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The Rough Stochastic Volatility Model – Calibration to Market Data and Robustness Analysis

Jan Matas[1]

1 Introduction

We study an extension of the Black-Scholes model (Black and Scholes, 1973) that instead of assuming constant volatility, considers volatility being modeled as a stochastic process driven by the fractional Brownian motion (fBm) with Hurst parameter $H < 1/2$. It is thus called the rough fractional stochastic volatility (RFSV) model and it was first proposed by Gatheral et al. (2018) by showing that the volatility is rough and that the process (1) is a suitable model.

Mathematically, the RFSV model can be expressed in the following form:

$$\begin{aligned} \mathrm{d}S_t &= rS_t\mathrm{d}t + \sigma_t S_t \mathrm{d}W_t, \quad t \geq 0, \\ \sigma_t &= \sigma_0 \exp\left\{\xi B_t^H\right\}, \\ \mathrm{d}Z_t &= \rho \mathrm{d}W_t + \sqrt{1-\rho^2}\mathrm{d}\widetilde{W}_t, \end{aligned} \tag{1}$$

where $(B_t^H, t \geq 0)$ is the fBm with Hurst parameter $H \in (0, 0.5)$ that is represented by some of its integral representation featuring the Bm $\widetilde{W}_t$, parameter $\rho \in [-1, 1]$ is the correlation between W_t and $\widetilde{W}_t$, and $\sigma_0, \xi > 0$ are parameters.

The solution of the RFSV model is not known in any semi-analytical form because the standard stochastic calculus cannot be used since the fBm is involved. Hence, we have to use Monte Carlo simulations for pricing a derivative under the model. For simulation of the fBm, we use the Hybrid scheme (Bennedsen et al., 2017) with implemented "turbocharging" method for variance reduction (McCrickerd and Pakkanen, 2018).

AARE [%]	April 1	April 15	May 1	May 15
Heston model	5.15	3.79	6.58	3.39
Bates model	3.73	3.57	5.77	3.41
AFSVJD model	2.21	2.16	5.89	3.20
RFSV model	6.65	7.36	7.73	6.26

Table 1: The results of the overall calibration to market data.

2 Results

We calibrated the RFSV model to a real market dataset consisting of call option prices of the Apple Inc. stock from 2015. We then compared the fit with other stochastic volatility models using the average absolute relative error ($AARE$). The results are presented in Table 1, where we can see that the RFSV model did not outperform any of the other models. However,

[1] student of the master's degree program Mathematics for Business Studies, e-mail: janmatas@students.zcu.cz

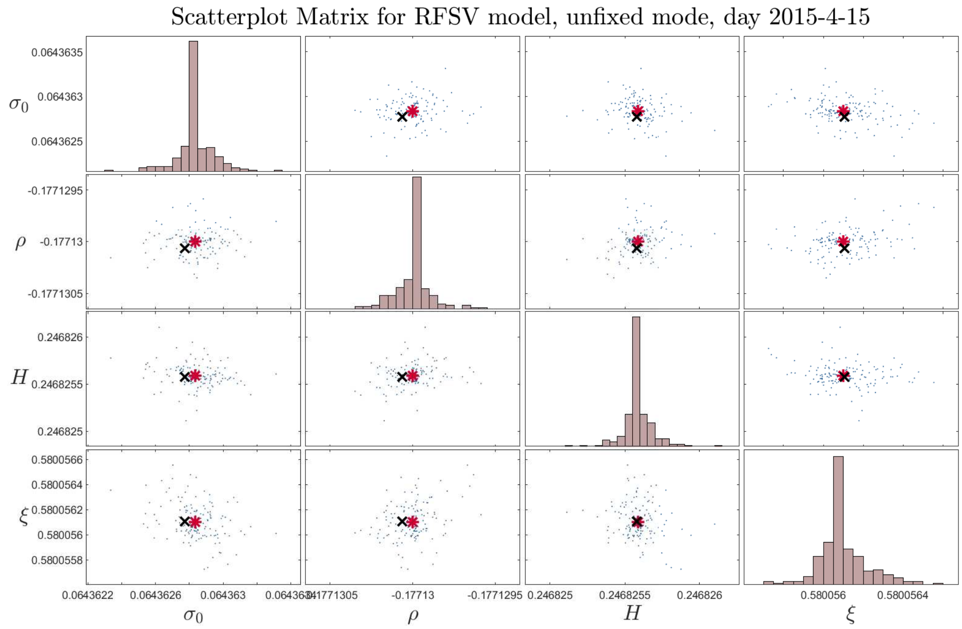

Figure 1: Scatterplot matrix for the results of the calibrations of the RFSV model to the bootstrapped samples for May, 1. Blue dots represent the coefficient estimates, the red star denotes their mean, and the black cross depicts the result of the overall calibration.

we examined the source of the errors and we realized that the errors occurred mainly for deep-out-of-the-money options, whose price is near zero. Therefore, the RFSV model still provides a decent fit, despite the higher $AARE$.

Then, we analyzed robustness, i.e., the property of a model that conveys the sensitivity to the uncertainty in the option structure when it is calibrated. We ran calibrations to 200 bootstrapped samples and we examined the resulting variation in the estimated prices and variation in the estimated coefficients. In both cases, the variation appeared to be remarkably small. In Figure 1, we can see the coefficient estimates, which are represented as blue dots, for one day. Both the narrow ranges and the fact that the mean of the estimates (red star) is very near the result of the overall calibration (black cross) suggest that the RFSV model, in comparison with the other models, is very robust.

References

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