

## Flutter boundary assessment for a blade cascade using developed discontinuous Galerkin code

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This contribution builds on the last year contribution [5], where the flutter boundary was assessed for a single aerofoil with the aid of the fluid-structure interaction (FSI) code developed for the purpose of aeroelastic flutter analysis as a part of FlowPro. FlowPro is a multipurpose numerical software developed by the authors of this contribution. Flutter boundary established in [5] was compared with the results of Kirshmann [4] and Hall [2]. Since the results agreed reasonably well to say the least, the FSI code is now considered as validated. The aim of this contribution is to establish the flutter boundary for a blade cascade.

The developed algorithm for fluid flow modelling is based on the discretisation of the system of Navier-Stokes equations by the discontinuous Galerkin method with the Lax-Friedrichs numerical flux. Furthermore, the time integration is performed by the second order backward difference formula (BDF2), which is an unconditionally stable implicit method. In its general form, it also allows us to adaptively change the size of time steps.

Let us move on to the flutter boundary prediction. The flutter boundary is a curve in  $\mathbb{R}^2$  which determines the threshold for the occurrence of the instability of type flutter depending on two variables, namely the far-field Mach number  $M$  and speed index  $U_F$ . Here, we study the flutter boundary for a blade cascade consisting of four flat plates with periodic boundary condition as show in Fig. 2. For each blade, we considering the wing model of Isogai [3] shown in Fig. 1. The dynamics of each blade is determined by the following system of two differential equations

$$\begin{aligned} m\ddot{y} + S_\alpha\ddot{\alpha} + K_y y &= L, \\ S_\alpha\dot{y} + I_\alpha\ddot{\alpha} + K_\alpha\alpha &= M_A, \end{aligned}$$

where

$$\begin{aligned} m &= \mu\pi\rho_\infty b^2, & S_\alpha &= mbx_a, \\ I_\alpha &= mr_a^2, & K_y &= m\omega_y^2, \\ K_\alpha &= I_\alpha\omega_\alpha^2, & \omega_\alpha &= U_\infty/U_F b\sqrt{\mu}. \end{aligned}$$

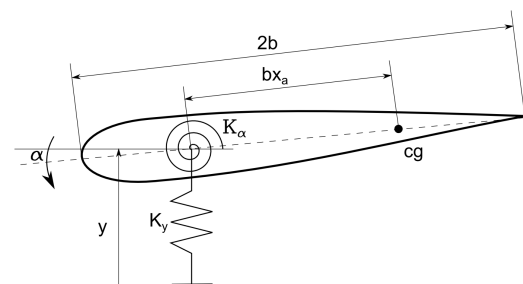


Fig. 1. Isogai wing model

On the right-hand side of the equations of motion we have the lift  $L$  and torque  $M_A$  acting on the aerofoil due to the aerodynamic forces. The structural parameters are  $x_a = 1.8$ ,  $r_a^2 = 3.48$ ,  $\mu = 60$  and the heat capacity ratio is  $\kappa = 1.4$ . Far-field velocity  $U_\infty$  and density  $\rho_\infty$  can be readily calculated using  $M$  and  $\kappa$ . In order to solve the system of second order ordinary differential equations, we first rewrite it as a system of first order ordinary differential equations

and consequently apply the two-step Adams-Bashforth scheme. We solve the FSI problem as a whole by solving the flow field and the rigid body dynamics in turns using the weak coupling. The fluid flow influences the rigid body through the lift  $L$  and torque  $M_A$ . Conversely, the rigid body influences the fluid flow by changing its own position given by the displacement  $y$  and angle  $\alpha$ . Hence, the mesh vertices must be recalculated at each time step, which we achieve by the blending function approach [1].

The flutter boundary is established using an iterative process in which aeroelastic simulations are performed for many combinations of values of the far-field Mach number  $M$  and speed index  $U_F$ . If the resulting motion grows in an unbounded fashion with time, the system is considered unstable and prone to aeroelastic flutter. If the disturbances are damped with time, the system is stable and flutter does not occur. If the system continues to oscillate with constant amplitude, the system is neutrally stable and the flutter boundary is established. The flutter boundary determined by the developed FSI software is plotted in Fig. 3. The green and red dots correspond to the combinations of  $U_F$  and  $M$  for which the system is stable and unstable, respectively. The flutter boundary drawn by the black line was found using bisection method.

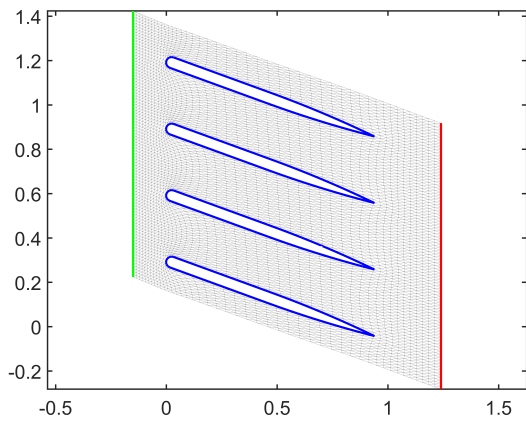


Fig. 2. Geometry and mesh

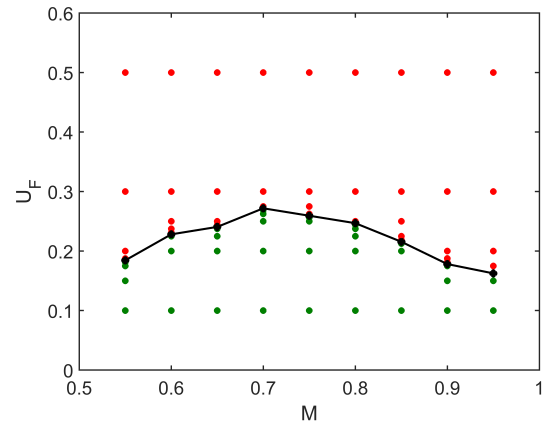


Fig. 3. Flutter boundary

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