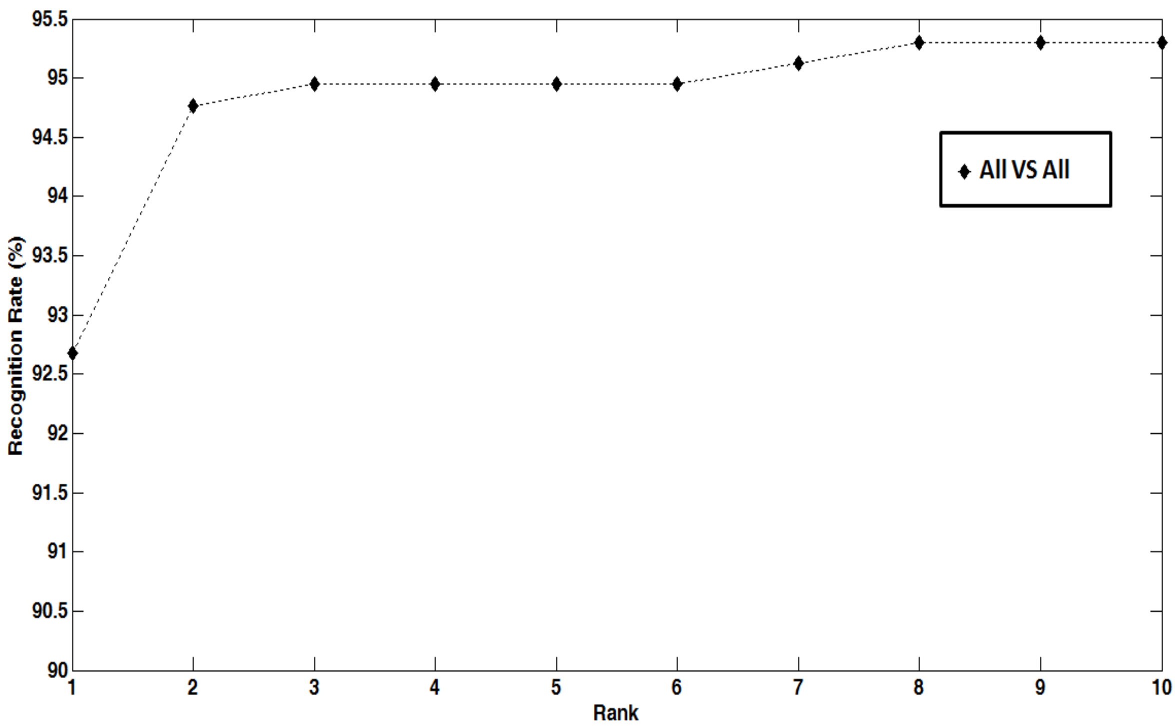

Figure 5: The CMC curve of the proposed approach for the scenario: All vs. All.

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