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Combining adaptive mesh refinement with a parallel multilevel BDDC solver

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Adaptive mesh refinement and domain decomposition. Adaptive mesh refinement is an important part of solving problems with complicated solutions or when a prescribed accuracy needs to be achieved. In this approach, solution is found on a given mesh and its local error is estimated. Regions where the estimated error is high are then refined to improve the accuracy, and the solution is recomputed. This strategy leads to accumulation of degrees of freedom to regions with abrupt changes in the solution, such as boundary or internal layers.

The growing size of the problems, especially in 3D, more and more often requires solving these on a parallel computer. To this end, partitioning of the computational mesh into subdomains is required. If the partitioning is not adjusted to the new meshes in the adaptive process, large imbalances in subdomain sizes quickly emerge, and utilization of the parallel computer becomes inefficient.

A viable way to maintain the subdomain sizes balanced is repartitioning using the space-filling curves, which maintains approximately equal number of elements in each subdomain. Such approach is offered by the *p4est* library [1]. However, this repartitioning strategy typically produces subdomains composed of several disconnected components.

Another important ingredient of simulations based on FEM is the solver for the arising system of linear equations. A good match for a parallel computation is using a domain decomposition method, and a recent member of this family is the multilevel Balancing Domain Decomposition based on Constraints (BDDC) [2, 4]. This method is implemented in our parallel solver *BDDCML*[1] [5].

We have developed a custom implementation of the FEM to study the impact of the special structure of the subdomains created by the *p4est* library on the BDDC solver. Disconnected components of subdomains are detected using subdomain mesh graphs while hanging nodes are incorporated naturally into the computation by the non-overlapping domain decomposition.

Numerical results and discussion. The solver has been applied to a number of benchmark Poisson and linear elasticity problems. The problem geometry was a unit cube in all our experiments. These experiments were performed using up to about 1 billion unknowns and 4 096 CPUs of the *Salomon* supercomputer at the IT4Innovations supercomputing centre.

In the first set of experiments, we have generated a mesh refined in several predefined regions. The behaviour of the solver was compared to uniformly refined meshes resulting in approximately half of the iterations, yet comparable in time to solution.

[1] `http://users.math.cas.cz/~sistek/software/bddcml.html`

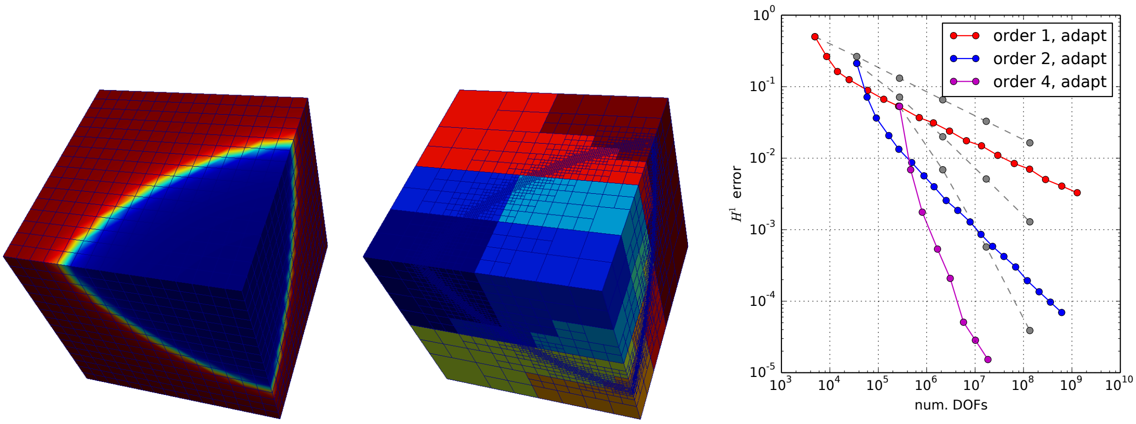

Fig. 1. A benchmark for a parallel adaptive computation: visualization of the exact solution (*left*), adaptively refined mesh partitioned into subdomains (*middle*), and convergence of the H^1 norm of the error on 2048 subdomains for different polynomial orders (*right*), adaptive (*colour lines*) vs. uniform (*grey lines*) mesh refinements.

The next experiment was running a benchmark adaptive computation, see Fig. 1. This test was performed for the Poisson problem using linear, quadratic, and fourth-order finite elements. The solver has allowed us to verify the convergence rate for problems refined up to 1 billion unknowns using 2 048 subdomains (and CPU cores). More details can be found in our paper [3].

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References

[1] Burstedde, C., Wilcox, L., Ghattas, O., p4est: Scalable algorithms for parallel adaptive mesh refinement on forests of octrees, SIAM Journal on Scientific Computing 33 (3) (2011) 1103-1133.

[2] Dohrmann, C.R., A preconditioner for substructuring based on constrained energy minimization, SIAM Journal on Scientific Computing 25 (1) (2003) 246-258.

[3] Kůs, P., Šístek, J., Coupling parallel adaptive mesh refinement with a nonoverlapping domain decomposition solver, Advances in Engineering Software 110 (2017) 34-54.

[4] Mandel, J., Sousedík, B., Dohrmann, C.R., Multispace and multilevel BDDC, Computing 83 (2-3) (2008) 55-85.

[5] Sousedík, B., Šístek, J., Mandel, J., Adaptive-Multilevel BDDC and its parallel implementation, Computing 95 (12) (2013) 1087-1119.