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Marek Klimko

Theoretical Study of Turbine Stage Characteristics in Off-design Conditions

Marek Klimko[1, a)]

[1]*University of West Bohemia, Department of Power System Engineering, Univerzitni 8, 306 14 Pilsen, Czech Republic*

[a)]*klimko@kke.zcu.cz*

Abstract. The article deals with the theoretical analysis of the basic axial turbine stage characteristics. In the introduction, the analytical derivation of the stage efficiency is analysed, and the influence of the change of reaction, the velocity ratio u/c_f and the absolute velocity exit angle from the stator blades are monitored. The next part consists of a 1D calculation in the middle blade radius with VT-400 (air turbine) input parameters, which is part of the experimental research at the Department of Power System Engineering. The calculation was carried out for 3 blading variants (low, medium and full reaction). Modes outside the turbine optimum operating point were induced by changing the velocity ratio u/c_f, especially by changing the absolute isentropic velocity c_f at constant circumferential velocity u. This action results in a change of the relative velocity vector flowing at the rotor blade. This leads to a redistribution of velocity triangles and causes additional losses (incidence losses).

INTRODUCTION

The thermodynamic blade efficiency of the turbine stage is characterized by the ratio of the mechanical work of 1 kg of the working substance to the energy supplied to the stage. In this context, it is important to evaluate whether the output absolute velocity from the bucket c_2 will be further utilized in the next step or the last (or one) stage in which the kinetic energy of the output absolute velocity is no longer applied and is therefore considered to be a loss. We then distinguish the efficiency of the stage defined on the basis of the total state T-T (total-to-total) and T-S (total-to-static), which is determined from the total input and static output state.

$$\eta_{T-T}^{ST} = \frac{H^{ST}}{H_{iz}^{ST} + \frac{c_0^2}{2} - \frac{c_2^2}{2}} \tag{1}$$

$$\eta_{T-S}^{ST} = \frac{H^{ST}}{H_{iz}^{ST} + \frac{c_0^2}{2}} \tag{2}$$

These relationships for calculating the efficiencies are evident from the following *h-s* flow diagram of the working substance expansion in the turbine stage.

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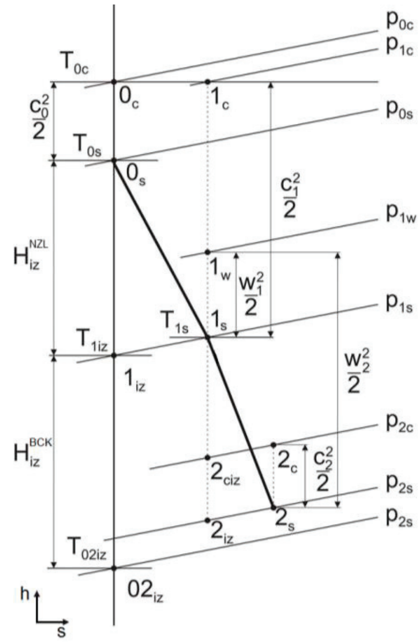

FIGURE 1. *h-s* diagram of turbine stage expansion

THEORETICAL ANALYSIS OF THE TURBINE STAGE PERFORMANCE

In turbine practice, the dimensionless parameter u/c_f is very often used. This is the ratio of the circumferential velocity u_s at the centre profile of the rotor blade to the isotropic (fictional) expansion speed c_f.

$$c_f = \sqrt{2 \cdot \left(H_{iz}^{ST} + \frac{c_0^2}{2}\right)} = \sqrt{2 \cdot H_0} \tag{3}$$

For further analysis, it is important to define the method of marking and orientation of the input and output angles of the individual components of the velocity vectors (Figure 2).

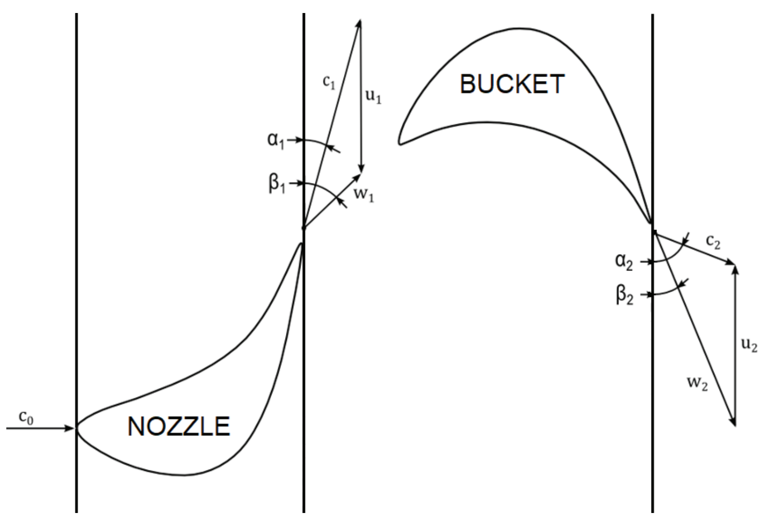

FIGURE 2. Velocity triangles

Circumferential efficiency of the stage is dependent on a number of parameters. In the next section, we try to analytically analyse the efficiency relation, which will be the function of the velocity ratio u/c_f, the reaction ρ, the absolute velocity exit angle from the nozzle α_1, the output angle of the relative velocity from the stage β_2 and the loss coefficients in the nozzle φ and the bucket ψ.

$$\eta_u = f(\varphi, \psi, \rho, \alpha_1, \beta_2, u/c_f) \tag{4}$$

We will consider a one turbine stage, so we start from the definition of T-S (2). The useful enthalpy gradient of the H^{ST} grade can be rewritten using Euler's turbine theorem as a product of the circumferential velocity and the difference of the circumferential (tangential) components of the absolute velocity at the input and output of the rotor blade row. Then we get the equation (5).

$$\eta_u = \frac{u \cdot (c_{1u} - c_{2u})}{\frac{c_f^2}{2}} \tag{5}$$

From the velocity triangles (Fig. 2) it is possible to define the necessary velocity components and by successive modifications we get the final expression for the circumferential efficiency of the stage (7).

$$c_{1u} = c_1 \cdot \cos\alpha_1 \,, c_{2u} = w_{2u} + u \,, w_{2u} = w_2 \cdot \cos\beta_2$$

$$\varphi = \frac{c_1}{c_f} \;\rightarrow\; c_1 = \varphi \cdot c_f \,, \psi = \frac{w_2}{w_{2iz}} \;\rightarrow\; w_2 = \psi \cdot w_{2iz}$$

$$\eta_u = \frac{u \cdot [(c_1 \cdot \cos\alpha_1) - (w_2 \cdot \cos\beta_2 + u)]}{\frac{c_f^2}{2}}$$

$$c_1 = \sqrt{2 \cdot (h_0 - h_1) + c_0^2} = \varphi \cdot \sqrt{2H_{iz}^{NZL} + c_0^2}$$

$$H_{iz}^{NZL} = f(\rho) \;\rightarrow\; H_{iz}^{NZL} = H_{iz}^{ST} \cdot (1-\rho)$$

$$c_1 = \varphi \cdot \sqrt{2H_{iz}^{ST} \cdot (1-\rho) + c_0^2} = \varphi \cdot \sqrt{1-\rho} \cdot c_f$$

$$w_2 = \psi \cdot \sqrt{2\rho \cdot H_{iz}^{ST} + w_1^2} = \psi \cdot \sqrt{\rho \cdot c_f^2 + w_1^2}$$

$$\eta_u = \frac{2u \cdot \left\{\varphi \cdot \sqrt{1-\rho} \cdot c_f \cdot \cos\alpha_1 - \psi \cdot \sqrt{\rho \cdot c_f^2 + \left[\varphi^2 \cdot (1-\rho) \cdot c_f^2 + u^2 - \right.}\right.}{c_f^2}$$
$$\frac{\left.\left. -2u \cdot \varphi \cdot \cos\alpha_1 \cdot \sqrt{1-\rho} \cdot c_f\right] \cdot \cos\beta_2 - u\right\}}{c_f^2} \tag{6}$$

Expression (6) is modified to include parameter $u/c_f = x$.

$$\eta_u = 2x \cdot \left[\psi \cdot \cos\beta_2 \cdot \sqrt{\rho - \varphi^2 \cdot (\rho - 1) + x^2 - 2\varphi \cdot x \cdot \cos\alpha_1 \cdot \sqrt{1-\rho}} - x + \right.$$
$$\left. + \varphi \cdot \cos\alpha_1 \cdot \sqrt{1-\rho}\right] \tag{7}$$

Relation (7) is plotted in the graph (Fig. 3), which shows the curves of the stage efficiency in relation to u/c_f for the different values of the reaction with neglected losses ($\varphi = \psi = 0$). The extreme of this parabolic function corresponds to the maximum efficiency value and the corresponding optimal velocity ratio u/c_f. These values can be easily determined by calculating the local extreme function, i.e. by determining the partial efficiency derivative according to u/c_f, which is set to zero. For equal impulse blading ($\rho = 0$), relation (8) is valid, for the reaction blading ($\rho = 0.5$) relation (9) applies.

$$\left(u/c_f\right)_{opt} = \frac{\varphi \cdot \cos \alpha_1}{2} \tag{8}$$

$$\left(u/c_f\right)_{opt} = \frac{\varphi \cdot \cos \alpha_1}{2 \cdot \sqrt{0,5}} \tag{9}$$

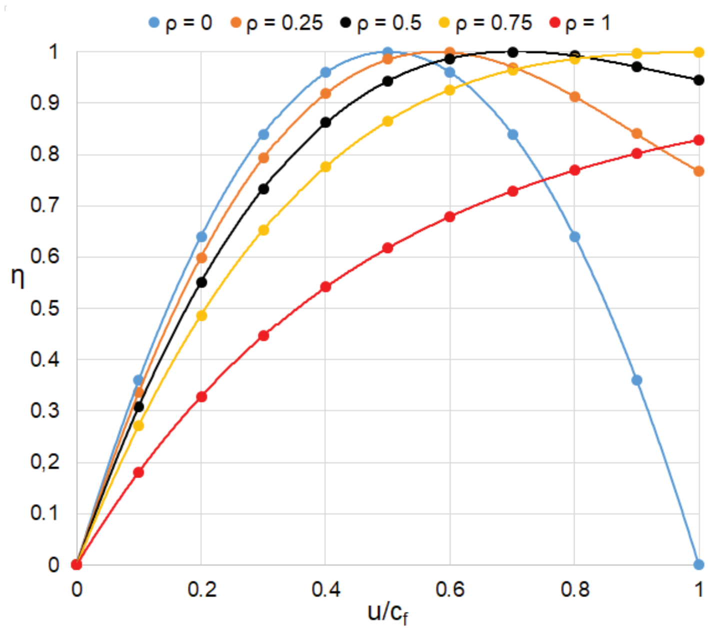

FIGURE 3. Dependency of turbine stage efficiency in relation to u/c_f for different reaction values

Relationship (7) serves as a theoretical outline of the problem, but it does not correspond to the actual flow ratios in the stage. If we change the u/c_f ratio so that c_f is constant, and if we only increase the circumferential velocity, the velocity triangles will be redistributed. The flow will load the rotor blade at a different angle (β_1), thereby changing the losses in the rotor blade row. This fact does not take into account relationship (7). Also, losses in the blade rows, especially the bucket, are not constant throughout the range u/c_f. The curves will, in fact, be steeper than for the theoretical relationship.

In relation (7), parameter α_1 represents the absolute velocity exit angle from the nozzle. This angle is based on the designed geometry of the particular blade profile. It is interesting to observe how the $\eta = f(u/c_f)$ curve deforms as it changes. On (Figure 4) the stage characteristic with 50% of the reaction for different α_1 values is indicated.

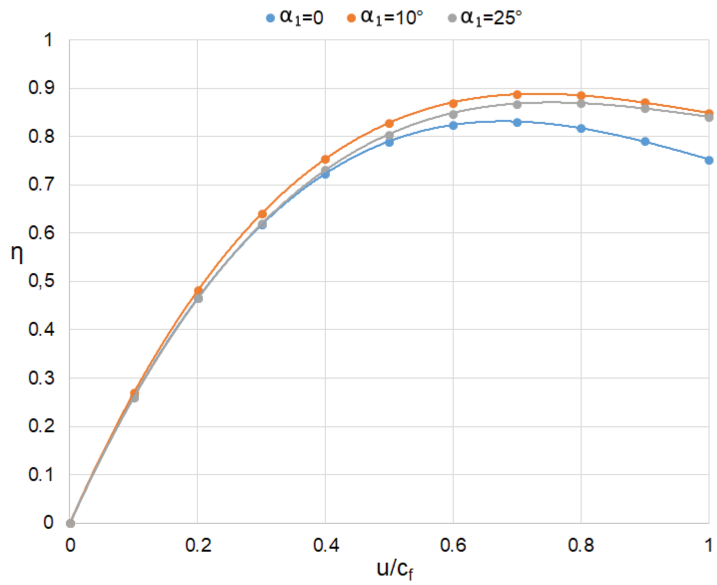

FIGURE 4. Dependency of turbine stage efficiency in relation to u/c_f for different angles α_1

Fig. 4 shows that the angle α_1 should be in the range of 10° to 20°. For higher α_1 values, the efficiency will decrease.

1D CALCULATION OF TURBINE STAGE

The aerodynamic design of the flow part of the turbine is usually performed for the selected nominal operation, which shows the lowest loss. This means that the leading edges of the turbine blades are designed in such a way that they correspond to the direction of the flowing medium. Most turbines operate in real time outside the optimum mode.

The thermodynamic process in the stage during load variation is subject to significant changes, such as changes in flow velocity, reaction, mass flow, stage efficiency, etc. For stationary turbines operating at constant speeds, circumferential speeds remain constant even during changes to the load. For turbines used in the propulsion of boats, pumps and turbochargers, the speed changes, and of course also the circumferential speeds. In this paper we deal with the first variant, the case of a stationary turbine with a constant speed. Load change is only triggered by changing the pressure drop on the stage, resulting in an increase in the absolute exit velocity vector c_1 from the nozzle. This causes the relative velocity vector flowing on the rotor blade to deviate, which logically translates into the velocity triangles at the output of the stage.

The 1D calculation was performed for the input geometric parameters and initial conditions of our experimental *VT-400* air turbine. The calculation was carried out for 3 blade variants (low - LR, medium - MR and full reaction - FR). The input parameters are listed in the following table (Table 1).

TABLE 1. Input parameters

NOZZLE			BUCKET		
Root diameter	D_P^{NZL}	400	Root diameter	D_P^{BCK}	400
Blade length	L^{NZL}	45.5	Blade length	L^{BCK}	47
Output absolute velocity angle	α_1	14.5°	Bucket efficiency	η^{BCK}	85%
Nozzle efficiency	η^{NZL}	93%			

Output relative velocity angle (bucket)			Boundary conditions		
Low reaction (LR)	β_2^{LR}	22°	RPM	n	2300 min^{-1}
Medium reaction (MR)	β_2^{MR}	19°	Ambient pressure	p_{0s}	98 000 Pa
Full reaction (FR)	β_2^{FR}	15°	Ambient temperature	t_{0s}	25 °C

The calculation was performed on a median radius. The efficiency of the stage, the entry angle of the relative velocity on the bucket, the absolute exit velocity angle from the stage and the stage reaction in relation to u/c_f in the range of 0.45 to 0.75 were monitored. The individual dependencies are evident from Fig. 5 (a), (b), (c), and a comparison of efficiencies is shown in Fig. 6.

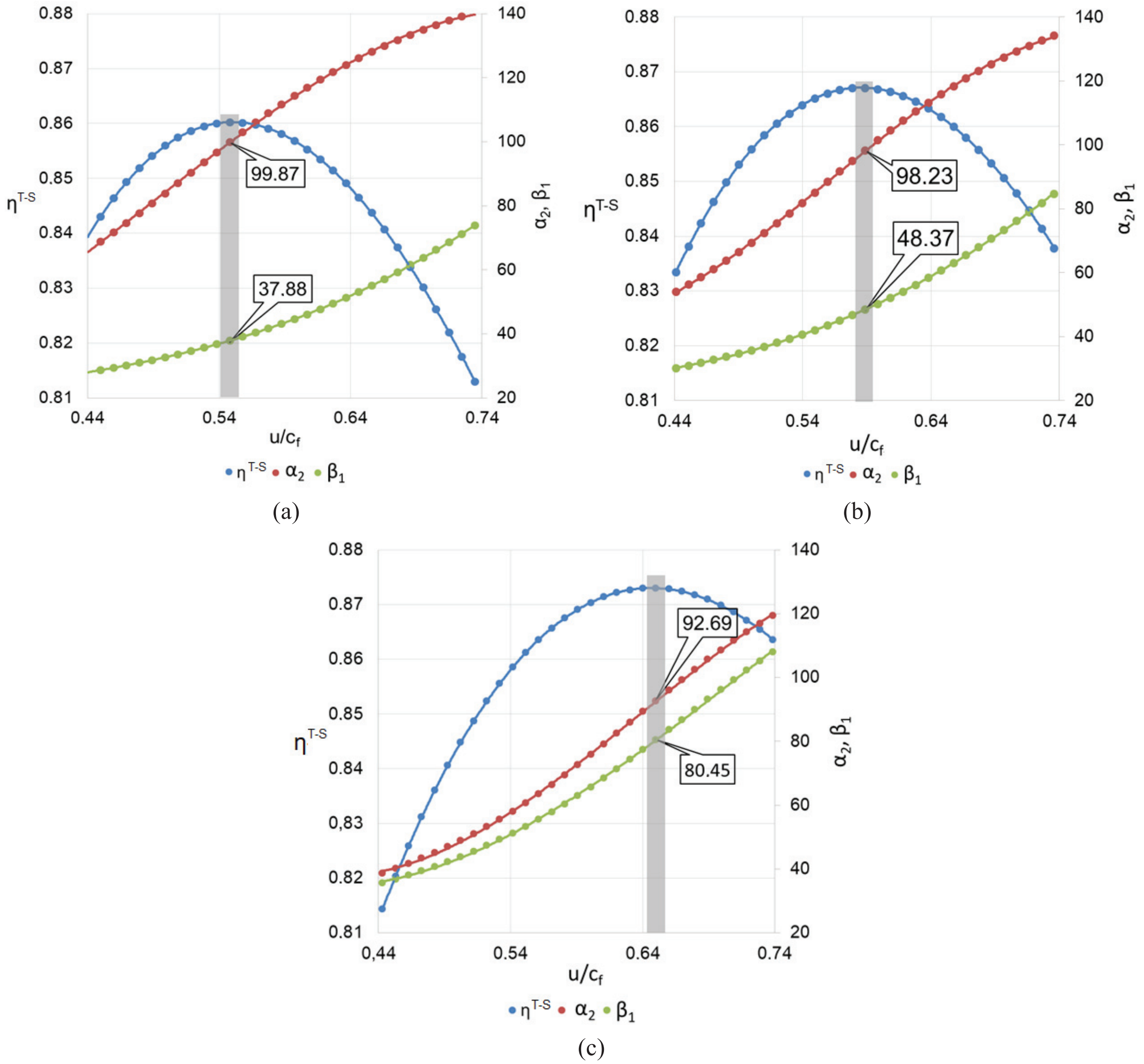

FIGURE 5. Dependencies of turbine stage efficiency in relation to u/c_f : (a) – LR; (b) – MR; (c) - FR

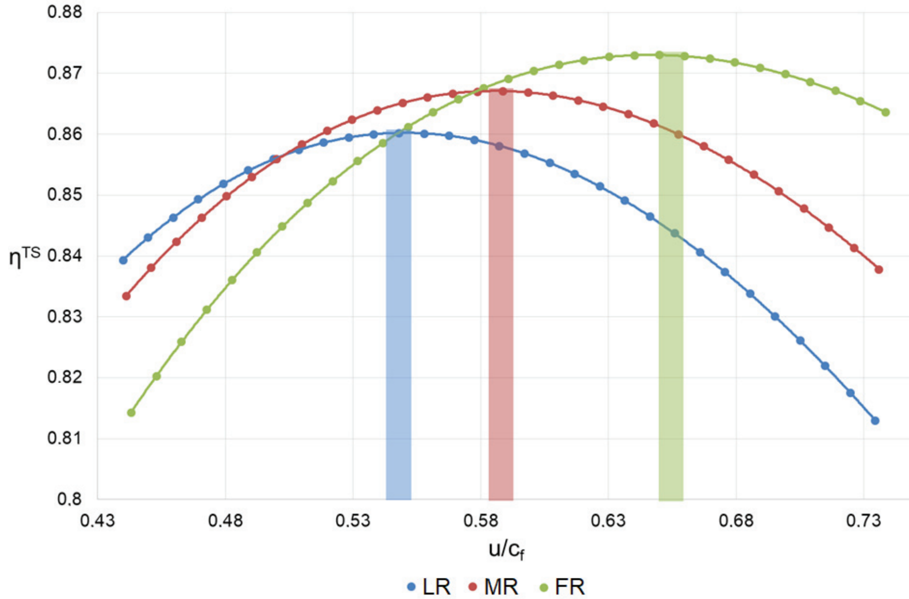

FIGURE 6. Comparison of turbine stage efficiency dependencies in relation to u/c_f

The graphs (Fig. 5, Fig. 6) show the optimum modes of the stage for maximum efficiency. For clarity, these points are summarized in the following table (Table 2).

TABLE 2. Optimal stage modes

	LR	MR	FR
ρ^{SS}	24 %	34 %	48 %
η^{TS}	86 %	86.7 %	87.3 %
$(u/c_f)_{opt}$	0.55	0.59	0.65
α_{2opt}	99.9°	98.2°	92.7°
β_{1opt}	37.9°	48.4°	80.5°

The course of the reaction is clear in Fig. 7.

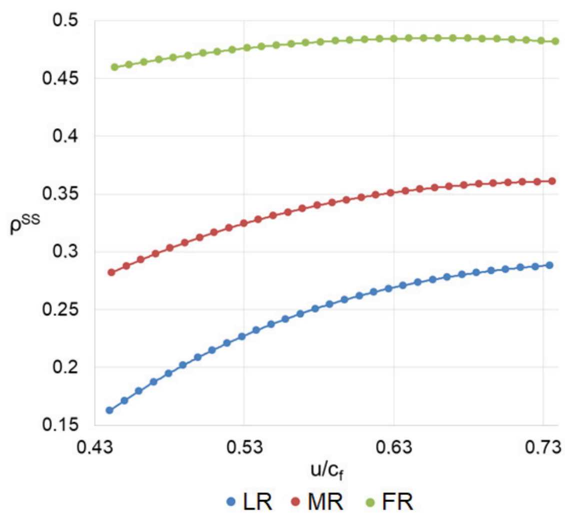

FIGURE 7. Comparison of turbine stage reaction in relation to u/c_f

Part of the calculation is the drawing of the velocity triangles (Figure 8). For each blade version, a lightweight, overloaded and optimal mode was selected. For each variant the rounded value of the velocity ratio u/c_f, the angle of the absolute output velocity from the stage α_2 and the angle of the input relative velocity to the bucket β_1 are shown.

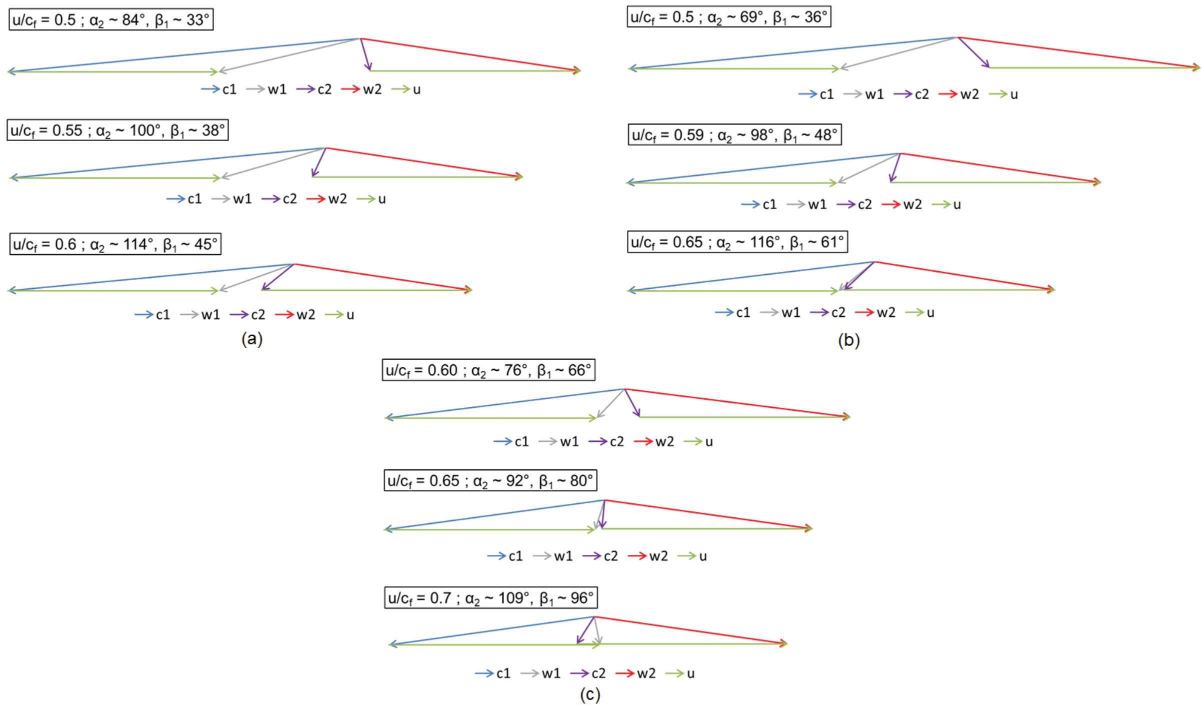

FIGURE 8. Velocity triangles: (a) – LR; (b) – MR; (c) - FR

A blade designer is particularly interested in the course of the loss coefficient ζ depending on the angle of attack at the entrance to the blade row. Attempts are made to find a blade row that exhibits a low value of the loss coefficient over a wide range of angles of attack. The loss coefficient is usually plotted in relation to the angle of incidence i, which is the difference between the optimal (designed) angle β_{1D} (β_{1opt}) and the angle of the flow entering the blade row β_{1f}. The incidence orientation is illustrated in Fig. 9.

$$i = \beta_{1D} - \beta_{1f} \tag{10}$$

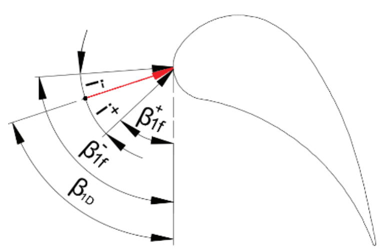

FIGURE 9. Incidence orientation

Stepanov's hypothesis is based on the assumption that losses at the off-design angle of attack are proportional to the square of the vector difference between the reference (design) and non-reference input velocity (11).

$$\zeta^S = 0.058 \cdot \left[\left(\frac{\sin \beta_2}{\sin \beta_{1f}} \right)^2 - \left(\frac{\sin \beta_2}{\sin \beta_{1D}} \right)^2 \right] + 0.265 \cdot \left[\frac{\sin i \cdot \sin \beta_2}{\sin \beta_{1D} \cdot \sin \beta_{1f}} \right]^2 \tag{11}$$

The distribution of losses for the blade variants is shown in Fig. 10. It can be seen that the optimal angle of attack on the rotor blade is found to be several degrees in the negative incidence area. It can also be seen that the low reaction stages resists off-design modes the least. Increasing the degree of reaction extends the range of the angles of attack around the optimum mode, with the most advantageous option being the full-reaction (FR) option.

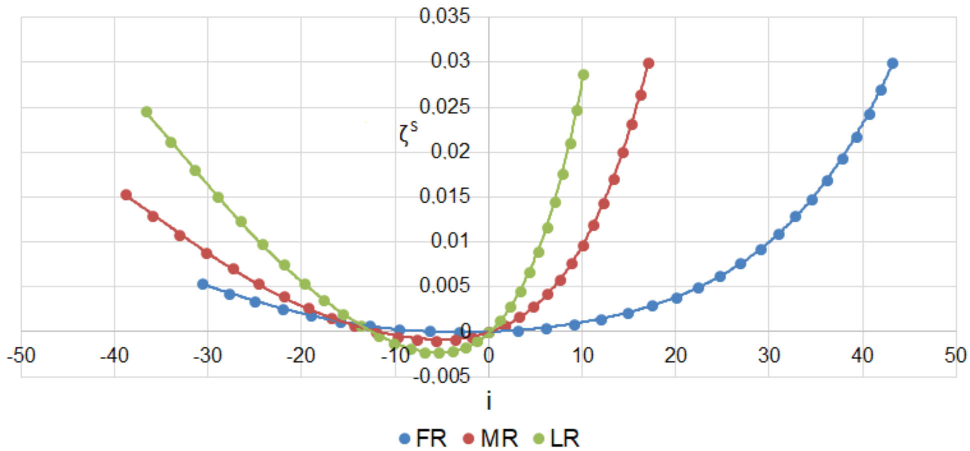

FIGURE 10. Loss distribution by Stepanov´s loss model

CONCLUSION

This contribution describes the analysis of the efficiency of a turbine stage in terms of fundamental theory followed by the 1D calculation of three blade variants (stages with low - LR, medium - MR, and full reaction - FR). The effect of the change of the absolute isentropic velocity c_f at constant speed was monitored. The result is the dependencies of inputs and outputs, i.e. the output angles. In terms of modes outside optimal operation, it is interesting to observe the changes of the input angle of the relative velocity on the rotor blades. An incorrect incidence angle contributes to increased losses (incidence losses) and may lead to the flow separation from the profile or to the formation of separation bubbles, swirl flow, etc. The analysis of the influence of incidence on the profile losses revealed a significant influence on the reaction size. The full reaction variant fits within the wider range of incidence. Improvements in performance outside the optimal operation for the lower reaction variant could be achieved by, for example, turning the stator blades, as this may favourably affect the flow rotor blade row within a wider range of u/c_f. From a design point of view, however, this alternative is quite demanding and is used more in gas turbines where such regulation is used as an effective means of controlling the working regimes of individual rotors of multi-rotor turbo-compressor engines.

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