

## Numerical simulation of free-surface flow over a weir with non-reflective outlet boundary conditions

J. Musil<sup>a,b</sup>

<sup>a</sup> Faculty of Mechanical Engineering, CTU in Prague, Karlovo nám. 13, 121 35 Praha, Czech Republic  
<sup>b</sup> Institute of Thermomechanics, Czech Academy of Sciences, 182 00 Praha, Dolejškova 5, Czech Republic

The non-reflective boundary conditions are essential ingredient in the procedure of computational domain restriction where presence of artificial/open boundaries requires prescription of appropriate boundary conditions in order to obtain numerical solution equivalent to the original one. Thus, these boundary conditions not only lead to reduction of computational cost by means of computational domain restriction but also allow to prescribe physical quantities on artificial boundary in correspondence with analytic form of solution.

This contribution is focused on non-reflective boundary conditions for two-phase incompressible VOF [4] method, which is mathematical model for Navier-Stokes equations described by following system of equations:

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

$$\partial_t \alpha + \nabla \cdot (\alpha \mathbf{u}) = 0. \quad (3)$$

Here  $0 \leq \alpha \leq 1$  is the liquid fraction in the mixture ( $\alpha = 0$  corresponds to air,  $\alpha = 1$  corresponds to water) and the density of the mixture is  $\rho = \alpha \rho^{water} + (1 - \alpha) \rho^{air}$ . The symmetric tensor  $\boldsymbol{\tau}$  expresses tangential stresses and  $\mathbf{g}$  is the acceleration due to gravity.

The non-reflective boundary conditions for VOF model have been developed by employing similarity with theory of one-dimensional shallow water flows and afterwards have been implemented into the open-source package OpenFOAM [3]. This procedure was already presented at the Computational Mechanics conference in 2017 [1] or also in [2].

Whereas performance of these boundary conditions was not yet tested on outlet boundaries the aim here is to present results of numerical simulations of water flow over a weir with artificial boundary at the outflow. The computation has been performed on two-dimensional quadrilateral structured grid with 31 720 finite volume cells. The initial condition was set to  $\mathbf{u} = \mathbf{0}$  and to  $p + h\rho g$  as hydrostatic pressure in entire domain,  $\alpha$  according to Fig. 1. At the bottom the no-slip boundary condition for  $\mathbf{u}$  and homogeneous Neumann condition for  $p + h\rho g$  and  $\alpha$  was prescribed. At the inlet boundary (left) there was set homogeneous Neumann condition for  $p + h\rho g$  and coupled boundary conditions for  $\mathbf{u}$  and  $\alpha$  respecting prescribed volumetric flow as  $25 \text{ m}^3/\text{s}$ . The top boundary is equipped with modified homogeneous Neumann conditions for  $\mathbf{u}$  and  $\alpha$  and total pressure is set as zero here. At the outlet (right) there were prescribed four variants of boundary conditions for  $\mathbf{u}$  and  $\alpha$ :

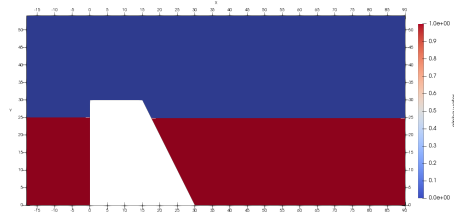


Fig. 1. Initial condition for  $\alpha$

- I) homogeneous Neumann conditions for both  $\mathbf{u}$  and  $\alpha$
- II) OpenFOAM coupled boundary conditions allowing water height (i.e.  $\alpha$ ) to vary according to the prescribed volumetric flow rate (here set as negative value of inlet flow rate)
- III) homogeneous Neumann condition for  $\alpha$ , non-reflective boundary condition for  $\mathbf{u}$
- IV) non-reflective boundary conditions for both  $\mathbf{u}$  and  $\alpha$ .

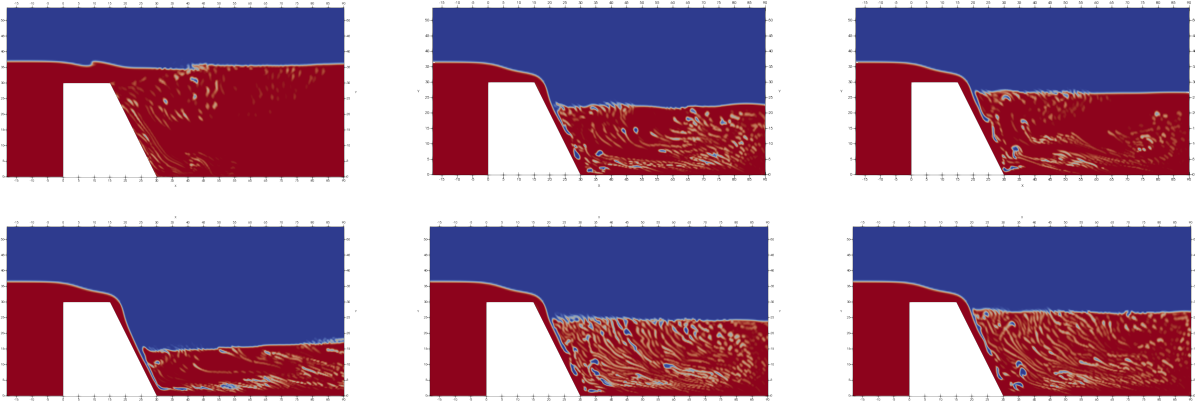


Fig. 2. Water height ( $\alpha$ ) in  $t = 50$  s (upper row) and  $t = 130$  s (lower row)

In the first column of Fig. 2 one can see I) variant of boundary conditions which lead to un-physical solution with large oscillations of water level. The second column shows variant II) which seems to perform well, but there is a problem with velocity at boundary being fixed according to the inflow and thus not respecting solution in the vicinity of boundary. In the third column III) variant is shown. Here the non-reflective condition for velocity gives satisfying results. Note that in all three aforementioned cases homogeneous Neumann condition (or its equivalent) for  $\alpha$  was prescribed, though it is incorrect due to the fact that transport equation for  $\alpha$  is 1st order PDE. The last, IV) variant of boundary conditions led to solver failure. That might have been caused by segregated approach employed in OpenFOAM for solving equations (1) and (3).

Further, the III) variant of boundary conditions was tested at the outlet of 3D case of a weir with more complex geometry.

## Acknowledgement

This work was supported by the Grant Agency of the Czech Technical University in Prague, grant No. SGS16/206/OHK2/3T/12.

## References

- [1] Fürst, J., Musil, J., Development of non-reflective boundary condition for free-surface flows, Proceedings of the conference Computational Mechanics, Špičák, University of West Bohemia, 2017.
- [2] Musil, J., Non-reflective boundary conditions for free-surface flows, Master thesis, Czech Technical University in Prague, Prague, 2018.
- [3] Weller, H.G., Tabor, G., Jasak, H., Fureby, C., A tensorial approach to computational continuum mechanics using object-oriented techniques, Computers in Physics 12 (6) (1998), doi: 10.1063/1.168744.
- [4] Yeoh, G.H., Jiyuan, T., Computational techniques for multiphase flows, Elsevier, 2009, pp. 215-232.